Pion Double Charge Exchange and the Nuclear Shell Model

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The theory of analog pion double charge exchange on semimagic even-even nuclei is examined from the point of view of the nuclear shell model. It is shown that the amplitude for the reaction can be expressed as the sum of two independent functions for an entire shell. A more general relation is also discussed which is true for any shell in which the generalized seniority scheme is valid.

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The pion-nucleus double-charge-exchange (DCX) reaction has been recognized for many years as a useful tool for studying nucleon-nucleon correlations because of its nature of inherently involving (at least) two nucleons. Early calculations neglected any such correlations and, while they were reasonably successful at high pion energies (above the 3-3 resonance), they were unable to explain the data in the (3,3) region.[1]

There are at present several outstanding problems. The one most directly addressed by this Letter is the variation of the DCX cross section to the double isobaric analog state (DIAS) with the number of excess neutrons in a given shell. The most naive expectation based on the assumption that the nuclear ground-state wave function describes a state of independent particles is that the cross section should vary as the total number of excess neutron pairs in the shell; $n(n-1)/2$, where $n \equiv N-Z$. Measurements of DCX for the calcium isotopes in the resonance region showed this not to be the case, with the $^{48}Ca/^{42}Ca$ ratio being $\simeq 6$ instead of the 28 expected and the $^{44}Ca/^{42}Ca$ ratio being about 1 instead of the 6 expected.[2] The analysis of this phenomenon in the resonance region is complicated by the fact that distortion effects are very important, multinucleon processes may be important, and even multi-Δ processes may play a role.[3]

Recent data at low energy have shown that this expectation is even more strongly violated at low energies, where these uncertainties in the reaction mechanisms are expected to be much less severe. This leads us to believe that the answer to the problem lies in the nuclear structure. Indeed, recently Bleszynsky and Glauber[4] have shown that there are significant shell-model corrections for DCX on ^{48}Ca. The principal result reported here is an expression for the cross section for DCX to the DIAS as a function of the number of excess neutrons for any shell-model orbit j, under the assumption that only two-body processes are important. The remarkable feature of the expression is that no more than two amplitudes are involved regardless of the size of the shell. One of these amplitudes corresponds to monopole transitions through intermediate nuclear states and alone would give the same result as previously calculated.[1] This is the term to survive in the limit of an independent-particle wave function. The second amplitude vanishes in the uncorrelated limit and so we identify it as a measure of the correlated part of the shell-model wave function.

Another problem in pion DCX is that of the angular distribution in the resonance region. There, a two-amplitude model has been suggested as a solution to the problem of the minimum which is observed experimentally to occur at too small an angle to be a purely diffractive phenomenon.[5] We note that the two amplitudes which naturally arise in the present work provide such a system. The amplitude corresponding to uncorrelated nucleons is the same in the present work and in Ref. 5. The second amplitude is assigned there to the core, whereas our second amplitude comes purely from transitions within the valence neutrons.

In order to obtain the desired relation, we consider the reaction to be caused by a two-body operator between two identical (in the same isospin multiplet) 0^+ states. Thus we need to calculate the matrix element for the transition:

$$M = \langle 0^+ | \textstyle\sum_{i,j} \theta_{ij}(\mathbf{r}_i, \mathbf{r}_j, \mathbf{k}, \mathbf{k}') | 0^+ \rangle. \quad (1)$$

The $|0^+\rangle$ denotes the ground state. *The two-body transition operator* θ_{12} contains, of course, all the dependence of the reaction mechanism and is a function of energy and momenta of the incoming $\mathbf{k}$ and outgoing $\mathbf{k}'$ pion waves, in addition to the dependence on the coordinates of the two nucleons involved in the charge-exchange process. The θ_{12} operator is, in general, complex. In the framework of the shell model, one can write matrix elements for a many-body wave function in terms of two-body matrix elements if the operator is a sum of two-body terms. These shell-model techniques were applied extensively in the calculation of energy levels and are well known.[6] We shall apply some of these techniques to the DCX.

Take the simple case when the shell-model state is described by a single j^n configuration of identical particles coupled to total angular momentum $J=0^+$ and seniority $v=0$. In this case, the diagonal matrix element (for even n) is given by[6]

$$\langle j^n, v=0, J=0^+ | \sum \theta_{ij} | j^n, v=0, J=0^+ \rangle = \tfrac{1}{2} n(n-1)\alpha + \tfrac{1}{2} n\beta, \quad (2)$$

where α and β are constants independent of n. This strikingly simple formula holds for any two-body $\sum \theta_{ij}$ interaction. The DCX transition operator is not necessarily, of course, a scalar in space; however, for matrix elements involving a transition from a $J=0^+$ to $J=0^+$ state only the scalar part of θ_{12} survives and one can use Eq. (2) to calculate the DCX. In calculating the DCX cross section, one has to take into account the fact that we go from the ground state (g.s.) to the DIAS, i.e., from $T_Z=T$ to $T_Z=T-2$. This introduces an additional factor $[n(n-1)/2]^{-1/2}$ in the amplitude. The DCX cross section is then

$$\sigma_{\rm DCX}(\theta) = \tfrac{1}{2} n(n-1) | \alpha + \beta/(n-1) |^2. \quad (3)$$

