Monte Carlo Modeling of Ionospheric Oxygen Acceleration by Cyclotron Resonance with Broad-Band Electromagnetic Turbulence

John M. Retterer
Boston College, Chestnut Hill, Massachusetts 02167

Tom Chang and G. B. Crew
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

J. R. Jasperse
Air Force Geophysics Laboratory, Hanscom Air Force Base, Massachusetts 01731

and

J. D. Winningham
Southwest Research Institute, San Antonio, Texas 78284

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We demonstrate that cyclotron resonance with observed electric field fluctuations is responsible for production of the oxygen-ion conics that are observed by the Dynamics Explorer 1 satellite in the central plasma-sheet region of the Earth's magnetosphere. The ion-velocity distribution is described by a quasi-linear diffusion equation which is solved by the Monte Carlo technique. The acceleration produced by the observed wave spectrum agrees well with the ion observations, in both form and magnitude. To our knowledge, this represents the first successful comparison of an observed conic with any theoretical model.

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Ion acceleration through wave-particle interaction with plasma turbulence in the auroral regions of the Earth's magnetosphere has commonly been invoked[1] to explain the origin of observed[2] intense fluxes of energetic ions flowing out of the ionosphere into the outer magnetosphere, which are known as "ion conics" because of the form of the ion distribution in velocity space. Until recently,[3] however, attempts to verify the theories by correlating ion-flux and plasma-turbulence data have been unsuccessful[4]: Because of the small spatial scale of energetic conics and the regions in which they form, it has proven difficult to observe simultaneously both an ion conic and the waves which are responsible for its generation. By the examination of a special class of conic events, namely, the oxygen-dominated, less-energetic conics produced in the broad central plasma-sheet (CPS) region of the auroral zone, we are able to report here the first successful description of an observed ion conic by a theoretical model of ion acceleration through wave-particle interaction. The theoretical ion-velocity distribution is calculated with use of a Monte Carlo technique which allows us to compare not only the overall magnitude of the acceleration produced by the observed turbulence, but also to compare the form of the ion-velocity distribution produced by the wave-particle interaction combined with the effect of the static, but inhomogeneous, geomagnetic field.

Such comparisons can be realized only where it is possible to measure the plasma turbulence and ion fluxes simultaneously. Intense, broad-band, low-frequency, electric and magnetic field noise has been commonly observed at low altitudes over the Earth's auroral zone by nearly all the satellites that have flown in this region.[5] Particle measurements[6] performed on board the Dynamics Explorer 1 (DE-1) satellite have revealed the existence of a population of oxygen-dominated ion conics that extend in latitude throughout the equatorward portion of the auroral zone (which maps out to the CPS in the Earth's magnetotail) during times of magnetic storm activity. These energetic particle fluxes are coincident with intense, low-frequency auroral-zone turbulence,[7] and it has been suggested[8] that wave-particle interaction with this turbulence is responsible for the transverse acceleration of the ions to form the observed conics. Based on the laboratory study of transverse heating of ions through cyclotron resonance,[9] Chang *et al.*[8] found that if the turbulence contained a modest fraction of left-hand-polarized waves, then its observed amplitude could easily account for the energies of the measured conics, making this even an ideal one for our detailed scrutiny.

In the absence of wave-particle interaction, the evolution of the ion-velocity distribution, f, would be described by the Liouville equation[10];

$$df/dt \equiv \partial f/\partial t + v_\parallel \partial f/\partial s + (v_\perp/2B)(dB/ds)(-v_\perp \partial f/\partial v_\parallel + v_\parallel \partial f/\partial v_\perp) = 0, \tag{1}$$

where s is a coordinate denoting position along a geomagnetic field line, $v_\parallel$ is the velocity along the field line, B is the magnitude of the geomagnetic field, and q and m are the charge and mass of an ion. The ions move adiabatically, conserving their magnetic moment and energy in the static geomagnetic field. In the presence of the observed broad-band turbulence, the effect on the ions of their interaction with the turbulence is adequately described by a diffusion equation,

$$df/dt = v_\perp^{-1}(\partial/\partial v_\perp)(v_\perp D_\perp \partial f/\partial v_\perp), \qquad (2)$$

