Estimating the Chiral-Symmetry–Restoration Temperature in Two-Flavor QCD

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We combine measurements of the chiral-symmetry–restoration transition temperature and the hadron masses in lattice QCD with two flavors of light quarks to estimate the transition temperature in megaelectronvolts. We compare this estimate to results of "quenched" QCD to estimate the effects of our large lattice spacing on the results. We find that the dynamical quarks lower the temperature of the phase transition relative to the pure gauge theory.

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Recent simulations of lattice QCD with dynamical fermions have clarified the behavior of the theory at high temperature. With four flavors of light quarks the theory has a first-order phase transition which is separated from the well-known deconfinement phase transition of pure gauge QCD (the infinite-quark-mass limit) by a region of intermediate quark masses in which there is a rapid crossover in the behavior of thermodynamic quantities but no phase transition.[1,2] With two flavors of light fermions it is clear that the theory is approaching a phase transition as the quark mass is lowered.[2-4] The phase transition for light quarks can be characterized as a chiral-symmetry restoration, while the transition in the pure gauge theory can be characterized as a deconfinement transition. (Recall that for any nonzero quark mass chiral symmetry is explicitly broken, while for any noninfinite quark mass the Z_3 symmetry associated with confinement is explicity broken.) It is important to estimate the temperature of the chiral-symmetry–restoration phase transition for the study of heavy-ion collisions and of the early Universe.

In principle it is straightforward to estimate T_c in megaelectronvolts. (T_c will be used to denote the "crossover" temperature, which is well defined even at values of the quark mass for which there is no phase transition.) The procedure is to simulate QCD on a lattice of size N_t in the Euclidean time direction, which corresponds to a temperature of $1/aN_t$, where a is the lattice spacing. Then some known mass scale is measured at zero temperature with the same lattice spacing, or the same value of the plaquette coupling g. An obvious choice is the mass of some hadron, such as the ρ or the nucleon. The hadron masses are measured in units of the lattice spacing, which can be used to fix a and hence T_c.

Since the chiral-symmetry–restoration phase transition is driven by very low mass quarks, we have studied the theory with two flavors. Using staggered fermions, we have measured the crossover values of $6/g^2$ for quark masses of 0.1, 0.05, and 0.025 with $N_t=4$ (accurately) and $N_t=6$ (roughly).[2,5] We used the version of the "hybrid-stochastic" algorithm[6] described previously.[7] The results are displayed in Table I along with previous results for the pure gauge theory.[8] Note that within the accuracy of our measurements the crossover values of $6/g^2$ for $N_t=6$ are obtained from those for $N_t=4$ by simply adding 0.15.

We have also studied the hadron spectrum at these same values of g and m. Our procedure differs from most hadron-mass calculations in that we vary $6/g^2$ with the quark mass so as to keep T_c fixed in units of the lattice spacing. For the values of $6/g^2$ corresponding to $aT_c=\frac{1}{4}$ we used $6^3\times 24$ and $8^3\times 24$ lattices. For

TABLE I. $6/g^2$ at the high-temperature crossover, with and without dynamical fermions.

N_t	m_q	$6/g^2$
4	0.1	5.375(20)
4	0.05	5.320(10)
4	0.025	5.2875(25)
6	0.1	5.525(40)
6	0.05	5.470(40)
6	0.025	5.438(40)
4	∞	5.680(10)
6	∞	5.865(15)
8	∞	6.02(2)
10	∞	6.18(2)
12	∞	6.33(2)
14	∞	6.45(2)

TABLE II. Hadron masses in units of the lattice spacing. The first three rows are at the high-temperature crossover values of $6/g^2$ for $N_t=4$, and the last three rows for $N_t=6$.

m_q	$6/g^2$	m_π	m_ρ	m_N
0.1	5.375	0.808(1)	1.41(1)	2.26(3)
0.05	5.32	0.581(1)	1.41(5)	2.21(5)
0.025	5.2875	0.417(1)	1.35(5)	2.20(5)
0.1	5.525	0.827(1)	1.19(1)	1.85(1)
0.05	5.47	0.614(1)	1.12(4)	1.54(3)
0.025	5.4375	0.449(2)	0.96(5)	1.33(7)

$aT_c=\frac{1}{6}$ we used $8^3\times 24$ and $10^3\times 24$ lattices. Thus the spatial size of our lattices was large enough to ensure that we did not get ordering in the spatial directions, and we were able to check directly the finite-size effects on the masses. In most cases we ran for 1000 molecular-dynamics time units and computed hadron propagators on 500 configurations. The details of this simulation and analysis will be reported elsewhere.[5] In Table II we display preliminary estimates for the π, ρ, and nucleon masses in lattice units for $aT_c=\frac{1}{4}$ and $\frac{1}{6}$. These masses are in good agreement with two recent smaller simulations using two flavors of dynamical fermions.[9,10]