This formula is remarkable in its simplicity. The DCX cross section to the DIAS contains now two, and in the case of a pure j^n configuration only two, amplitudes. The first one represents transitions which occur in the absence of shell-model correlations in the nuclear wave function. The cross section due to this term is simply proportional to the number of neutron pairs to be made into pairs of protons when going from the ground state to the DIAS. This counting rule is independent of the mutual location of the two nucleons in each pair, and therefore this term will dominate the cross section if the transition DCX operator is of long range. In the case of a transition operator which is constant (over the volume of the nucleus), only this term will contribute. (Note, however, that a short-range interaction will contribute also to the α term.) *The second term, proportional to β, is a new term that did not receive much attention in the past.* It represents DCX transitions which take place when the nuclear wave function is more than just that of independent particles. The shell-model state in the seniority scheme has a correlated wave function formed by the combination of pairs of nucleons coupled to $J=0^+$. Each time such a pair is added, one has to antisymmetrize the wave function. The β term will contribute when the DCX transition operator is of short range. By examining Eq. (3), one sees that the relative contribution of the pairing term is largest when n is the smallest, i.e., when $n=2$ or, equivalently, $T=1$. As one increases the number of excess neutrons, the field effects, represented by the parameter α, become more important in comparison with the two-body pairing term represented by $\beta/(n-1)$. If one prefers to discuss the DCX reaction in terms of a sequential process, the term proportional to α corresponds to the transition in which the intermediate state is the single isobaric analog state (IAS), while the second term in Eq. (3) would correspond in the case of sequential transitions to processes in which the intermediate states are nonanalog states.

The expression in Eq. (3) can explain a substantial amount of experimental findings which seem puzzling. Before proceeding with the application of this two-amplitude expression, we will rewrite it in a different form so as to be able to compute the coefficients α and β in terms of the radial wave function of the orbit j. With use of standard techniques of the shell model,[6] one may for a single j^n configuration write the matrix element in Eq. (1) as

$$\langle j^n, v=0, J=0^+ | \sum \theta_{ij} | j^n v=0, J=0^+ \rangle = \sum_{L=0}^{2l} f_L F_L(\mathbf{k},\mathbf{k}'), \quad (4)$$

where

$$F_L(\mathbf{k},\mathbf{k}') = \frac{(2l+1)(2j+1)}{4\pi} \left[\begin{Bmatrix} L & l & l \\ \tfrac{1}{2} & j & j \end{Bmatrix} C^{lLl}_{000} \right]^2 \int d^3r_1\, d^3r_2 \sum Y_L^M(\hat{\mathbf{r}}_1) Y_L^{M*}(\hat{\mathbf{r}}_2) \phi_l^2(r_1) \phi_l^2(r_2) \theta_{12}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{k},\mathbf{k}'),$$

where ϕ is the radial wave function of the j orbit. The constants f_L are composed of $3j$, $6j$, and fractional parentage coefficients. Not surprisingly, in view of Eq. (2), one finds after performing all the algebra that only two combinations of F_L appear in the above matrix elements for all the seniority-zero j^n ($n=2,4,\ldots$) states. These are the monopole F_0 (we will denote it as A) and the *sum of all higher multipoles* with unit coefficient, $B=\sum_{L>0} F_L$. One can connect Eqs. (3) and (4) by making the identification

$$A = \alpha + [2/(2j+1)]\beta, \quad B = [(2j-1)/(2j+1)]\beta, \quad (5)$$

and arrive at a closed expression for the DCX cross section to the DIAS for the case of a pure j^n configuration of identical particles and lowest seniority $v=0$:

$$\sigma_{\rm DCX}(\theta) = \frac{n(n-1)}{2} \left| A + \frac{(2j+3-2n)}{(n-1)(2j-1)} B \right|^2, \quad (6)$$

where A and B are, in general, complex and dependent on the scattering angle θ and the pion energy. This is the central result of our work. This formula is more general than its derivation and holds also for spin-dependent interactions, but then A and B are not given anymore by Eq. (4). We deal therefore with a three-parameter problem, the absolute values $|A|$ and $|B|$, and a phase ϕ between them.

