Charmless Decays of Bottom Mesons and a Fourth Generation

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B decays to charmless inclusive final states are discussed. The generic process is the loop-induced $b \to sg^*$, g^* being an on-shell or off-shell gluon. We include the cases when the gluon momentum is lightlike, timelike, and spacelike. The timelike case includes $g^* \to q\bar{q}$ and gg splitup. Our findings are as follows: (1) $B(b \to sg^*) \simeq 1\%$ for three generations; (2) an experimental observation of charmless branching ratio $\gtrsim 5\%$ would (conservatively) imply the possibility of a fourth generation; and (3) charmless branching ratio $> 20\%$ would be difficult to accommodate in the standard model even with four families.

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Loop-induced weak decays provide an important framework for testing the standard model (SM) and for seeking signals for new physics. Compared with the s quark, loop decays of the b quark are especially promising as (1) they are generally not as rare, (2) they are more readily amenable to perturbative QCD analysis, and (3) the b quark, being a member of the third family, is likely to be more sensitive to the presence of the fourth family. In this work we confine ourselves to QCD-induced loop decays often called "penguin graphs." These decays lead to charmless final states. Indeed, both[1,2] CLEO and ARGUS collaborations have given results for the measurements of B mesons decaying into inclusive charm final states. In principle, that result can be used in conjunction with theoretical expectations to deduce the charmless branching ratio (BR). Unfortunately, the current data do not yet have the necessary sensitivity to give useful information on the charmless BR. However, more data are expected with the upgrade of existing machines and with the coming of new machines (such as the Stanford Linear Collider, the Fermilab Tevatron, CERN ACOL, etc.) that can study B physics. Furthermore, with the advent of vertex detectors an accurate determination of the charmless BR (perhaps by vetoing against charm) may be feasible. As we show below, the standard-model prediction for the inclusive b-to-s weak decays is 1%–2%. While a measurement of this precision will be extremely difficult to make, should the actual branching ratio be substantially larger it could be measured and would be circumstantial evidence for the existence of one or more additional generations of quarks.

Penguin-type processes have been discussed in the literature before for b decays.[3-6] However, the treatments that exist are incomplete and indeed an important subset of graphs has never been included. The discussions in the literature are also somewhat fragmentary. We will therefore try to present a comprehensive treatment of the subject, except that in this paper we will confine ourselves to the discussion of inclusive decays. This is because we feel that the exclusive (say, two-body) modes[7] that result are suppressed as the B meson has many channels available to it. Furthermore, the calculation of exclusive modes contains additional sources of uncertainties. Besides, the aforementioned subset of graphs that is important for our inclusive discussion has never been, and would be much harder to be, incorporated in the exclusive calculations. To reiterate, we think that although the inclusive calculations that we report here have their own uncertainties, they are much less than in the corresponding exclusive calculations. Our results are that the inclusive branching ratios is of order 1% for three generations and if the Kobayashi-Maskawa (KM) mixing elements $V_{t's}$ and $V_{t'b}$ are not suppressed, it can become of order 10% or so if there is an additional family.

To calculate the total rate for an experimental signal consisting of $B \to K+X$ (no charm), we incorporate $b \to s+g^*$ where the gluon may be (1) lightlike,[6] i.e., on its "mass shell," (2) timelike and branches into a $q\bar{q}$ [3-6] pair (where $q=u$, d, and s) or into two on-shell gluons (gg), and (3) spacelike, and the process is analogous to the original penguin process of Shifman, Vainshtein, and Zakharov[8] (from now on we call this the "penguin"). In addition, for consistency we also discuss $b \to sgg$ where the gluons do not come from a single virtual timelike gluon as was the case in (2), but rather are each emitted from quark lines.

Let us first make a qualitative point regarding process (1), i.e., lightlike gluon with four-momentum (q) such that $q^2=0$ versus process (2) where $q^2>0$. The second process is of higher order in α_s and one may momentarily think that, as in most parton-model type of calculations, the first reaction represents adequately the process of hadronization. However, this usual philosophy fails here as $b\to sg$ with $q^2=0$ is a magnetic transition involving only the magnetic (i.e., F_2) form factor while the electric (F_1) form factor does not contribute. For the processes under consideration the contribution of the $O(\alpha_s^2)$ F_1^2 term far exceeds the $O(\alpha_s)$ F_2^2 term and overcompensates by a considerable amount the extra power of α_s. As a result we find that the $O(\alpha_s^2)$ process where the gluon is virtual and timelike and branches into quarks and gluons dominates the full $b\to s$ transition.