The π mass appears to be going to zero with the square root of the quark mass. With this form it is straightforward to compute the quark mass at which the π/ρ or π/nucleon mass ratios take their physical values. ($6/g^2$ is also to be extrapolated.) For $aT_c=\frac{1}{4}$ we find that the π/ρ mass ratio is correct at $m_q=0.009\pm 0.001$, while the π/nucleon mass ratio is correct at $m_q=0.015\pm 0.001$. These numbers are different because the ρ/nucleon mass ratio is unphysical on our lattices. Similarly for $aT_c=\frac{1}{6}$, the mass ratios are correct at $m_q=0.0036\pm 0.0004$ and 0.0044 ± 0.0005, respectively.

To estimate the critical temperature in megaelectronvolts, we extrapolate the ρ and nucleon mass in Table II to zero quark mass and use $aT_c=1/N_t$. Strictly speaking, we should extrapolate ρ and nucleon masses to the physical quark mass where m_π/m_ρ or m_ρ/m_N is correct. Because this physical quark mass is very small relative to the quark masses used in present simulations, the difference between extrapolating to zero or to the physical quark mass is negligible. From the $N_t=4$ data we find $T_c=143\pm 9$ MeV using the ρ mass as a standard and 108 ± 5 MeV using the nucleon mass. From the $N_t=6$ data, we find 135 ± 19 MeV and 123 ± 17 MeV, respectively. The quoted errors are statistical only. The statistical errors are large for the $aT_c=\frac{1}{6}$ results because of the large uncertainties in the critical value of $6/g^2$. The errors do not include possible systematic errors from the fitting procedure for the mass spectrum, from the finite time step and finite accuracy of the Green's-function computation in the simulation algorithm, and (obviously) from the fact that the lattice spacing is large. We used the same step size and conjugate-gradient accuracy in the mass-spectrum calculation and in the high-temperature simulation, so that these errors probably cancel in the final result. Because we did the spectrum calculation on two different spatial lattice sizes we know that the effect of the spatial size is small. There is a shift of approximately 1% in the π masses between the $N_s=8$ and the $N_s=10$ results for $aT_c=\frac{1}{6}$. The discrepancies between the ρ and nucleon masses provide some indication of the effects of large lattice spacing and the still too large quark mass. It is also apparent in our spectrum calculations that we do not have good flavor symmetry. This means that masses measured in two channels on the lattice which couple to the same particle in the continuum do not agree. (In the notation of Bowler *et al.*,[11] we quote the π and ρ masses from the *PS* and *VT* operators, respectively, which in our simulation do not agree with the more difficult to measure *PV* and *SC* operators.)

In view of the absence of flavor symmetry and the unphysical mass ratios in our simulation, we must be somewhat cautious in interpreting the results. Clearly much smaller lattice spacing is needed. To estimate these effects we can study results from the pure gauge theory

TABLE III. Hadron masses in the quenched approximation. The values of aT_c are obtained by interpolation in Table I. The last two columns are the deconfinement temperature in megaelectronvolts with the given ρ and N masses as standards.

Ref.	$6/g^2$	aT_c	am_ρ	am_N	$T_c(\rho)$	$T_c(N)$
11	5.7	0.241(4)	1.00(1)	2.34(10)	186(4)	97(8)
12	5.7	0.241(4)	0.98(11)	1.21(13)	189(21)	187(20)
13	5.895	0.159(3)	0.587(51)	0.962(54)	209(19)	155(9)
12	5.9	0.157(3)	0.75(9)	0.88(12)	161(20)	168(23)
11	6.0	0.130(5)	0.56(3)	1.12(23)	179(12)	109(23)
14	6.0	0.130(5)	0.41(1)	0.52(5)	244(11)	235(24)
13	6.0	0.130(5)	0.391(32)	0.621(44)	256(23)	197(16)
15	6.15	0.105(3)	0.355(27)	0.537(14)	228(19)	184(7)
16	6.2	0.098(2)	0.31(3)	0.48(3)	243(24)	192(13)
15	6.30	0.087(2)	0.303(40)	···	220(29)	···

