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Realism and Quantum Flux Tunneling

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Leggett and Garg have argued that the phenomenon of quantum flux tunneling oscillations will, according to the predictions of quantum mechanics, contradict the pair of assumptions "macroscopic realism" and "noninvasive measurability." It is argued here that there can be no contradiction of realism in such a case, and that the contradiction is between quantum mechanics and noninvasive measurability.

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Discussions about the compatibility or incompatibility of *quantum mechanics* (QM) with *realism* (essentially the doctrine that objects have properties independently of the observation process) have been going on at the conceptual level for a very long time. The discovery of Bell's inequality seemed to make it possible to move the discussion to the level of mathematical demonstration and experimental test, at least for quantities such as spin components and polarizations. In a paper provocatively entitled, "Quantum Mechanics versus Macroscopic Realism: Is the Flux There when Nobody Looks?", Leggett and Garg[1] (LG) have attempted to do the same for the magnetic flux in a SQUID (superconducting quantum-interference device). To do so they compare the predictions that follow from QM for the autocorrelation of the flux at different times with the predictions of an alternative theory. The alternative theory is characterized by two postulates: (A1) *Macroscopic realism*: "A macroscopic system with two or more macroscopically distinct states available to it will at all times *be* in one or the other of these states." (A2) *Noninvasive measurability at the macroscopic level*: "It is possible, in principle, to determine the state of the system with arbitrarily small perturbation of its subsequent dynamics." This alternative theory is said to lead to inequalities which are analogous to those of Bell[2] and of Clauser *et al.*,[3] and which are violated by the predictions of QM.

No attempt was made by LG to separate the roles of the two assumptions (A1) and (A2) in producing a result contrary to QM, but the general tone of their discussion and especially the title of their paper strongly suggest a contradiction between macroscopic realism and QM. I shall argue that the contradiction of QM is produced solely by the assumption of noninvasive measurability (A2), and that no contradiction between QM and realism can be demonstrated in this kind of experiment.

The analogy between the Bell-type inequalities and the inequalities (2a) and (2b) of LG can be misleading because the physical principles from which the former were derived are not applicable to the SQUID. A key role is played in Refs. 2 and 3 by the *locality* postulate: If two measurements are carried out on parts of a system at spacelike separation from each other, then one measurement should not affect the other. This postulate motivates a conditional factorization of the joint probability distribution for the two measurements, which in turn leads to the Bell-type inequalities. Such an argument is not applicable to the successive measurements of flux in a SQUID, which are carried out on a single localized system and have timelike separations. Thus, in spite of their similar mathematical forms, the LG inequalities have an entirely different physical significance from the Bell-type inequalities.

Although the derivation of their inequalities was not presented in Ref. 1, it would appear that LG used a method similar to that of Garg and Mermin.[4] If pair distributions are given for a set of statistical variables, these may or may not be compatible with the existence of nonnegative higher-order distribution functions of which the pair distributions are marginals. If appropriate higher-order distribution functions exist, then data are compatible with realism. If not, then one may conclude that the entire set of variables cannot be simultaneously real. Apparently LG wish to apply that method to the values of the flux in the SQUID at three or more different times. [The paragraph of Ref. 1 containing Eq. (1) supports this interpretation of their intent.]

The compatibilty of QM with (A1) and its incompatibility with (A2) can be illustrated by the calculation of the distribution function for the results of successive flux measurements. To do this one must make a model of the SQUID interacting with the measurement apparatus. Quantum coherence will be maximized, and so also will the chance of obtaining a nonclassical result, if we neglect dissipative effects, even though they may be important in a real experiment.

Following LG, the flux trapped in the SQUID is idealized as a two-state system which can pass from the "positive" to the "negative" flux state by tunneling. This two-valued flux variable is represented by the Pauli operator σ_z, whose eigenvalues are ± 1. The Hamiltonian of the isolated SQUID can then be written as $H_0 = \frac{1}{2}\omega\sigma_x$, where σ_x is a Pauli operator and ω is the tunneling frequency. (That is to say, $\hbar\omega$ is the energy separation between symmetric and antisymmetric states of the double-minimum potential well in Fig. 1 of Ref. 1. I choose units of $\hbar = 1$ for convenience.) Again following LG, the flux can in principle be measured by propelling a neutron through the SQUID ring at a speed such that the magnetic field causes its spin to precess by $\pm\pi/2$, depending upon whether the flux is positive or negative. If we idealize the magnetic field experienced by a moving neutron as a square pulse, then the interaction between the SQUID and the jth neutron can be written as $H_{0j} = \alpha\sigma_z\sigma_x^{(j)}$ for the time interval $t_j \le t \le t_j + \tau$ during which the neutron is in the magnetic field of the SQUID, and $H_{0j} = 0$ at all other times. Here $\sigma_x^{(j)}$ is the x component of spin of the jth neutron. The coupling parameter α is essentially the product of the field strength and the neutron magnetic moment. The full Hamiltonian is $H = H_0 + \sum_j H_{0j}$, where the sum is over the neutrons that are used, one at a time, in the successive measurements.

