# Charge Neutrality of Atoms and Magnetic Monopoles

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Under the assumption that charge conservation and charge quantization are exact, it is pointed out that the *expectation values* of the charges can still deviate from the exact quantized *eigenvalues*, as long as the charge superselection rule is not absolute. The deviations can be suppressed by a large energy scale, identified as the mass of the magnetic monopole. Existing experimental results are consistent with $\Delta Q_p + \Delta Q_e \approx 3\times 10^{-20}e$ and $\Delta Q_n \approx -3\times 10^{-20}e$, where $\Delta Q_i$ is the deviation for particle $i$. Implications, in particular $\Delta Q_{\nu_e} \approx 6\times 10^{-20}e$, are discussed.

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The history of experimental measurements of the electrical neutrality of matter goes back many decades, and so does the spectulation that there might be small deviations from strict neutrality.[1] The experimental situation regarding the proton-electron charge difference, $\Delta Q_p + \Delta Q_e$, where $\Delta Q_i$ denotes the deviation of the charge of particle $i$ from the quantized eigenvalues, was reviewed by Stover, Moran, and Trischka[2] in 1967. The upper bound on $|\Delta Q_p + \Delta Q_e|$, without the assumption of $\Delta Q_p + \Delta Q_e = \Delta Q_n$, was (1) from the gas-efflux method, $4\times 10^{-20}e$, (2) from the molecular-beam method, $36\times 10^{-18}e$, and (3) from the isolated-body method, $1\times 10^{-19}e$. Since then, the main improvements have come from quark searches.[3-5]

The motivation to question the validity of the neutrality of matter, in the face of such experimental accuracy, comes from an examination of its theoretical foundation. Strictly speaking, the proton-positron charge equality depends not only on the conservation and quantization of charge but also on the charge superselection rule. If the charge superselection rule is *not* absolute, then the proton and/or the positron may not be eigenstates of the charge operator. The *expectation values* of their charges can then be of different magnitudes, even though the *eigenvalues* of the charge operator are *exactly quantized.*

The conservation of charge and the quantization of charge have profound theoretical functions in gauge invariance and the compactness of the gauge group.[6] By contrast, the charge superselection rule was originally *postulated* in analogy to the superselection rule between states of integral spins and those of half-odd-integral spins.[7] The authors of Ref. 7 stated, "We are thus led to the postulate that multiplication of the state vector $F$ by the operator $e^{i\alpha Q}$ produces no physically observable modification of the state of a system of (mutually interacting) charge fields. We can give no conclusive evidence for this assertion, and such evidence may in fact depend on a deeper understanding of the meaning of electric charge which we still lack."

In order to examine the validity of the charge superselection rule, it is instructive to compare the charge operator $Q$ with $J_z$, the $z$ component of the total angular momentum operator. Both operators are conserved observables with quantized eigenvalues, but there is no superselection rule for $J_z$. In fact, $e^{i\alpha J_z}$ rotates a given state vector by an angle $\alpha$ around the $z$ direction.

Consider the following hypothetical situation: Imagine that there were only neutral particles with anomalous magnetic moments interacting with a superstrong uniform magnetic field in the $z$ direction. Let the energy of the interaction between the magnetic moments of all the particles and this strong field be $10^{20}$ times larger than any other energies in the system. The lowest energy states for all the particles are eigenstates of $J_z$. An observer unaware of the presence of the strong magnetic field might postulate that there was a "superselection rule for $J_z$." In this hypothetical case the "superselection rule of $J_z$" would be valid with an accuracy of $10^{-20}$. The possibility I would like to suggest, by the above example, is the following: There is actually *no* charge superselection rule. However, because all the deviations from such a rule are suppressed by an extremely large energy scale, we are led to the opposite conclusion. From this point of view, $e^{i\alpha Q}$, in complete analogy with $e^{i\alpha J_z}$, rotates a state vector by an angle $\alpha$ around the "charge" direction in some internal space and is in principle an observable.