We now give a more detailed discussion. The loop-induced bsg coupling is (all external momenta ignored whenever possible)

$$g_s(2\sqrt{2}G_F/16\pi^2)v_i\bar{s}(\lambda^a/2)[F_1^i(q^2\gamma_u-q_u\mathbf{q})L+F_2^i i\sigma_{\mu\nu}q_\nu m_bR]b, \tag{1}$$

where g_s is the strong coupling constant, λ^a is the Gell-Mann matrix, $v_i\equiv V_{is}^*V_{ib}$, where $i=u,c,t,(t')$, and q_μ is the gluon momentum. The form factors can be extracted from the effective $bs\gamma$ coupling calculated in Inami and Lim[9]:

$$F_1^i=x_i(y_i+13y_i^2-6y_i^3)/12+[4y_i+(-4y_i^2-5y_i^3+3y_i^4)x_i](\ln x_i)/6,$$

$$F_2^i=-(-y_i+3y_i^2+6y_i^3)x_i/4-\tfrac{3}{2}x_i^2y_i^4\ln x_i,$$

where $x_i=m_i^2/M_W^2$, $y_i=(x_i-1)^{-1}$, and m_i is the internal quark mass. Note that to this order in g_s, for small x_i, $F_1^i\simeq-\frac{2}{3}\ln x_i$ and $F_2^i\simeq\frac{1}{2}x_i$ and is power suppressed.

There are three types of processes that follow from Eq. (1).

(1) $q^2=0$: The quark-parton level process is $b\to sg$, where the gluon is "on-shell" (a parton, and therefore fragments to real hadrons with probability unity), and only the magnetic form factor F_2 contributes. The rate is [for SU(N) color group]

$$\Gamma^{q^2=0}(b\to sg)=\frac{N^2-1}{N}\frac{3\alpha_s}{4\pi}|v_iF_2^i|^2\Gamma_0, \tag{2}$$

where $\Gamma_0=G_F^2m_b^5/192\pi^3$.

(2) $q^2>0$: When the gluon is timelike, the process should be $b\to sg^*$ followed by $g^*\to q\bar{q}$, gg, i.e., the virtual gluon decays into a $q\bar{q}$ pair *or* a pair of on-shell gluons, gg. These "partons" subsequently fragment into real hadrons. Note that in principle (if q^2 is hard) this parton fragmentation process is distinct from process (1). Furthermore, as alluded to earlier, the $\alpha_s^2F_1^2$ term far exceeds the $\alpha_sF_2^2$ term since the F_1 form factor is leading.

The two-gluon process $b\to sgg$ has not been included in previous discussions in the literature. However, not only is there the contribution coming from $g^*\to gg$, but also, because of (QCD) gauge invariance, the two (on-shell) gluons could each get emitted directly from quark lines. The latter process has two types of contributions, the one-particle irreducible (1PI) graphs where both the gluons are radiated from the internal quark line within the W loop, and the one-particle reducible (1PR) graphs where one essentially has an effective color magnetic (F_2) gluon emission followed (or preceded) by a gluon bremsstrahlung. The discussion of these processes is analogous to that of the $K^0\to s\bar{d}\to\gamma\gamma$ transition in the literature,[10] but more complicated.

The form factor associated with the 1PI term vanishes rapidly[10] with m_i for $m_i>m_b$ and thus is very insensitive to heavy internal quark (t,t') masses. The charm contribution is subleading compared with the F_1^2 term, and a direct calculation indicates that the 1PI contribution is orders of magnitude below that of Eq. (3) (comparing the two incoherently).

The 1PR terms are proportional to F_2 in amplitude, which may interfere with the F_1 term in the $g^*\to gg$ process. It turns out that with the inclusion of the 1PR process the F_1F_2 interference term for $b\to sgg$ cancels out, and one is left only with the interference term coming from $b\to sq\bar{q}$ (which comes only from $b\to sg^*$). We get the result

$$\Gamma(b\to sq\bar{q})+\Gamma(b\to sgg)=[(N^2-1)/2N](\alpha_s^2/16\pi^2)[(n_f+N)|v_iF_1^i|^2-4n_f(v_iF_1^i)(v_jF_2^j)]\Gamma_0. \tag{3}$$

The terms proportional to n_f are from incoherent summing over n_f quark flavors for $b\to sq\bar{q}$. Since we are concerned with charmless final states, we shall use $n_f=3$ (i.e., $u\bar{u}$, $d\bar{d}$, $s\bar{s}$[11]) in subsequent discussions. The term proportional to N comes from $b\to sgg$, which cannot be ignored, since the gluon color "charge" is bigger than that of the quarks. Indeed, for the SU(3) color group and three noncharm final-state quark flavors, the $b\to sq\bar{q}$ and $b\to sgg$ processes are equally important. The F_2^2 terms have been dropped[12] in Eq. (3).

