Effect of Repulsive and Attractive Scattering Centers on the Magnetotransport Properties of a Two-Dimensional Electron Gas

R. J. Haug, R. R. Gerhardts, K. v. Klitzing, and K. Ploog

Max-Planck-Institut für Festkörperforschung, D-7000 Stuttgart 80, Federal Republic of Germany

(Received 15 June 1987)

GaAs/AlGaAs heterostructures systematically doped with additional Be or Si impurities near the twodimensional electron gas show distinct shifts of the quantum Hall plateaus and of the minima in the longitudinal resistivity ρ_{xx} relative to the expected values of the Landau-level filling factor. A quantum Hall plateau $\rho_{xy} = h/e^2$ which does *not* cross the classical straight line $\rho_{xy}^0 = B/n_s e$ is observed. The results are explained by a microscopic transport calculation emphasizing non-Born scattering of electrons by individual impurities.

PACS numbers: 72.20.My, 72.10.Fk, 73.40.Lq

Low-temperature transport properties of semiconductors depend strongly on the impurities in the materials. The highest low-temperature mobility in GaAs/AlGaAs heterojunctions is nowadays limited by the lowest density of residual impurities achievable in the GaAs. The presence of impurities is also important for the quantum Hall effect $(QHE)^1$ and the fractional QHE^2 observed in a two-dimensional electron gas (2DEG) at low temperatures, since the width of the plateaus depends strongly on the impurity concentration.³ For precision measurements of the QHE, it is important to know where in the plateau region the Hall resistance has the exact quantized value, and this also depends on charge and distribution of the impurities. For most of the samples, little is known about the actual distribution of the impurities and about their influence on density of states and transport properties of the 2DEG. For Si metal-oxide-semiconductor field-effect transistors, Furneaux and Reinecke⁴ investigated the effects of driftable Na⁺ ions in the oxide on the width and the position of the Hall plateaus and interpreted their results in terms of an asymmetric distribution of localized states in the tails of overlapping Landau levels.

In the present Letter we report the first systematic investigation of the influence of well-defined charged impurities on the magnetotransport properties of the 2DEG in GaAs/AlGaAs heterostructures, which in recent years have become the model system for studies of the QHE and especially of the fractional QHE. Since these heterostructures are grown by molecular-beam epitaxy (MBE), it is possible to dope with different types of atoms at an arbitrary stage of the growth process and thus to introduce either positively or negatively charged impurities at arbitrary, well-defined distances from the 2DEG. This opens the possibility to vary the effective strength of the interaction between the 2D electrons and the charged impurities in a wide range.

We have prepared and studied two types of intentionally doped samples. The first contained additional Si atoms in the GaAs, which act as donors; the second type, Be atoms, which act as acceptors. The different impurities are found to have distinct and opposite effects on the resistivity components measured as functions of the magnetic field B. Because of the presence of the charged impurities, the plateaus of the Hall resistivity $\rho_{xy}(B)$ do not occur symmetrically with respect to the classical free-electron result $\rho_{xy}^0 = B/en_s$, where e is the elementary charge and n_s is the area density of the 2DEG. For the donor-dominated samples, the plateaus are shifted towards lower magnetic field values, i.e., larger values of the filling factor $v = n_s h/eB$, whereas the Be-doped samples exhibit the plateaus at smaller filling factors. We are able to explain all these experimental results qualitatively by a microscopic transport calculation based on the so-called single-site approximation^{5,6} or selfconsistent T-matrix approximation (STMA).^{7,8} In the samples with repulsive Be - scatterers, we observe for filling factors near v=1 a Hall plateau which does not cross the classical straight line $\rho_{xy}^0 = B/en_s$. From our model calculation, we understand this effect, which to the best of our knowledge has never been reported before, as a result of an overlap of localized states belonging to the lowest Landau and spin level with extended states of the same Landau level and opposite spin.