I shall try to establish the case against the charge superselection rule, which invalidates the strict electrical neutrality of matter, in four steps: (1) I discuss the implications of the existence of a large energy scale which suppresses the deviation from the charge superselection rule. These implications are valid independent of the origin of the energy scale. (2) I suggest that the large energy scale is the mass of the magnetic monopole.[8,9] More explicitly, I show that a *real* monopole will induce violations of the charge superselection rule. But because the monopole mass is so large, only the violations induced by *virutal* monopole-antimonopole pairs are observable. By the uncertainty principle, the probability of the creation of a virtual monopole pair is inversely proportional to the monopole mass. Therefore the deviations are suppressed by the monopole mass. I then estimate the magnitude of the deviations from strict neutrality in this case. (3) I reexamine previous experiments[3-5,10] and find that they are consistent with small but *nonzero* values for $\Delta Q_p+\Delta Q_e$ and $\Delta Q_n$. (4) I discuss some of the ramifications of the values I obtained for $\Delta Q_p+\Delta Q_e$ and $\Delta Q_n$.

*Consequences of a large but finite energy scale which suppresses the violations of the charge superselection rule.*— Because charge is conserved, the commutator $[H,Q]=0$. Hence if $|m\rangle$ is an eigenstate of $H$ with mass eigenvalue $m$, then $Q|m\rangle$ is also an eigenstate of $H$ with exactly the same eigenvalue $m$. Now if $|m\rangle$ is not an eigenstate of charge, which is allowed because the charge superselection rule is violated, then $Q|m\rangle$ is linearly independent of $|m\rangle$. Therefore, if any particle is not an eigenstate of charge, then there must be at least one other particle with the same mass.

Because of $CPT$ invariance, the mass of every particle and its antiparticle are always degenerate. Hence, every charged particle by mixing with its antiparticle can, in general, have a charge expectation value different from the quantized eigenvalue. By the same token, if experimentally we find that the charge expectation value of a particle deviates from the quantized value and there is no degeneracy other than its antiparticle, then $CPT$ invariance must be exact. Conversely, if the charge superselection rule turns out to be exact after all, one possible explanation, although an unattractive one, is that $CPT$ invariance is violated.

Particles that are their own antiparticles will be exactly neutral if there is no other charged particle with the same mass. The only known massless particles are photons and gravitons (and gluons if color is absolutely conserved). Hence the charge expectation values of the photon and the graviton are exactly equal to zero. Since a particle and its antiparticle can annihilate into photons, the charge expectation value of a particle is exactly equal and opposite to that of its antiparticle.

If the charge expectation values of particles, in general, can be different from the quantized charge eigenvalues, it is possible that the lepton-number and baryon-number conservation follow from charge conservation.[11]

*Identification of the mass of the magnetic monopole as the energy scale for the violation of the charge superselection rule.*— It has been shown by Wilczek that in the presence of magnetic monopoles, the charge operator $Q$ is not a chiral singlet.[12] Since the charge operator does not commute with chiral rotation, at least one of the particle states with definite chirality will *not* be an eigenstate of charge, which violates the charge superselection rule. There are models involving monopoles that violate the charge superselection rule.[13,14]

In the usual grand unification schemes, the monopole mass is estimated to be $1/\alpha$ times the unification scale; then[15]

$$M \gtrsim 10^{16}\ \mathrm{GeV}/c^2.$$

If one expects all the interactions including gravity to be unified at the Planck mass, then a reasonable estimate is

$$M \approx (1/\alpha)(\hbar c/G)^{1/2} \approx 10^{21}\ \mathrm{GeV}/c^2.$$

Denote the probability of violation of the charge superselection rule for a particle of mass $m$ by $\epsilon^2$. The expected magnitude of $\epsilon^2$ depends on whether the particle is elementary or composite. (i) If it is an elementary particle, then either $\epsilon^2 \approx m/M$ or $\epsilon^2 \approx E_0/M$, where $E_0$ is some universal scale. (ii) If it is a composite particle, then either $\epsilon^2 \approx m_c/M$ or $\epsilon^2 \approx E_0/M$, where $m_c$ is the mass scale of the constituents. We expect $E_0$ to be approximately equal to or smaller than the electroweak unification scale of 100 $\mathrm{GeV}/c^2$. Above this latter scale, the charge direction is not singled out. If the nonzero violations of the superselection rule are of the same order of magnitude for all the particles, then we expect, for $M \approx 10^{21}\ \mathrm{GeV}/c^2$,