(quenched approximation) where results are available for T_c and hadron masses with much smaller lattice spacings. It seems reasonable that the effects of the large lattice spacing with dynamical fermions should be similar to those in the pure gauge theory. This idea is supported by the similarities in the mass spectrum with and without dynamical quarks, such as the large nucleon-to-ρ mass ratio, and by the similar deviations from perturbation theory in the scaling of $6/g^2$ at the high-temperature transition with and without dynamical quarks. We quote the earlier results for $6/g^2$ at the deconfinement transition of pure gauge theory in Table I.[8] As with the dynamical fermions, we will take the T_c measurements to fix the lattice spacing. For example, since in the pure gauge theory with $N_t=4$ the critical value of $6/g^2$ is 5.68, a quenched spectrum calculation with $6/g^2=5.7$ can be compared with our full QCD calculations with $aT_c=\frac{1}{4}$. In Table III we give a selection of quenched-approximation results for the ρ and nucleon masses with Kogut-Susskind fermions extrapolated to zero quark mass.[11-16] Where the authors quoted an extrapolation we have used it; otherwise, we made a linear fit to the quoted results, usually using the three smallest values of m_q. For each quenched-approximation calculation we have assigned a lattice spacing in units T_c by interpolation in Table III. As with the dynamical fermions, this leads to estimates of T_c in megaelectronvolts.

Finally, in Figs. 1(a) and 1(b) we plot our estimates for T_c in megaelectronvolts together with the quenched-approximation estimates. It can be seen that T_c with dynamical fermions lies below the pure-gauge-theory estimates (with the exception of the "Edinburgh nucleons"). Within the rather large scatter of the different results there does not seem to be much tendency for the quenched T_c to change as the lattice spacing decreases (it might increase slightly, especially if the nucleon is used to set the scale). Therefore we think that our results suggest a chiral-symmetry restoration at zero baryon density at a temperature in the range of 100 to 160 MeV, significantly lower than in the pure gauge theory.

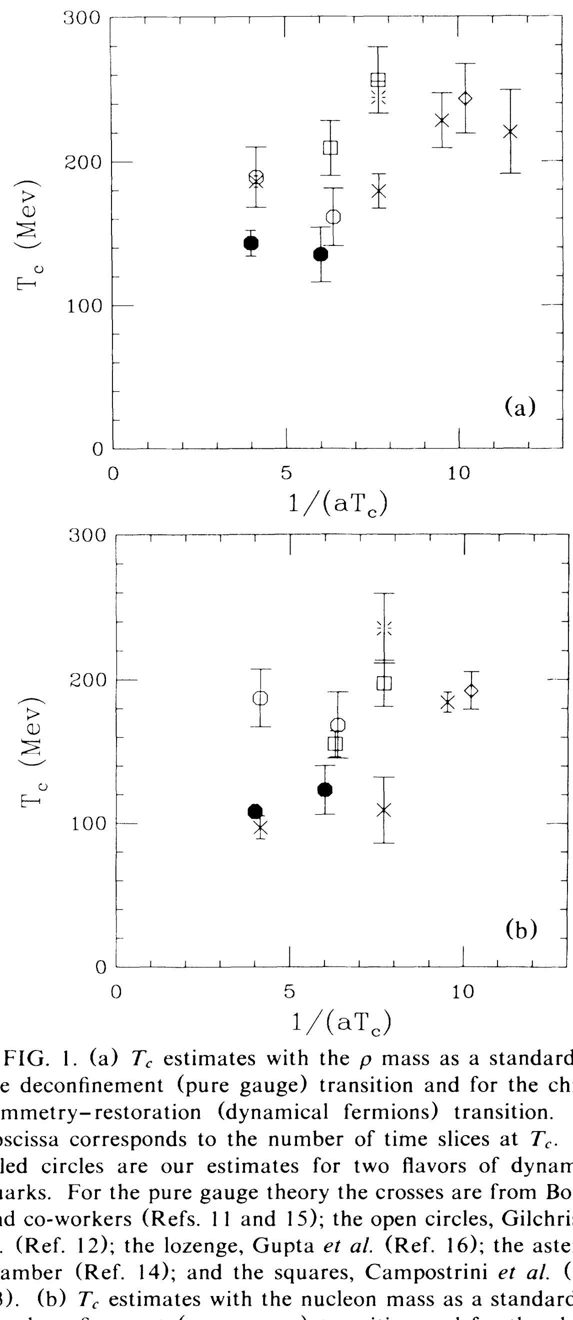

FIG. 1. (a) T_c estimates with the ρ mass as a standard for the deconfinement (pure gauge) transition and for the chiral-symmetry–restoration (dynamical fermions) transition. The abscissa corresponds to the number of time slices at T_c. The filled circles are our estimates for two flavors of dynamical quarks. For the pure gauge theory the crosses are from Bowler and co-workers (Refs. 11 and 15); the open circles, Gilchrist *et al.* (Ref. 12); the lozenge, Gupta *et al.* (Ref. 16); the asterisk, Hamber (Ref. 14); and the squares, Campostrini *et al.* (Ref. 13). (b) T_c estimates with the nucleon mass as a standard for the deconfinement (pure gauge) transition and for the chiral-symmetry–restoration (dynamical fermions) transition. The symbols are the same as in (a).

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