Possible Nonconservation of Electric Charge

Rabindra N. Mohapatra
Department of Physics and Astronomy, University of Maryland, College Park, Maryland, 20742
(Received 9 July 1987)

This paper points out some implications of the possibility that electric-charge nonconservation may occur in nature via an interaction that changes an electron to a positron (i.e., $\Delta Q = 2$).

PACS numbers 11.30.Er, 13.10.+q, 14.60.Cd

The advent of unified gauge theories has led physicists in recent years to question the absoluteness of many conservation laws such as baryon number, lepton number, etc. In fact, most grand unified theories lead in a natural manner to interactions mediated by superheavy particles, which change baryon as well as lepton number. The fact that the particles mediating these interactions are superheavy explains why these selection rules appear in low-energy physics. There is, however, another sacred quantum number, the electric charge, which remains conserved in all grand unified theories considered to date and is related to the assumption that electric charge is an exact gauge symmetry, with the associated gauge boson, the photon, being massless. It is, nevertheless, interesting to speculate on the possibility that conservation of electric charge may break down in some future theory that unifies all interactions, and to discuss its implications. In fact, this possibility has been discussed in the literature from time to time by theorists[1,2] as well as experimentalists,[3] where the electric-charge-nonconserving process $e^- \to \nu\gamma$ has been the main focus of attention. The latest bound on the lifetime for this process is given by $\tau(e^- \to \nu\gamma) \gtrsim 10^{25}$ years.[4] If this nonconservation is assumed to be described by an effective Lagrangean $\mathcal{L}^{\Delta Q=1}$,

$$\mathcal{L}^{(1)} = \delta e\, \bar{e}\gamma_\mu \nu A_\mu + \text{H.c.}, \quad (1)$$

the decay width of the electron is given by

$$\Gamma(e^- \to \nu\gamma) = (\delta e)^2 m_e/16\pi^2, \quad (2)$$

leading to an upper bound on δe as follows:

$$\delta e \lesssim 2.6\times 10^{-26}.$$

An immediate consequence of the breakdown of electric-charge conservation is that the photon must acquire a mass. Since empirical limits exist on the allowed value of the photon mass,[5] one may like to relate these two phenomena in such a theory. If we taken a phenomenological Lagrangean of the form given in Eq. (1), it will induce a photon mass at the one-loop level. However, the one-loop graph turns out to be quadratically divergent and we get

$$m_\gamma^2 \approx (\delta e^2/4\pi^2)\Lambda^2, \quad (3)$$

where Λ is the ultraviolet cutoff. The quadratic divergence is easily understood since $e^- \to \nu\gamma$ happens to be a dimension-four coupling and we do not respect gauge invariance anymore. The induced photon mass is therefore very sensitive to the value of Λ chosen. In fact, for the present upper limit on m_γ of 10^{-22} MeV, and $\delta e \lesssim 10^{-26}$, we can tolerate a cutoff $\Lambda \simeq 100$ GeV or so. Instead one might conjecture the Planck mass as a natural value for the cutoff Λ, in which case, one would expect m_γ in disagreement with observations unless δe is at most of order 10^{-43}, a limit which is experimentally unreachable. I will call this type of breaking of electric charge by dimension-four terms to be "hard" breaking.

In this Letter, I discuss an alternative mode of charge nonconservation, where an electron can oscillate into a positron in vacuum.[6] This can be represented by an effective Lagrangean of the form

$$\mathcal{L}^{(2)} = \delta m_e e^{-\mathrm{T}} c^{-1} e^- + \text{H.c.} \quad (4)$$

First, I argue that the existing experimental limits obtained from atomic K-shell experiments that limit δe can also be used to limit δm_e. Secondly, I argue that the connection between m_γ and δm_e involves only a logarithmic divergence; therefore, experimental bounds on one of the parameters perhaps can be translated into reliable bounds on the other. At this point it is worth pointing out that a natural way to produce such a Lagrangean would be through spontaneous symmetry breaking; however, a unified gauge theory with spontaneous breaking of electric charge is ruled out by results on $g-2$ of the muon and electron.[7] The main culprit is a charged physical Higgs boson with mass of order of the photon mass, which contributes a big amount to the $g-2$ through a two-loop graph. Also, it is not clear how such a tiny vacuum expectation value would naturally arise in a gauge theory. Supersymmetric models where superpartners have different electric charges are a possibility.[8] Kaluza-Klein models also might be worth exploring since $(m_\gamma)_{\max} \approx m_e^2/M_{\mathrm{Pl}}$ numerically.