Our samples were prepared from modulation-doped GaAs/AlGaAs heterostructures grown by MBE. On top of 2-µm nominally undoped GaAs, a 21-nm undoped Al-GaAs spacer layer was grown followed by 40 nm of Sidoped AlGaAs. During the growth process either Si or Be impurities were introduced in the GaAs at a fixed distance (2 nm) from the interface with the method of δ doping.⁹ Four heterojunctions with different area densities of additional Si impurities were grown. Before and after these δ -doped heterostructures, undoped reference structures were grown which had nearly the same carrier concentration $n_s = 3.0 \times 10^{11}$ cm⁻² and mobility $\mu = 5$ $\times 10^5$ cm²/V·s at a temperature of T = 4 K. The rather high mobility shows that the density of residual impurities in the GaAs is very small. The same procedure was applied to produce δ -doped samples with Be impurities. Here the reference structures had $n_s = 2.5 \times 10^{11}$ cm⁻² and $\mu = 3.5 \times 10^5$ cm²/V·s. The mobilities of the δ doped heterostructures were drastically lower than those of the reference structures, whereas the carrier concentrations changed only slightly. An additional concentration of 3×10^9 impurities/cm² reduced the mobility by a factor of 5 without changing n_s noticeably. The highest impurity concentration of 4×10^{10} cm⁻² reduced the mobility to $\mu = 3 \times 10^4$ cm²/V·s and changed the carrier concentration by 10%.

Hall-bridge specimens with a width of 100 μ m have been used to measure the longitudinal resistivity ρ_{xx} and the Hall resistance ρ_{xy} for different temperatures down to T = 1.4 K in a magnetic field perpendicular to the 2DEG. Figure 1 shows results for a δ -doped sample with a Be impurity concentration of about 4×10^{10} cm^{-2} . It is very remarkable that the plateau of the Hall resistance with the value $h/e^2 = 25813 \Omega$ does not cross the line for the classical Hall resistance $\rho_{xy}^0 = B/n_s e$ and that the plateau appears in the filling-factor range 0.65 < v < 0.95. At higher temperatures (20 K < T < 90 K), where the QHE is no longer observed, ρ_{xy} gets closer to the classical value. In samples with Si impurities a shift of ρ_{xy} with respect to the classical value was observed in the opposite direction (Fig. 2). This effect is more pronounced when a negative voltage at a backside gate is applied in order to push the wave function closer to the Si layer.¹⁰ Asymmetric structures are also observed in the ρ_{xx} data, with extrema shifted to smaller filling factors for the Be-doped sample (Fig. 1), and to larger filling factors for the Si-doped sample (Fig. 2).



FIG. 1. Measured resistivities ρ_{xx} and ρ_{xy} as functions of the magnetic field *B* for a sample $(n_s = 2.1 \times 10^{11} \text{ cm}^{-2}, \mu = 30000 \text{ cm}^2/\text{V} \cdot \text{s})$ with additional Be impurities $(n_i = 4 \times 10^{10} \text{ cm}^{-2})$. The thin straight line indicates $\rho_{xy}^0 = B/en_s$, with n_s calculated from the ρ_{xx} minima for v > 2.

For the samples with a lower concentration of additional impurities, these shifts are systematically less pronounced.

In our model calculations, we simulate Coulomb potentials by two-dimensional δ potentials of appropriately adjusted scattering strength. For repulsive δ potentials of arbitrary concentration exact results on the density of states (DOS) in the lowest Landau level were obtained by Brézin, Gross, and Itzykson.¹¹ For sufficiently small concentration, a fraction of the states shifts to higher energies and forms an impurity band with exponential-type tails.¹¹ All the splitoff states are believed to be localized, and this seems to be supported by numerical calculations for small systems.¹² One expects that the localized states in the impurity-band regime do not contribute to transport phenomena and give rise to the QHE: highprecision plateaus of ρ_{xy} accompanied with vanishing ρ_{xx} . There is, however, no reliable magnetotransport theory which can treat localization effects and yield explicit results to compare with experiment.

