Consequences of the Large Real-To-Imaginary Ratio at $\sqrt{s}=546$ GeV

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The remarkably large value of $\rho = \mathrm{Re}f/\mathrm{Im}f$ at $\sqrt{s}=546$ GeV found recently in an experiment at the CERN $Sp\bar{p}S$ collider is shown to have dramatic consequences.

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The first measurement of the ratio ρ of the real part to the imaginary part of the forward $p\bar{p}$ amplitude at CERN $Sp\bar{p}S$ energies ($\sqrt{s}=546$ GeV) was announced at the recent European Physical Society High-Energy Conference at Uppsala.[1] The value

$$\rho = 0.240 \pm 0.024$$

is twice as big as expected in all standard analyses of asymptotic behavior.

The change in ρ between CERN ISR (Intersecting Storage Ring) energies ($\sqrt{s}=53$ GeV) and $Sp\bar{p}S$ energies is even more dramatic when translated into the behavior of the real part $r_{p\bar{p}}$ itself, as shown in the following table[2]:

$\sqrt{s}$ (GeV)	$\sigma_{p\bar{p}}$ (mb)	$r_{p\bar{p}}$ (mb)
53	43.65 ± 0.41	4.41 ± 0.79
546	60 ± 2	14.40 ± 1.44

Thus while $\sigma_{p\bar{p}}$ changes by a factor of 1.36, $r_{p\bar{p}}$ grows by a factor of 3.27.

I analyze below, in a very simple fashion, the consequences of this rapid growth. In the following, all amplitudes are forward, spin-averaged amplitudes and are normalized so that their imaginary parts are given directly by the total cross sections, i.e., I take

$$f_{pp} = r_{pp} + i\sigma_{pp}, \qquad f_{p\bar{p}} = r_{p\bar{p}} + i\sigma_{p\bar{p}}. \tag{1}$$

For amplitudes normalized in this way the Froissart bound limits asymptotic growth to

$$|f| \lesssim C \ln^2 s, \tag{2}$$

(both imaginary *and* real parts of f are permitted to grow like $\ln^2 s$) and the natural variable to discuss high-energy behavior turns out to be

$$\eta \equiv (2/\pi)\ln(s/s_0), \tag{3}$$

where s_0 is a scale factor taken equal to 1 $(\mathrm{GeV})^2$.

We are interested in the rate of growth of $r_{p\bar{p}}$ with η. A trivial modification of known results[3,4] allows us to write

$$\frac{d}{d\eta} r_{p\bar{p}} = \sigma_- + \frac{d^2}{d\eta^2}(\sigma_+ - \tfrac{1}{3}\sigma_-) + \tfrac{1}{3}\frac{d^4}{d\eta^4}(\sigma_+ - \tfrac{1}{15}\sigma_-) \ldots, \tag{4}$$

where

$$\sigma_+ \equiv \tfrac{1}{2}(\sigma_{pp} + \sigma_{p\bar{p}}), \qquad \sigma_- \equiv \tfrac{1}{2}(\sigma_{pp} - \sigma_{p\bar{p}}). \tag{5}$$

Consider the various possibilities.

(i) In the standard picture, both σ_+ and σ_- are smooth, slowly varying, and monotonic at these high energies and $\sigma_- \approx 0$. Then we can neglect σ_- in (4), and also higher derivatives than $d^2/d\eta^2$, since the Froissart bound limits the growth of σ_+ to $\leq C\eta^2$. In this case one has $\sigma_+ = \sigma_{p\bar{p}}$ and with use of the measured cross section one fails by a factor of 2 to reproduce the growth of $r_{p\bar{p}}$. Thus the standard picture fails.

(ii) If we insist on having $\sigma_- \approx 0$, then we can generate a sufficiently large rate of change of $r_{p\bar{p}}$ in (4) by giving σ_+ a significant positive fourth derivative. Since σ_+ cannot grow like η^4 indefinitely, this must be a local effect, something like the opening up of a new channel—a phenomenon which, at these energies, would be extraordinary. In this option, the $p\bar{p}$ cross section at the Fermilab Tevatron ($\sqrt{s}=1.8$ TeV) should be considerably larger than expected from the usual extrapolation, perhaps $\sigma_{p\bar{p}} = 85$–95 mb.

(iii) If we are willing to contemplate $\sigma_- \neq 0$ and *positive* (recall that σ_- is negative up to ISR energies), then an adequate rate of change can be induced with slowly varying monotonic σ_+ and σ_-, and keeping only the second derivative term in (4). A rough calculation suggests that σ_{pp} and $\sigma_{p\bar{p}}$ cross soon after the ISR region and that $\sigma_{pp} - \sigma_{p\bar{p}} \approx 4$–$10$ mb at $\sqrt{s}=546$ GeV. At the Tevatron $\sigma_{p\bar{p}}$ should be about 75 mb, roughly as previously expected. The idea that $\sigma_- \not\to 0$ asymptotically is not new,[5] and has received some support from a comparison of pp and $p\bar{p}$ elastic scattering at ISR energies.[6] That σ_- might be *positive* runs somewhat counter to intuition, though it is perfectly permissible, and such a possibility was indeed studied some time ago.[4,7]

Whether it be (ii) or (iii) that nature chooses, we seem once again to have a result that suggests that the asymptotic behavior of amplitudes is far from dull. And once more we see the need to be able to compare pp and $p\bar{p}$ at very high energies.

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[1]P. Kluit, in Proceedings of the European Physical Society High Energy Physics Conference, Uppsala, Sweden, June 1987 (to be published).

[2]The value for $\sigma_{p\bar{p}}$ at $\sqrt{s}=546$ differs from the published value, since what is actually measured is $\sigma_{p\bar{p}}(1+\rho^2_{p\bar{p}})$.

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[4]K. Kang and B. Nicolescu, Phys. Rev. D **11**, 2461 (1975).

[5]L. Lukaszuk and B. Nicolescu, Nuovo Cimento Lett. **8**, 405 (1973).

[6]P. Gauron, E. Leader, and B. Nicolescu, Phys. Rev. Lett. **54**, 2656 (1985), and **55**, 639 (1985), and University of Paris Report No. IPNO/TH 86-55, 1986 (to be published); In this study the properties of the asymptotically growing crossing-odd amplitude (the odderon amplitude) were largely determined by the growth of the $p\bar{p}$ "shoulder" at $t \approx -1.4$ GeV/c^2 and it turned out that σ_- was negative. However, the forward data used played little role in the sign determination. It seems likely that the new result can be accommodated and a detailed analysis is under way.

[7]The "fit-9 with type-2 odderon" of M. M. Block and R. N. Cahn [Rev. Mod. Phys. **57**, 563 (1985)—see Fig. 17] is precisely of this type.