To deal with the first question, I discuss e^-e^+ oscillation in an atom in a manner similar to the case of neutron-antineutron oscillation in a nucleus.[9] I approximate the evolution of the e^+ amplitude from the existing electrons in the atom by the following equation:

$$\frac{d}{dt}\begin{pmatrix} e^- \\ e^+ \end{pmatrix} = \frac{-i}{h}\begin{pmatrix} E_e+m_e & \delta m \\ \delta m & m_e-E_e+iV \end{pmatrix}\begin{pmatrix} e^- \\ e^+ \end{pmatrix}, \quad (5)$$

where V denotes any possible absorption effect due to e^-e^+ annihilation to two photons. Detailed atomic-physics calculations are necessary to determine V and in what follows, I will assume $V \sim E_e$. Using standard techniques,[9] one can find the lifetime of atomic instability due to e^+e^- oscillation to be

$$\tau^{-1} \simeq V(\delta m)^2/(4E_e^2+V^2). \quad (6)$$

Since we do not know V, I will assume $V \approx E_e$ which leads to $\tau^{-1} \simeq (\delta m)^2/5E_e$. In a heavy atom, $E_e \simeq 1$ keV; assuming $\tau > 10^{25}$ years, we obtain $\delta m_e \leq 10^{-28}$ MeV. This translates to an electron-positron oscillation time in vacuum of about two months.

Let us now turn our attention to relating δm_e to the photon mass induced by it. Again, doing a simple one-loop calculation, we find that in this case, the photon mass diverges only logarithmically and we get

$$m_\gamma^2 \simeq (\delta m_e)^2(\alpha/2\pi)\ln(\Lambda/m_e). \quad (7)$$

It is easy to convince oneself that the logarithmic divergence is maintained in higher orders. The apparently dangerous graphs are multiloop graphs involving virtual longitudinal photons since their propagators adds an extra momentum integration (d^4k) and a factor from the propagator of the form $(1/m_\gamma^2)k_\mu k_\nu/k^2$ and two extra fermion propagators for each virtual photon, adding a net apparent degree of divergence of two; but one has to realize that each k_μ factor gets converted to a $\partial_\mu J_\mu^{\text{em}}$ which is equal to $2\delta m_e e^{-\text{T}} c^{-1} e^-$, thus removing two more momenta from the numerator and reducing the apparent quadratic divergence to a logarithmic one as in the one-loop case. I will call this "soft" breaking of electric-charge conservation. We see that, because of the logarithmic divergence, a reasonable bound (uncertain only within a factor of 10) can be derived on m_γ from the above bound on δm_e and we get

$$m_\gamma \leq 10^{-28\pm1}\ \text{MeV}. \quad (8)$$

This bound, though model dependent, is stronger than the bound 10^{-22} MeV.[5]

Let us now turn to cosmological implications of electron-positron oscillation. In discussing this, it is worth remembering that, if we take Eq. (8) seriously, then e^+e^- oscillation would lead to a finite range to the electromagnetic force of $\mathcal{R}_\gamma \gtrsim 2\times10^{17\pm1}$ cm which is roughly a light year, which is much smaller than the distance between typical galaxies. Thus, galactic stability would not be affected by e^+e^- oscillation corresponding to the upper limit of 10^{-28} MeV. Note that if electromagnetic forces were of infinite range, galactic stability would imply a bound on electric-charge–nonconserving decays,[10] as follows:

$$(e\tau_\nu/\tau_{\Delta Q\neq0})^2 \leq G_{\text{Newton}} m_p^2, \quad (9)$$

or $\tau_{\Delta Q\neq0} \geq 10^{28}$ years. This bound can be converted into a bound on $n_Q/n_B < 10^{-18}$, if we assume that $N_B \approx N_e$ within a factor of 2, that the electrons disappear at the rate of one per 10^{28} years, and the age of the universe is 10^{10} years. Again, I emphasize that this bound does not apply to our case since the original bound $\tau_{\Delta Q\neq0} \geq 10^{28}$ years does not hold for finite-range electromagnetic forces. For infinite-range electromagnetism the bound on n_Q/n_B follows from the simple consideration[11] that between two cosmological objects the electromagnetic force be weaker than the force of gravity, i.e., $\alpha N_Q^2/R < G_{\text{Newton}} m_p^2 N_B^2/R$. Since in our case the left-hand side has an additional factor e^{-R_γ}/R_γ reflecting the finite range of Coulomb force, the bound on n_Q/n_B does not hold.

Several other model-dependent but more stringent bounds on n_Q/n_B have been considered recently by Orito and Yoshimura.[12] Some of their bounds do not apply to our case since, in the presence of very weak interactions such as e^+e^- oscillation, the universe acquires all its charge only after the era of recombination. Again, because of the finite range of Coulomb forces, the bounds derived from the observed isotropy of high-energy (10^{11}–10^{18} eV) cosmic rays also do not apply.