Since the observed shifts persist at higher temperatures, we believe that the essential features of our experiments can be understood without an exact treatment of localization. Thus, we have performed model calculations within the STMA,⁷ which neglects the coherent multicenter scattering processes leading presumably to localization.¹³ The interaction of an electron with an individual impurity, which seems to be important for the observed differences between donor-doped and acceptordoped samples, is, however, treated exactly in the STMA. The Kubo formulas for the conductivity tensor can be written in the form

$$\sigma_{ij} = \int \sigma_{ij}(E)(-df/dE)dE, \qquad (1)$$



FIG. 2. Same as in Fig. 1, but for a sample $(n_s = 3.4 \times 10^{11} \text{ cm}^{-2}, \mu = 30\,000 \text{ cm}^2/\text{V} \cdot \text{s})$ with additional Si impurities $(n_i = 4 \times 10^{10} \text{ cm}^{-2})$.

where f(E) is the Fermi function, and $\sigma_{ii}(E)$ can be expressed in terms of a single-particle self-energy $\Sigma(E)$ and scattering amplitudes $t_{\mu}(E)$,⁷ which are related by $\Sigma(E) = \sum n_{\mu} t_{\mu}(E)$, with n_{μ} the area density of the μ th type of δ potentials. The scattering amplitude $t_{\mu}(E)$ is self-consistently determined by an equation which contains $\Sigma(E)$ and thus the effect of other impurities in an effective-medium approximation. For sufficiently low impurity concentrations, the STMA yields isolated impurity bands, 5-8,14 which for attractive (repulsive) impurity potentials split off on the low- (high-) energy side of the Landau levels and lead to an asymmetric DOS. In the gaps between the impurity bands and the main Landau levels, the diagonal conductivity $\sigma_{xx}(E)$ vanishes and the Hall conductivity assumes the quantized values $\sigma_{yx}(E) = Ne^2/h$ if the energy gap is located between the (N-1)th and the Nth Landau levels.⁶

For practical calculations, we use five types of δ potentials. Two of these model the donors in the doped Al-GaAs and two the residual acceptors in the GaAs. These impurities lead to a small acceptor band on the highenergy side of each Landau level and to a shift and broadening of the main Landau levels. The δ doping is taken into account by a plane of repulsive (Be) or attractive (Si) potentials with density $n_i = 2 \times 10^{10}$ cm⁻², which leads to an impurity band on the high- or the lowenergy side of each Landau level, respectively.

In Figs. 3 and 4 we show calculated resistivities versus

magnetic field for zero temperature and $n_s = 2.5 \times 10^{11}$ cm^{-2} . To facilitate comparison with experiments, we have included the enhancement of spin splitting by many-body effects¹⁵ in an approximate,¹⁶ but selfconsistent, calculation using the DOS from the STMA as an input. As a result, the spin splitting of the main Landau levels, which are calculated as small peaks with high DOS, is resolved, whereas the broad impurity bands overlap with the main Landau level or impurity bands of the other spin direction. At the Fermi level states with opposite spins from impurity bands and the main Landau levels may occur simultaneously, as indicated by the hatching in the lower parts of Figs. 3 and 4. The detailed structure of the $\rho_{xx}(B)$ and $\rho_{xy}(B)$ curves depends on the exact position of the impurity-band edges and is of little interest since, first, an exact calculation probably would yield no energy gaps but localized states in the impurity-band region, and, second, these localized states would not contribute to the transport coefficients.

For the interpretation of the STMA calculations, we thus ignore details and take the position of the impurity bands as a rough indication of where most of the localized states must be expected. In the cross-hatched regions of Figs. 3 and 4 we expect, e.g., only localized states and thus quantized Hall plateaus and vanishing ρ_{xx} . Because of our choice of model parameters, different Landau levels do not overlap and $\rho_{xy}(B)$ coincides with the classical straight line at filling factors v=2 and 4 (B=5.16 and 2.58 T). For acceptor-dominated, i.e., Be-doped, samples, localized states occur predominantly on the high-energy sides of the classi-



FIG. 3. Calculated resistivities in units of h/e^2 vs magnetic field for a model sample $(n_s = 2.5 \times 10^{11} \text{ cm}^{-2})$ with additional repulsive potentials $(n_i = 2 \times 10^{10} \text{ cm}^{-2})$ at T = 0 K. The broken line indicates ρ_{xy}^0 . The hatching /// (\\) indicates that impurity states of the lower (higher) spin level are at the Fermi energy.