The probability distribution for any number of successive measurements can be calculated explicitly for this simple model because the time development operator, $U(t) = \exp(-iHt)$, can be calculated exactly. For the isolated SQUID (before, between, and after measurements), it has the form

$$U_0(t) = \cos(\tfrac{1}{2}\omega t) - i\sigma_x \sin(\tfrac{1}{2}\omega t). \quad (1)$$

During the jth measurement, when the Hamiltonian will be equal to $H_0 + H_{0j}$, it will have the form

$$U_{0j}(t) = \cos(\beta t) - i\beta^{-1}(\tfrac{1}{2}\omega\sigma_x + \alpha\sigma_z\sigma_x^{(j)})\sin(\beta t), \quad (2)$$

with $\beta(\alpha^2 + \omega^2/4)^{1/2}$.

At $t=0$ the SQUID may have an arbitrary state, $a|+\rangle + b|-\rangle$, with $|a|^2 + |b|^2 = 1$. (Eigenvectors of σ_z are used as basis vectors in all cases.) The initial state of the jth neutron is chosen to be of the form $(|+\rangle^{(j)} + i|-\rangle^{(j)})/\sqrt{2}$, which corresponds to spin polarized in the y direction. During a measurement, a neutron will spend a time τ in the magnetic field of the SQUID, and its spin will precess towards either the $+z$ or the $-z$ direction, depending upon the value of the flux. The state of the SQUID+neutron system immediately following the passage of the first neutron is obtained from the initial state by means of the operator $U_{0j}(\tau)$. Because the magnetic flux is not constant, it is necessary to choose the transit time τ so as to maximize the correlation between the initial flux and final spin of the neutron. This optimum is achieved by choosing τ so that $\cos(\beta\tau) = (\alpha/\beta)\sin(\beta\tau)$, in which case the state will be

$$|\Psi(\tau)\rangle = \cos(\beta\tau)\sqrt{\tfrac{1}{2}}\{(2a - i\tfrac{1}{2}b\omega/\alpha)|+\rangle|+\rangle^{(1)} + (\tfrac{1}{2}b\omega/\alpha)|+\rangle|-\rangle^{(1)} - i(\tfrac{1}{2}a\omega/\alpha)|-\rangle|+\rangle^{(1)} + (\tfrac{1}{2}a\omega/\alpha + i2b)|-\rangle|-\rangle^{(1)}\}. \quad (3)$$

This is not a perfect measurement, since even if the initial value of the flux was definitely positive ($a=1$, $b=0$) there would be a probability of order $(\omega/\alpha)^2$ that the final value of the neutron spin would be negative. This defect can, in principle, be overcome by taking the strong-coupling, *fast-measurement* limit: $\alpha \to \infty$, $\tau \to 0$, $\alpha\tau = \pi/4$. In this limit we have $\beta \to \alpha$, and $\omega/\alpha \to 0$, so that (3) becomes

$$|\Psi(\tau)\rangle = a|+\rangle|+\rangle^{(1)} + ib|-\rangle|-\rangle^{(1)}, \quad (4)$$

and the correlation between the initial value of the flux and the final value of the spin becomes perfect. Moreover the value of the flux is not changed during the measurement, and so the measurement is as close as possible to being nondisturbing.