I conclude this Letter with the following comments of theoretical nature. First, I present some additional mechanisms of both soft and hard breakings of electric charge. As another example of a soft $\Delta Q\neq0$ Lagrangean, consider the following Lagrangean:

$$\mathcal{L}^{(3)} = f_3 \bar{e}\phi^- \nu + \delta m_\phi^2 \phi^- \phi^- + \text{H.c.} \quad (10)$$

This also leads to a logarithmically divergent photon mass at three-loop level, with

$$m_\gamma^2 \simeq [f_3^4 4\pi\alpha/(16\pi^2)^3]\delta m_\phi^2[\ln(\Lambda/m_\phi^2)]^3. \quad (11)$$

If this interaction is combined with a Majorana mass for the neutrino, it can lead to electron-positron oscillation with

$$\delta m_e \simeq (f_3^2/16\pi^2) m_{\nu_e}(\delta m_\phi^2/m_\phi^2);$$

since the present limit on m_ϕ from the DESY e^+e^- storage ring PETRA is 22 GeV, with $m_{\nu_e} < 10$ eV, we infer for the $\Delta Q\neq0$ coupling parameter $\delta m_\phi \leq (10^{-8}/f_3)$ GeV. Choosing f_3 as a typical Yukawa coupling parameter ($f_3 \simeq 10^{-3}$–10^{-5}), we find $\delta m_\phi \leq 10^{-3}$–$10^{-5}$ GeV. On the other hand, if we chose $f_3 \simeq 10^{-8}$, we can have $\delta m_\phi \approx 1$ GeV. If such a Higgs particle of mass 40–45 GeV existed, with $\delta m_\phi \simeq 1$, we may expect to see $\Delta Q\neq0$ signals of type $e^-e^- \to \phi^-\phi^- \to e^-e^-\bar{\nu}\bar{\nu}$ at the CERN collider LEP, if ϕ^- has no other significant decay mode than $e^-\bar{\nu}$.

As another example of hard electric-charge violation, we consider the four-fermion Lagrangean which leads to $e^- \rightarrow 3\nu$. If we write for instance, a vector interaction of the form

$$\mathcal{L}^{(4)} = (1/M_{\Delta Q}^2)\bar{e}\gamma_\mu \nu\, \bar{\nu}\gamma_\mu \nu, \tag{12}$$

present experiments would imply $M_{\Delta Q} \gtrsim 10^9$ GeV.

I would like to acknowledge useful discussions with S. Barr, M. Goldhaber, J. C. Pati, and F. Reines on the subject matter of this Letter. This work was supported by a grant from the National Science Foundation.

[1]L. B. Okun and Ya. B. Zeldovich, Phys. Lett. **78B**, 597 (1978); L. B. Okun and M. B. Voloshin, Pis'ma Zh. Eksp. Teor. Fiz. **28**, 156 (1978) [JETP Lett. **28**, 145 (1978)].

[2]V. Kuzmin, A. Yu. Ignatiev, and A. N. Tavkhelidze, paper at the Seminar on Gauge Field Theories, Moscow, 1978 (unpublished).

[3]M. K. Moe and F. Reines, Phys. Rev. **140B**, 992 (1965); M. Goldhaber, private communication; R. I. Steinberg *et al.*, Phys. Rev. D **12**, 2582 (1975); E. Belloti *et al.*, Phys. Lett. **124B**, 435 (1983); A. A. Pomansky, in *Proceedings of the International Neutrino Conference, Aachen, West Germany, 1976,* edited by H. Faissner *et al.* (Vieweg, Braunschweig, Germany, 1976), p. 641.

[4]F. T. Avignone *et al.*, Phys. Rev. D **34**, 97 (1985).

[5]A. S. Goldhaber and M. M. Nieto, Rev. Mod. Phys. **43**, 277 (1971); L. Davis, A. S. Goldhaber, and M. M. Nieto, Phys. Rev. Lett. **35**, 1402 (1975).

[6]Preliminary discussion of these ideas was presented by the author in the Proceedings of the Eighth Workshop on Grand Unification, Syracuse, New York, 1987, edited by K. Wali (World Scientific, Singapore, to be published).

[7]A. Yu. Ignatiev, V. Kuzmin, and M. E. Shaposnikov, Phys. Lett. **84B**, 315 (1979).

[8]Supersymmetric composite models of this type have been proposed by J. C. Pati and A. Salam, Nucl. Phys. **B214**, 109 (1983). Electric-charge nonconservation in this model has been pointed out by R. N. Mohapatra and J. C. Pati, to be published.

[9]R. N. Mohapatra and R. E. Marshak, Phys. Lett. **94B**, 183 (1980); M. Chetyrkin *et al.*, Phys. Lett. **99B**, 358 (1981); P. G. Sandars, J. Phys. G **6**, L161 (1980); D. Peaslee, unpublished; R. N. Mohapatra, in Proceedings of the Harvard Workshop on N-$\bar{N}$ oscillation, Cambridge, Massachusetts, 1982, edited by M. Goodman (unpublished); L. N. Chang and N. P. Chang, Phys. Lett. **92B**, 103 (1980); T. K. Kuo and S. Love, Phys. Rev. Lett. **45**, 93 (1980); V. A. Kuzmin, Pis'ma Zh. Eksp. Teor. Fiz. **12**, 228 (1970) [JETP Lett. **12**, 228 (1970)].

[10]M. Goldhaber, private communication.

[11]R. A. Lyttleton and H. Bondi, Proc. Roy. Soc. (London) Ser. A **252**, 313 (1959).

[12]S. Orito and M. Yoshimura, Phys. Rev. Lett. **55**, 2457 (1985).