where the left-hand side of the equation contains the terms of the Liouville equation and $D_\perp$ is the quasilinear velocity diffusion rate perpendicular to the geomagnetic field.[11] The diffusion coefficient $D_\perp$ depends on the electric field spectrum of the turbulence, but the generation and maintenance of the turbulent steady state is a complicated, nonlinear, nonlocal problem that is beyond the scope of this paper. In the present instance, we proceed using the observed wave spectrum to estimate the diffusion coefficient. Before we present the formula for $D_\perp$, we simplify it and express it in terms of the quantities measured by the instruments on board the satellite. For interaction between electromagnetic plasma modes and ions, we can take advantage of the approximation that wavelengths are sufficiently long so that the components of wave vector and ion velocity perpendicular to the magnetic field, $k_\perp$ and $v_\perp$, satisfy $k_\perp v_\perp \ll \Omega$, where Ω is the ion gyrofrequency. This implies that the dominant contribution to $D_\perp$ comes from the spectral density of the electric field in the left-hand polarization with frequencies ω near Ω. The satellite measures no wave-vector information for the electric field spectral density, so the result must be expressed in terms of the spectral density integrated over wave-vector space, leaving only its dependence on frequency. Polarization information from the satellite observations is also limited, so let us take the spectral density of left-hand-polarized waves, $|E_L(\omega)|^2$, to be some fraction η of $|E_x(\omega)|^2$, the observed spectral density of one orthogonal component of the electric field in the plane perpendicular to the geomagnetic field: $|E_L(\omega)|^2 = \eta |E_x(\omega)|^2$. Thus, we have

$$D_\perp \approx (\eta q^2/4m^2)|E_x(\omega = \Omega)|^2, \qquad (3)$$

where we have assumed that $|E_x(\omega)|^2$ is smooth enough near Ω that the small Doppler shift $k_\parallel v_\parallel$ can be neglected in evaluating $D_\perp$; to first approximation the diffusion coefficient is independent of velocity. The local heating rate, $2mD_\perp$, calculated from Eq. (3) agrees with the result obtained earlier[8] by a heuristic argument.

The diffusion coefficient $D_\perp$ is to be evaluated with use of electric field spectra observed simultaneously with the ion-velocity distribution measurements. Over the relevant range of frequencies, the observed electric field spectral density is well approximated by a power law: $|E_x(\omega)|^2 = |E_0|^2(\omega_0/\omega)^\alpha$. Because the spectral density is smaller at the gyrofrequencies of lighter-mass ion species, we expect that heavy species, such as oxygen, will be preferentially accelerated by this mechanism.[8] The survey of DE-1 observations of the low-frequency auroral-zone turbulence[5] indicates that $|E_0|^2$ is either roughly constant or grows with altitude. Following this observation, we assume that the magnitude and the spectral form of the left-hand polarized component of the electric field turbulence is constant over the ion-acceleration range. Although the velocity-diffusion rate is independent of velocity, it will depend on position through the variation of the ion gyrofrequency along the geomagnetic field line. Evaluating the spectral density at the ion gyrofrequency and taking the gyrofrequency to fall with the cube of the geocentric distance, $\Omega(s) \propto s^{-3}$, we find that the velocity-diffusion rate increases with altitude, $D_\perp \propto s^{3\alpha}$, in the relevant region of the geomagnetic field. It is this power-law form for $D_\perp$ which produces the interesting asymptotic properties of the solution of (2) for the ion-velocity distribution discussed below.

We will solve the kinetic equation for the ion-velocity distribution using a Monte Carlo model which was developed to investigate problems of this kind.[12] Because the number of accelerated ions is small (oxygen is a minority constituent of the plasma), we treat them as "test particles" in externally imposed fields, rather than calculating the fields self-consistently. (In addition, a self-consistent kinetic simulation of the microphysics could not practically describe the ion evolution over the distances of hundreds or thousands of kilometers that are relevant for the ion-conic phenomena.) From an initial distribution in velocity and space, the calculation of the evolution of the distribution proceeds by following the trajectories of a large number of ions with time. Between the velocity perturbations caused by interaction with the waves, it is assumed that the ions travel in the static geomagnetic field with constant energy and magnetic moment. The wave-particle interactions are taken into account by perturbing the ion velocities with random increments $\Delta\mathbf{v}$ such that $\langle \Delta\mathbf{v}\Delta\mathbf{v}\rangle = 2\mathbf{D}\Delta t$ where the time step is Δt, and $\mathbf{D}$ is the velocity-diffusion tensor.