$$\epsilon^2 \lesssim 10^{-19}.$$

*Experimental evidence for the deviations from strict neutrality.*— In the experiment by King,[10] by substracting the average voltage curve of the helium runs from that of the hydrogen runs, he obtained the following estimates:

$$2(\Delta Q_p+\Delta Q_e) \approx (7\pm 2.5)\times 10^{-20}e,$$

$$2(\Delta Q_p+\Delta Q_e)+2\Delta Q_n \approx (0\pm 2)\times 10^{-20}e.$$

In the experiment by LaRue, Fairbank, and Hebard,[4] they parametrize the chargelike forces on the niobium balls as

$$F_a = q_r E_a - P_z\,\partial E_a/\partial z - R^3 E_a\,\partial E_F/\partial z,$$

where $E_a$ is the applied field, $\partial E_F/\partial z$ the fixed field gradient due to patch effects, $P_z$ the permanent dipole moment, $R^3E_a$ the induced dipole moment, and $q_r$ the residue charge. They do not allow the possibility of a volume charge $CR^3$, which is needed if there is a deviation from neutrality. However, in fitting $\partial E_F/\partial z$, they subtract out a constant and add it back with $q_r$:

$$\partial E_F^*/\partial z = \partial E_F/\partial z - C,\quad q_m^* = q_r + CR^3,$$

which exactly simulates a volume charge. This procedure is independent of whether $q_r = \pm e/3$ or 0. If we interpret $\partial E_F^*/\partial z$ as the true fixed field gradient, instead of $\partial E_F/\partial z$, then $CR^3$ gives the volume charge. There were three runs in this experiment (see Table I of Ref. 4). Run $C$ used four balls of the same size, and therefore one cannot extract reliably the corresponding $CR^3$. For run $A$, balls $2ANT$ and $3AWT$ with mass $\approx 9\times 10^{-5}$ g and $R=0.014$ cm give $CR^3 = q_m^* - q_r \approx -0.15e$, with an error of about $0.05e$. For run $B$, balls $3BWT$ and $5BNT$ give $CR^3 \approx -0.25e$, with an error about $0.1e$. Using the average value $CR^3 \approx -0.2e$, we find[16]

$$41(\Delta Q_p+\Delta Q_e)+52\Delta Q_n \approx -3\times 10^{-19}e.$$

If we use King's result on helium, $(\Delta Q_p + \Delta Q_e) + \Delta Q_n \approx 0$, we get

$$\Delta Q_p + \Delta Q_e \approx 3\times 10^{-20}e.$$

This value is consistent with King's result on hydrogen,

$$2(\Delta Q_p + \Delta Q_e) \approx (7\pm 2.5)\times 10^{-20}e.$$

In the experiment by Marinelli and Morpurgo,[3] the largest steel ball used had $R\approx 0.3$ mm and $m\approx 1.1\times 10^{-4}$ g, and the residue charge after elimination of a subtle effect explained by Buckingham and Herring[17] was found to be $q' = -(0.15\pm 0.05)e$. For a smaller ball with radius 0.2 mm, $q' < -0.05e$.[18] The values of $q'$ include possible contributions from the patch effect. The expected value of the volume charge on the larger ball, with the values of $\Delta Q_i$ obtained above, is $q_v \approx -0.14e$, and for the smaller ball $q_v \approx -0.04e$, consistent with the values of $q'$ obtained in this experiment.

*Further consequences.*— From charge conservation in $\beta$ decay, and using $\Delta Q_{\nu_e} = -\Delta Q_{\bar{\nu}_e}$, I conclude that the expectation value of the electron neutrino charge is

$$\Delta Q_{\nu_e} \approx 6\times 10^{-20}e.$$

There must then be a charged particle with exactly the same mass as the neutrino. A possible candidate is the $u$ quark. The possibility that the $u$ quark can have a very small mass, actually even a zero mass, was pointed out by Kaplan and Manohar, if one includes the second-order effect in chrial-symmetry breaking.[19]

With a nonzero charge for $\nu_e$, we distinguish two possibilities. (i) If $\nu_e$ is a massive Dirac particle, then the present limit on the neutrino magnetic moment[20] of $\mu_{\nu_e} \le 1.4\times 10^{-9}\mu_B$ gives a lower limit on the mass of $\nu_e$:

$$m_{\nu_e} \ge 4\times 10^{-5}\,\text{eV}.$$

Conversely, the upper limit[21] $m_{\nu_e} \le 18$ eV gives a lower limit of $\mu_{\nu_e} \ge 10^{-15}\mu_B$ on $\mu_{\nu_e}$. The magnitude of $\mu_{\nu_e}$ may be large enough to solve the solar neutrino puzzle.[22] (ii) The other possibility is that the neutrino mass is strictly zero. In this case, the magnetic moment is also strictly zero.[23,24] If the $u$ quark is degenerate with $\nu_e$, then it will also be massless, which ensures that the strong interaction conserves *CP*.