FIG. 4. Same as in Fig. 3, but for a sample with additional attractive potentials $(n_i = 2 \times 10^{10} \text{ cm}^{-2})$.

cal straight line (Fig. 3). Near v=1 (B=10.3 T) extended states in the main Landau level of the higher spin energy overlap with localized states of the lowest spin level. As a consequence, the plateau occurs for v < 1and does not extend to the classical straight line which assumes the plateau value at v=1. Opposite shifts are obtained for donor-dominated samples (Fig. 4). The minima of $\rho_{xx}(B)$ in Figs. 3 and 4 are shifted in accordance with the corresponding Hall plateaus near v=1. All these results of the calculation are in nice agreement with the essential features of the experimental findings. The calculations for donor doping (Fig. 4) indicate that for filling factors well below 1, localized states of the "wrong" spin direction may be occupied, if the lowest spin level has a small region with high density of extended states. This will not happen for pure acceptor doping and may have interesting consequences. For higher temperatures where no QHE is observed, the asymmetrical shifts of the $\rho_{xv}(B)$ curves persist for both the measured and the calculated curves, which we do not show here. This indicates that the shifts and asymmetries have an origin independent of the localization, and supports our interpretation of the STMA results. A calculation based on a Gaussian random potential,¹¹ which corresponds to the Born approximation for individual impurity scattering, would, on the other hand, not be able to explain the asymmetries, even if localization effects could be included properly.

The asymmetries discussed in this Letter will not occur if the concentration of the relevant scatterers is very high¹⁴ or if these scatterers are very far away from the 2DEG. In both cases, the effective potential seen by the electron will fluctuate more or less symmetrically around its mean value and the Landau levels will be broadened nearly symmetrically, although around a shifted energy.

In conclusion, we have shown that in GaAs/AlGaAs heterostructures the additional doping of the GaAs near

the 2DEG with donor or acceptor impurities shifts the position of quantum Hall plateaus and of the corresponding ρ_{xx} minima to opposite directions. We understand these shifts as a consequence of strong, non-Born scattering of electrons by individual impurities.

We gratefully acknowledge the expert help of M. Hauser with the molecular-beam-epitaxy growth and the financial support of the Bundesministerium für Forschung und Technologie, West Germany (Grant No. NT-2718-C).

 1 K. v. Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980).

²D. C. Tsui, H. L. Störmer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).

³A. M. Chang, in *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, New York, 1987), p. 175.

⁴J. E. Furneaux and T. L. Reinecke, Phys. Rev. B 33, 6897 (1986).

⁵T. Ando, J. Phys. Soc. Jpn. **36**, 1521 (1974).

⁶T. Ando, Y. Matsumoto, and Y. Uemura, J. Phys. Soc. Jpn. **39**, 279 (1975).

⁷R. R. Gerhardts, Z. Phys. B 22, 327 (1975).

⁸W. Wiertz and R. R. Gerhardts, Z. Phys. B 25, 19 (1976).

⁹K. Ploog, J. Cryst. Growth **81**, 304 (1987).

¹⁰R. J. Haug, K. v. Klitzing, and K. Ploog, Phys. Rev. B **35**, 5933 (1987).

¹¹E. Brézin, D. J. Gross, and C. Itzykson, Nucl. Phys. **B235** [FS11], 24 (1984).

¹²T. Ando, J. Phys. Soc. Jpn. **53**, 3126 (1984).

¹³Y. Ono, J. Phys. Soc. Jpn. **51**, 3544 (1982), and **53**, 2342 (1984).

¹⁴G. Czycholl, Solid State Commun. **49**, 203 (1984).

¹⁵T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).

¹⁶Th. Englert, D. C. Tsui, A. C. Gossard, and Ch. Uihlein, Surf. Sci. **113**, 295 (1982).