In Table I, expressions are written for the DXC cross section for ^{42}Ca, ^{44}Ca, ^{46}Ca, and ^{48}Ca. One notes the following properties of Eq. (6) and the expressions in Table I. The contribution of the correlation term B is the largest in the $n=2$ (i.e., $T=1$) nuclei. As one goes to the heavier Ca isotopes, one finds that the contribution of the term declines and the $(N-Z)$ $(N-Z-1)$ behavior is violated in these nuclei to a lesser extent. This fact explains why the $T=1$ nuclei had the unexpectedly large value of $\sigma_{\rm DCX}$, at low energy.[7] Explicit calculations[8] of $|A|$ and $|B|$ for energies $T_\pi \simeq 35$ MeV show that the ratio $|B|/|A| \simeq 3.9$. For energies around and immediately above the resonance this ratio declines to about 1. This behavior of the ratio of $|B|/|A|$ can be presently understood[8] theoretically in terms of the behavior of the πN amplitudes. A fit to the forward-angle, low-energy (π^+,π^-) data[9] for ^{42}Ca, ^{44}Ca, and ^{48}Ca, with the expression in Table I, gives $|A|=0.33$ $(\mu\text{b/sr})^{1/2}$, $|B|=1.25$ $(\mu\text{b/sr})^{1/2}$, and $\phi=66°$. This compares reasonably well with the impulse-approximation computations of Ref. 8, where after using Eq. (4) one finds $|A|=0.2$ $(\mu\text{b/sr})^{1/2}$, $|B|=0.8$ $(\mu\text{b/sr})^{1/2}$, and $\phi=71°$.

The fact that the expression in Eq. (6) contains two amplitudes leads to several coherence effects observed experimentally. For example, shifts in the angular distributions of $\sigma_{\rm DCX}$ with respect to the predictions of an IAS-dominated sequential model are due to the interference between the A and B amplitudes. We present two predictions concerning such effects. The first deals with the problem of the minimum in the angular distribution mentioned before. If in ^{42}Ca (and other nuclei) the forward position of the first minimum[2] is caused by the interference of the amplitudes A and B defined here, then the effect should be much smaller in the rest of the shell because the contribution of B is much less. In fact, the minimum in ^{48}Ca would be shifted to *larger* angles relative to ^{42}Ca as in the predictions of the model that involves only the A term. The second general prediction follows from the observation in the table that only the two-neutron case in ^{42}Ca has the full strength for the B term. Thus, for the neutron numbers larger than two [especially for the larger values of j as seen in Eq. (6)], the characteristics of A should dominate. Since these have been calculated many times[10] we know that these

TABLE I. DCX cross sections for the even Ca isotopes that arise from the formula in Eq. (6) in the text.

Nucleus	$\sigma_{\rm DCX}$
^{42}Ca	$\lvert A+B\rvert^2$
^{44}Ca	$6\lvert A+\frac{1}{9}B\rvert^2$
^{46}Ca	$15\lvert A-\frac{1}{15}B\rvert^2$
^{48}Ca	$28\lvert A-\frac{1}{7}B\rvert^2$

cross sections should show a monotonic rise above 50 MeV, contrary to what is observed for the $T=1$ DCX reaction. In fact, there has been one $T>1$ reaction studied[11] (^{56}Fe), and the energy dependence is in good agreement with this prediction.

We note that, while Eq. (6) holds for a pure-j shell, one can also obtain relations among cross sections because of the fact that Eq. (2) holds also for quasispin formulations of the shell model.[12] Also, for the generalized seniority scheme,[13] one can apply the above formalism for DCX transitions. With abandonment of the attempt to relate α and β to A and B [Eq. (5)], there still exist relationships between cross sections. For example, labeling the cross section $\sigma(E,\theta)$ for n neutrons outside a closed shell by σ_n, we have

$$7\sigma_8 - 20\sigma_6 + 12\sigma_4 - 4\sigma_2 = 0 \tag{7}$$

[this relation is valid with distortion effects included but will be broken by the variation of distortions across the shell, by Q-value differences, and by three- (or higher-) step processes]. The Ni isotopes are well described in the framework of a generalized seniority scheme,[13] and an attempt should be made to measure the pion DCX reaction for the even Ni isotopes.

In conclusion, we have demonstrated in this work that only when the shell-model correlations in the nuclear wave function are taken into account can one explain the low-energy DCX data. The large discrepancies between experiment and previous theories were a result of neglecting such correlations.

A more general conclusion can be drawn from the success of the theory presented above. What this work shows is that the DCX reaction is sensitive to correlations in the initial and final states. In order to be able to reproduce the DCX cross sections, one must take into account details of the shell-model wave function, including configuration mixing, and not limit ourselves, as far as nuclear structure goes, to a description of nuclear states in terms of an independent-particle model. The hope that we had some 25 years ago, that the pion DCX reaction will provide information about two-body correlations in nuclei, seems to be indeed fulfilled.

Our work points also to a possibly new direction in the exploration of the DCX reaction. It shows that the method of effective interactions, applied in the past to energy levels,[6] can also be used in the analysis of the DCX results in terms of effective two-body transition operators. We believe that a substantial body of existing data and of data to come out of future experiments will be analyzed in terms of such a theory. A comparison between the deduced "empirical," effective two-body DCX transition matrix elements and the ones calculated in the framework of a distorted-wave impulse approximation or coupled-channels theory will be able to tell us more about the importance of the truly short-range correlations in nuclei.

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