(3) $q^2<0$: When the gluon is spacelike, it gets attached to the spectator quark and one has the usual penguin graph. The decay goes through a two-body process, but the width is suppressed by two powers of the B-meson decay constant $f_B\lesssim\frac{1}{20}m_b$. For comparison, we

use the leading-log, all-orders result of Ref. 5 (now in the spacelike region), and find the rate

$$\Gamma^{q^2<0}(\text{penguin}) = (f_B/m_B)^2[(N^2-1)^2/N^3]6\pi^2|v_i c_5^i|^2\Gamma_0, \quad (4)$$

where the Wilson coefficient c_5^i can be found in Ref. 5.

Having collected the formulas for the various processes, we now discuss the results for the SM with three and four generations of quarks. Clearly, when the partons fragment into hadrons, there should be considerable interference between processes (1)–(3), since they would share many common final states. Unfortunately, we do not have an adequate method at present to model this, which is why we shall take the incoherent sum over the processes $b \to sg$, $b \to sq\bar{q}$, $b \to sgg$, and $b \to s$ "penguin," viz., the rates presented in Eqs. (2)–(4), as the full result for hadronic charmless $b \to s$ transitions.

The three-generation result is rather insensitive[13] to the (already well constrained) KM mixing parameters and depends only very mildly on m_t. The total hadronic (charmless) $b \to s$ BR, as well as the incoherent individual contributions, are plotted in Fig. 1. It is clear that process (2) dominates, and the contributions of process (1) and (3) are typically more than an order of magnitude smaller. The smallness of the real (i.e., $q^2=0$) gluon bremsstrahlung contribution for small m_t is because F_2^i is suppressed for small m_i. Actually, to be consistent with the $O(\alpha_s^2)$ analysis that we are giving, one should include QCD loop corrections to process (1). This will bring in logarithms at the $O(\alpha_s^2)$ order, which is known to give rise to enhancements to the $b \to s\gamma$ process,[14] especially for small m_t when the $O(\alpha_s)$ term is suppressed. The overall effect would flatten out the real gluon curve,[15] while slightly enhancing it, making its appearance in Fig. 1 similar to the other two processes.

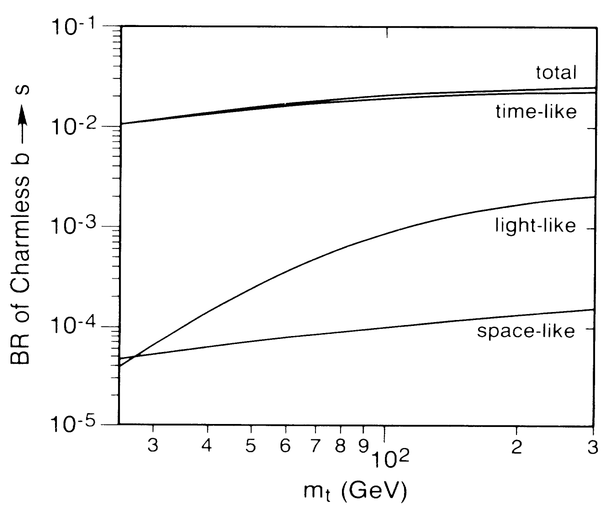

FIG. 1. BR for $b \to s$ in three-generation case.

The curves for the full rate given in Fig. 1, however, will not change significantly. We thus see that the total BR ranges from 1% to 2% for $m_t = 25$–200 GeV. (Note that, in comparison, the tree graph contribution to the charmless $b \to s$ transition occurring via $b \to uW$, $W \to us$ has a BR $\leq 0.2\%$ even if $|V_{ub}/V_{cb}| \simeq 0.20$.) We emphasize once again that the timelike gluon splitup dominates, and the $g^* \to gg$ contribution (which has never been included before) is as important as, if not more so than, the $g^* \to q\bar{q}$ contribution.

If there exists a fourth generation, then the KM matrix is less constrained, and in particular there would also be a virtual t' contribution. We follow Ref. 13, and note that $|v_u| \ll |v_c| \simeq 0.05$; hence, v_u is ignorable. Then taking v_c as known (it is important that it is also small), we find from $v_c + v_t + v_{t'} \approx 0$ that there is one single controlling KM factor, which we choose to be $v_{t'} = V_{t's}^* V_{t'b}$. We plot the BR's versus $v_{t'}$, for $-0.3 \leq v_{t'} \leq 0.3$, in Fig. 2. We choose two typical top quark masses $m_t = 50$ or 80 GeV and three t' masses, i.e., $m_{t'} = 150$, 200, and 300 GeV, and present only the total rate.