If a time t_{12} is allowed to elapse before the second measurement, the state will become

$$|\Psi(\tau + t_{12})\rangle = U_0(t_{12})|\Psi(\tau)\rangle = a\cos(\tfrac{1}{2}\omega t_{12})|+\rangle|+\rangle^{(1)} - ia\sin(\tfrac{1}{2}\omega t_{12})|-\rangle|+\rangle^{(1)} + b\sin(\tfrac{1}{2}\omega t_{12})|+\rangle|-\rangle^{(1)} + ib\cos(\tfrac{1}{2}\omega t_{12})|-\rangle|-\rangle^{(1)}. \quad (5)$$

The second measurement can now be carried out with use of the second neutron, and the joint probability distribution for the results of the two measurements is

$$P_{12}(+,+)=|a\cos(\tfrac{1}{2}\omega t_{12})|^2,\quad P_{12}(+,-)=|a\sin(\tfrac{1}{2}\omega t_{12})|^2,$$
$$P_{12}(-,+)=|b\sin(\tfrac{1}{2}\omega t_{12})|^2,\quad P_{12}(-,-)=|b\cos(\tfrac{1}{2}\omega t_{12})|^2. \tag{6}$$

Because of the simple form of the time development operators, it is easy to extend the calculation to any number of successive measurements. For example, in the case of three fast measurements ($\tau \ll t_{12}, t_{23}$) we have

$$P_{123}(+,+,+)=|a\cos(\tfrac{1}{2}\omega t_{12})\cos(\tfrac{1}{2}\omega t_{23})|^2,\quad P_{123}(+,-,+)=|a\sin(\tfrac{1}{2}\omega t_{12})\sin(\tfrac{1}{2}\omega t_{23})|^2,\ \text{etc.} \tag{7}$$

(General rule: a or b factor corresponds to $+$ or $-$ result in the first measurement; cosine factor if successive results agree, sine factor if successive results disagree.) Having a probability distribution for three successive measurements, we can compute the probability distributions for any subset of them as marginal distributions, for example,

$$P_{12}(+,+)=P_{123}(+,+,+)+P_{123}(+,+,-).$$

There is no possibility of obtaining any conflict with realism by the criteria of Ref. 4.

However, one would obtain a contradiction if one were to require, as is suggested by Eq. (1) of LG, that the pair distribution

$$P_{13}(Q_1,Q_3)=P_{123}(Q_1,+,Q_3)+P_{123}(Q_1,-,Q_3)$$

should have the *same functional form* as $P_{12}(Q_1,Q_2)$. Such a requirement would follow from assumption (A2) *noninvasive measurability,* according to which the correlation between the first and third measurements of the flux would be *the same as if the intervening second measurement had not taken place.* However, it is apparent from (6) and (7) that QM does not satisfy this condition. Even if all possible outcomes of the second measurement are summed over, the fact that a physical interaction took place may still be relevant to the probability for an outcome of a subsequent measurement. According to QM it does make a difference, contrary to the assumption (A2). It is perhaps worth pointing out that this disturbance of the dynamics by the measurement, predicted by QM, has nothing to do with the uncertainty principle, whose origin is in the noncommutativity of certain operators. It is rather an instance of the equality of action and reaction, which is evident from the symmetry of the interaction, $H_{0j}=\alpha\sigma_z\sigma_x^{(j)}$, with respect to the flux variable σ_z and the neutron spin $\sigma_x^{(j)}$.

Leggett[5] has proposed a different experimental design from that described above. Instead of measuring the flux at successive times t_1, t_2, and t_3 on each member of an ensemble prepared appropriately at $t=0$, he proposes that on similarly prepared members of the ensemble one should perform a pair of measurements at *either* t_1 and t_2, *or* t_2 and t_3, *or* t_1 and t_4, *or* t_2 and t_4. Different pairs would be chosen for different members of the ensemble. In this way one could measure the various pair distributions that enter into Eq. (2a) and (2b) of LG, without at the same time having measured any higher-order distributions. Whether or not this design will lead to different results depends upon whether or not the principle (A2) of *noninvasive measurability* holds in nature. If it holds, then the correlation between flux measurements at times t_2 and t_4 will be unaffected by whether or not there was a previous measurement at t_1 or an intervening measurement at t_3. According to QM [for which (A2) fails] it should make a difference, and the two experimental designs should yield different results, the former agreeing with the LG inequalities (as shown above), and the latter disagreeing with them (as was shown by LG).

I conclude that should the proposed flux correlation experiments confirm QM, we should conclude only that *noninvasive measurability* fails, even at this seemingly macroscopic level. (All bets are off in the more exciting possibility that QM might fail.) But no outcome of the experiment could cast any serious doubt on *realism.* On the one hand, this may be viewed as a disappointment, since we have been unable to bring the metaphysical principle of *realism* within the reach of experimental test. On the other hand, we may be reassured that mystics, psychics, and other irrationalists who have attempted to use misinterpretations of QM in support of their views have nothing to gain from this subject.

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