To study the event observed at 23:46 UT on day 318, 1981, by DE-1, we follow the ion trajectories along a portion of a CPS geomagnetic field line at an invariant latitude of 60°, extending up to the geocentric altitude of the satellite, roughly $2.0R_E$. Because of the form of the velocity-diffusion rate, most of the ion acceleration observed will have occurred near the altitude of the observation point, and the results of the calculation are insensitive to the initial conditions chosen for the oxygen ions; we started the ions at $s = 1.2R_E$, thermally distributed with a temperature of 0.2 eV. Although the Monte Carlo model used the actual electric field spectral density measured by the PWI (plasma wave) instrument on DE-1, the fit of a power law gives a convenient means of characterizing the spectrum. The fitted power-law parameters are $|E_0|^2 = 1.2\times10^{-6}$ $(\mathrm{V/m})^2/\mathrm{Hz}$ at $\omega_0/2\pi = 5.6$ Hz, with $\alpha = 1.7$. (These fitted values differ slightly from the values adopted by Chang *et al.*[8]) Mean-particle calculations[8] indicate that $\eta = \frac{1}{8}$ will produce approximately the observed level of heating; this is the value of η used in the Monte Carlo calculations. As ions pass the observation point at $2.0R_E$, statistics on their velocities are accumulated to calculate the ion-velocity distribution at this point. Several velocity moments are also calculated from the particle velocities, to compare with the mean-particle calculations of Chang *et al.*[8]

The ion-velocity distribution, calculated as described above, is presented in the top panel of Fig. 1, in the same format as the satellite data to be discussed later. Positive $v_\parallel$ denotes ions that are traveling up the geomagnetic field line, a transverse velocity of 5 km/s was added to place the theoretical distribution in the satellite reference frame. The velocity distribution is plotted with the use of contours of constant phase-space density in the $v_\parallel$-$v_\perp$ plane, uniformly spaced with an increment of 0.4 in the logarithm of phase-space density between contours. Without a simultaneous measurement of the ion density in the source region, the absolute normalization of the Monte Carlo velocity distribution is arbitrary and has been chosen to match the level of the observed distribution. We do find that normalization of the calculated density to the observed density[13] (~ 10 cm^{-3}) yields a density of ~ 500 cm^{-3} at the source, which is typical of the oxygen-ion density at that altitude. In Fig. 1, we see that the theoretical velocity distribution indeed has the characteristic conic form, with the energetic ion flux peaking at pitch angles between 110° and 140°. The form of the velocity distribution is determined by the competition of the transverse acceleration and magnetic mirroring processes. For the case of a power-law heating rate considered here, the velocity distribution exhibits similarity scaling: One may obtain f at another altitude by scaling the velocity coordinates in magnitude by the power $(3\alpha+1)/3$ of the ratio of the geocentric altitudes.

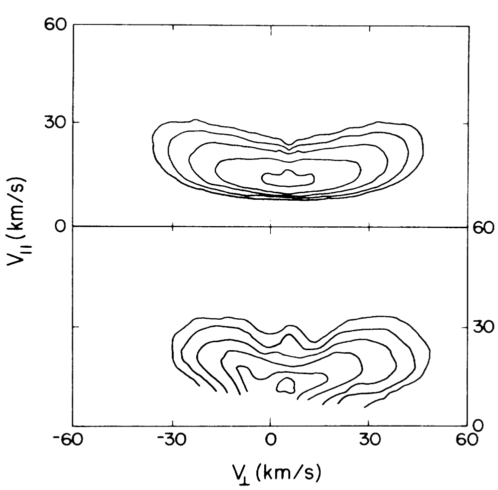

FIG. 1. The bottom panel presents a contour diagram of the observed ion-conic distribution function, measured by the HAPI instrument on DE-1, while the top panel presents our theoretical ion-velocity distribution, plotted in the same way as the observed conic distribution.