The total rate plotted in Fig. 2 shows that, for $v_{t'}$ of opposite sign with respect to v_c, the BR for QCD-induced $b \to s$ transitions may well reach above 10% if $v_{t'}$ is of the order of the Cabibbo angle or more. This is almost an order of magnitude larger than the corresponding three-generation result for $m_t = 50$ GeV. If the top-quark mass is heavier, say $m_t = 80$ GeV, the enhancement is less, not only since the three-generation result is bigger, but also because the four-generation result is smaller due to more effective cancellations between t and t' contributions.

We close with some brief remarks. We have plotted BR versus $v_{t'}$ in the four-generation case for $|v_{t'}| < 0.3$. To saturate this range would seem alarming from the point of view of the prevailing prejudice that off-diagonal elements such as $V_{t's}$ ought to be small. However, if one keeps an open mind, current experimental information together with four-generation unitarity does not rule out a large $V_{t's}$ ($|V_{t's}| \lesssim 0.7$ at 90% confidence level) while $V_{t'b}$ is effectively unconstrained.[16] Indeed, loop-induced rare B decays may be viewed[13,15] as the prime place where one can set limits on the KM elements $V_{t's}$, $V_{t'b}$, etc., before the study of $V_{ub'}$, $V_{cb'}$, etc., become possible. At present, for example, no realistic bound can be drawn from the current experimental limits[17] on $B \to K^*\gamma$. Even if the experimental situation improves in the next several years, it is not clear whether one can reliably extract limits on $V_{t's}^* V_{t'b}$ from measurements on $b \to s\gamma$, since there are very large QCD corrections and other technical difficulties.[15] In contrast, the inclusive QCD-induced hadronic $b \to s$ transitions should suffer less from QCD corrections. Therefore, a reliable measurement of charmless B decays is important and is strongly urged.

The main points of the paper are as follows: (i)

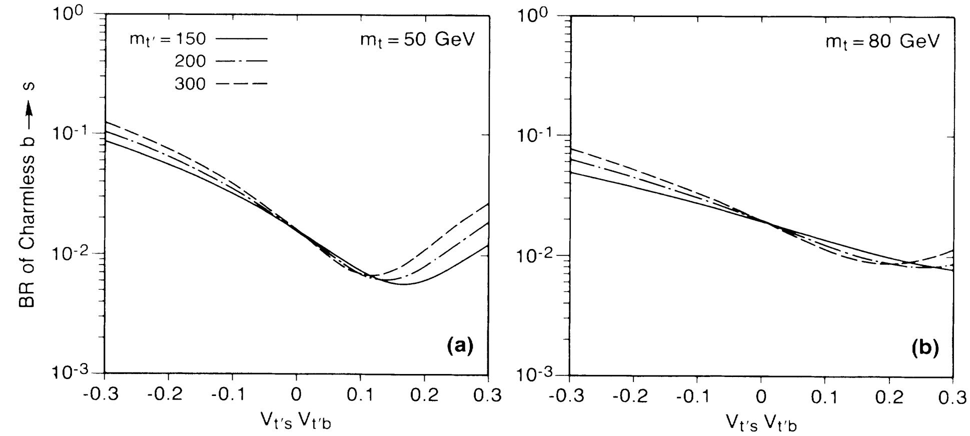

FIG. 2. Same as Fig. 1, but for four-generation case.

Charmless hadronic $b \to s$ transitions are at the level of 1%–2% within the three-generation SM; (ii) a BR $\geq 5\%$ cannot be accounted for by the three-generation SM and would be an indication for the existence of a fourth generation with relatively large mixing angles $V_{t's}$, $V_{t'b}$; and (iii) a BR $\geq 20\%$ would be very difficult to accommodate even with four generations. Although our discussion is not absolutely rigorous, these main points should hold.

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Note added.—While we have emphasized $b \to s$ transitions, via loop graph the BR for the corresponding $b \to d$ transitions will be down roughly by $\sin^2\theta_c$, i.e., $\simeq 10^{-3}$ or, at most, $\simeq 10^{-2}$ for three- or four-generation SM, respectively. These $b \to d$ transitions provide an irreducible background to the experimental determination of V_{ub} (via the direct tree-graph nonleptonic $b \to u$ decay) as in the recent ARGUS observation of B decays to $\bar{p}p\pi$, $\bar{p}p$, $\pi\pi$.

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