The fact that $\Delta Q_{\nu_e}$, $\Delta Q_n$, $\Delta Q_p$, and $\Delta Q_e$ may all be experimentally of the same order of magnitude suggests that a universal ratio may apply to all particles such that

$$\epsilon^2 \approx E_0/M \approx 10^{-19}\text{–}10^{-20}.$$

If one takes $E_0$ to be the electroweak unification scale, $E_0 \approx 100\,\text{GeV}/c^2$, then

$$M \approx 10^{21}\text{–}10^{22}\,\text{GeV/c}^2$$

which is consistent with grand unification at the Planck mass.

Deviations from strict charge neutrality at the level of $10^{-20}$ suggest that there may also be deviations from strict color neutrality at the same level. It also means that color confinement may not be absolute. If this is the case, one expects to see fractional charge at the level of about one in $10^{20}$ nucleons,[4,5] because additional quarks may be needed to neutralize the excess in color.

The experimental values of $\Delta Q_i$ suggest that $\Delta Q_p = \Delta Q_n \approx -3\times 10^{-20}e$ and $\Delta Q_e = \Delta Q_{\nu_e} \approx 6\times 10^{-20}e$. These two and similar equalities, together with charge conservation, may make it unnecessary to invoke lepton-number conservation and baryon-number conservation.

These and other consequences of the deviations from strict charge neutrality will be discussed in detail in subsequent communications.

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*Note added.*— It should be emphasized that the possibility suggested here is the following: If experimentally $\Delta Q_i$ turns out to be small but nonzero, it does *not* necessarily imply that charge conservation or charge quantization is violated. Violation of the charge superselection rule may be such a possibility, where charge quantization is exact. At the same time, charge conservation together with a nonzero $\Delta Q_p + \Delta Q_e$ can still ensure the stability of the proton, without the assumption of an independent baryon-number conservation law, provided that the Lagrangean is locally gauge invariant. Local gauge invariance is essential in order to maintain the path independence of the phase of the proton state vector in the presence of nonvanishing electromagnetic field strength, and the orthogonality between the proton and the antiproton state vectors at all space-time points, when charge superselection rule is assumed to be not absolute. Similar considerations may apply to lepton-number conservation laws.

It has been pointed out to me that E. Strocchi and A. S. Wightman[25] showed that every quasilocal operator of a bounded region commutes with $Q$ because of the Gauss theorem. If $\gamma_5$ does not commute with $Q$ in the presence of a magnetic monopole,[12] then $\gamma_5$ must not satisfy their definition of a quasilocal operator. Whether this is possible can be tested experimentally. According to Case[23] the electron neutrino $\nu_e$ can be a two-component field, say, strictly left handed and an eigenstate of $\gamma_5$, but still have a nonzero mass. But if the left-hand $\nu_e$ has a nonzero charge, then Case showed that the mass must be zero. If $\nu_e$ has a small charge, it cannot be an eigenstate of $Q$, and the superselection rule is violated. In a double-$\beta$-decay experiment, if a neutrinoless double-$\beta$-decay event is observed, then the neutrino has a mass and no charge. On the other hand, if no such event is observed, then it is caused by the nonzero charge or zero mass of $\nu_e$ as long as one does not assume a lepton-number conservation law independent of charge conservation. If the charge of the neutrino is not zero, then $\gamma_5$ does not commute with $Q$.

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[16]To confirm or reject these values experimentally, one may consider using many balls of different but precisely measured radii. The ideal maximum radius is $R \approx 0.035$ cm, giving $CR^3 \approx -0.5e$. One should also use different materials so that one can unambiguously differentiate between the true volume charge coming from the deviations of strict neutrality and the patch effect.

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