This fact is being pursued in an effort to obtain an analytical solution to the kinetic equation.[14]

Despite the absence of a parallel electric field in this calculation, noticeable parallel acceleration can be observed in Fig. 1. All transverse acceleration schemes must cause some parallel acceleration, because the effect of the mirror force is to convert the perpendicular energy gained into parallel energy. The fact that the velocity-distribution contours in this case resemble the hyperbolic curves that are a signature of the effect of a parallel electric field is a consequence of the simultaneous action of the magnetic mirror force and a heating process which is effective at all perpendicular velocities.[15] Because of this alternative means of parallel acceleration, evidence for parallel electric field acceleration of ion conics[16] must be interpreted carefully, at least for conics produced by this mechanism. Flatter spectra (i.e., smaller α) produce a wider spread in $v_\parallel$, while steeper spectra produce flatter velocity distributions, resulting from the greater dominance of local transverse heating in the latter case.

The mean-particle theory[8] predicts the values of the parallel and perpendicular energies as a function of geocentric distance by including the mean heating rate per ion in a set of equations which describe the motion of an ion guiding center in the geomagnetic field. Solutions of these equations[8] show that the ratio of perpendicular to parallel energies $W_\perp/W_\parallel$ rapidly approaches the constant value $(6\alpha+2)/9$. This behavior is consistent with the similarity scaling of the velocity distributions discussed above. In this limit, the equations for the ion energies can be easily integrated, providing results that are insensitive to the choice of initial conditions. The result for the total ion energy, $W=W_\parallel+W_\perp$, is

$$W(s)=(3\alpha+11/2)^{1/3}m[sD_\perp(s)/(3\alpha+1)]^{2/3}. \qquad (4)$$

The predictive power of the asymptotic mean-particle calculations is quite good. For the case modeled here, a Monte Carlo calculation using a power-law spectrum gives $W_\perp=40.7$ eV and $W_\parallel=28.5$ eV at $s=2.0R_E$, while the asymptotic mean-particle formulas give 40.1 and 30.8 eV, respectively. The small differences between these results can be attributed to the statistical error inherent in the Monte Carlo results and the asymptotic approximation in the mean-particle formulas.

The bottom panel of Fig. 1 contains a contour plot of phase-space density, from the particle observations made with the HAPI (high-altitude plasma) instrument on board DE-1. The velocity scales are drawn with the assumption that the ions are singly charged oxygen, as determined by the DE-1 EICS (energetic ion composition spectrometer) instrument.[13] In comparing the theory and the observations, we should note the following points. The contour plot of the observed velocity distribution is distorted by the analysis procedure for $v_\parallel < 8$ km/s, where no ion flux was observed. Along with the main ion-conic component of the velocity distribution, there is a superimposed, unresolved, field-aligned component. There is also a slight asymmetry between the two lobes of the observed conic, which is probably due to spatial and/or temporal variations along the satellite trajectory which we have not attempted to model. These considerations do not detract from the overall conclusion that the main ion-conic component of the observed ion-velocity distribution is well represented by the theoretical calculation, both in terms of the magnitude of the acceleration as well as the form of the distribution. This conclusion has been reinforced by the study of other observed conics,[17] in which equally good or better fits were obtained. Taking advantage of simultaneously observed wave-spectrum and ion-flux data, we have obtained the first (to our knowledge) successful comparison of an observed conic and a theoretical conic model.

Although the intense, low-frequency electric and magnetic field noise has been observed over the auroral zones for many years, the nature and origin of the turbulence are still not thoroughly understood and are the subjects of ongoing research. During the event which we have modeled, there were no obvious local sources of free energy for the turbulence; the field-aligned currents measured by the magnetometer and the plasma instrument on DE-1 were weak, and varied irregularly without correlation with variations in the ion conics.[18] The event did occur during a moderate magnetic storm (similar events have been observed during other magnetic storms), and the conics were observed equatorward of intense, high-energy electron precipitation, accompanied by ion beams, in the boundary plasma sheet. The polarization data for the low-frequency auroral-zone turbulence in the DE-1 survey appears to be consistent with a nonlocal source mechanism for the turbulence,[5] in which disturbances generated at higher altitudes propagate to low altitude as Alfvén waves.[19] Such waves could accelerate heavy ions as the waves approached gyroresonance after being reflected from the ionosphere. But the propagation of left-hand-polarized waves at frequencies near the ion gyrofrequencies, with the resulting possibility of mode-conversion phenomena,[20] remains largely unexplored in the suprauroral region. The uncertainty in the origin of the low-frequency turbulence does not alter the conclusion, based on the success of our modeling work, that this turbulence can explain the observed oxygen CPS conics.

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