Implications of Large B_d^0-$\bar{B}_d^0$ Mixing

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Implications of the large B_d^0-$\bar{B}_d^0$ mixing measured by the ARGUS Collaboration are examined. The possibly large top-quark mass is estimated. In addition, large CP-nonconservation effects in the two-body nonleptonic decays of B_d^0-$\bar{B}_d^0$ and B_s^0-$\bar{B}_s^0$ are recalculated. The results show that we still need $\cong 10^6$ $B_{d,s}^0$-$\bar{B}_{d,s}^0$ pairs for the testing of these CP-nonconservation effects in the $B_{d,s}^0$-$\bar{B}_{d,s}^0$ system.

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Recently, the ARGUS Collaboration[1] reported a new result on B_d^0-$\bar{B}_d^0$ mixing,

$$r=(N^{++}+N^{--})/N^{+-}\cong(20.7\pm5.8\pm2.7)\%, \quad (1)$$

where N^{++}, N^{--} denote the numbers of the like-sign lepton events, and N^{+-} that of unlike-sign events. Obviously, this value of the mixing is much larger than the previous estimates in the literature based on the standard model. Of course, this does not mean the failure of the standard model, because the previous estimations set the top-quark mass at less than 40 GeV, and there are also some uncertainties about B, f_B, and the mixing angles, etc.

Within the three-generation standard model, the large mixing of Eq. (1) implies that the top-quark mass should be larger than 40 GeV. But how large is it? In order to answer this question, we first compute the mixing r. Following Sanda and co-workers,[2] for $B_d^0\bar{B}_d^0$ states which are charge-conjugation eigenstates, we have

$$r=\frac{N^{++}+N^{--}}{N^{+-}}=\begin{cases} z^2/(2+z^2), & \text{for } C \text{ odd},\\ (3z^2+z^4)/(2+z^2+z^4), & \text{for } C \text{ even},\end{cases} \quad (2)$$

where $z=\Delta m/\gamma$ and we have neglected $y^2\equiv(\Delta\gamma/2\gamma)^2$ terms because $(\Delta\gamma/\Delta m)^2 \ll 1$ holds even better for larger top-quark mass. Notice that if both z^2 and y^2 are small (i.e., $\ll 1$), we get the expression of r that is extensively used in the literature:

$$r=\begin{cases} (z^2+y^2)/(2+z^2-y^2), & \text{for } C \text{ odd},\\ 3(z^2+y^2)/(2-z^2+y^2), & \text{for } C \text{ even}.\end{cases} \quad (3)$$

But in the present case z^2 is not small; we have to use Eq. (2).

Because the ARGUS measurement was taken on the resonance $\Upsilon(4S)$, the $B_d^0\bar{B}_d^0$ state has $J^{PC}=1^{--}$. Thus, according to Eqs. (1) and (2), we get

$$z^2/(2+z^2)\approx 20.7\%. \quad (4)$$

So we finally have

$$z\equiv\Delta m/\gamma\approx 0.72. \quad (5)$$

If we use the experimental value $\tau_B\cong1.11\times10^{-12}$ sec, then the total width is $\gamma\cong5.9\times10^{-13}$ GeV, and

$$\Delta m=z\gamma\cong 7.02\times10^{-13}\ \text{GeV}. \quad (6)$$

Theoretically, even for large top-quark mass, we still have

$$\Delta m\approx 2|M_{12}|. \quad (7)$$

Now we must use the expression[3] of M_{12} appropriate for large m_t. Because $|M_{12}|$ depends on m_t, combining Eqs. (6) and (7) we can extract the possible values of m_t.

We define $B_d^0=\bar{b}d$, $\bar{B}_d^0=b\bar{d}$, and take the phase convention $CP|B_d^0\rangle=|\bar{B}_d^0\rangle$. So in our case

$$M_{12}=-(G_F^2Bf_B^2m_BM_W^2/12\pi^2)\eta_2^{(B)}U_2\lambda_t^2, \quad (8)$$

$$\Gamma_{12}=(G_F^2Bf_B^2m_B/8\pi)\{m_b^2\lambda_t^2+\tfrac{8}{3}m_c^2\lambda_t\lambda_c\}, \quad (9)$$

where

$$\lambda_t=V_{tb}V_{td}^*,\quad \lambda_c=V_{cb}V_{cd}^*, \quad (10)$$

$$U_2=A_{uu}+A_{tt}-2A_{ut}, \quad (11)$$

$$A_{ij}=B_{ij}-\tfrac{5}{8}C_{ij}. \quad (12)$$

TABLE I. Top-quark mass for different values of f_B and B.

		Case (a)		Case (b)		Case (c)	
f_B (GeV)	B	U_2	m_t (GeV)	U_2	m_t (GeV)	U_2	m_t (GeV)
0.16	$\frac{1}{3}$	4.77	265	1.68	135	1.36	118
	1	1.59	130	0.559	68	0.453	60
	$\frac{3}{2}$	1.06	100	0.373	54	0.302	47
0.20	$\frac{1}{3}$	3.06	200	1.08	101	0.871	89
	1	1.02	98	0.359	52	0.290	46
	$\frac{3}{2}$	0.68	76	0.239	42	0.194	37

The expressions for B_{ij} and C_{ij} can be found in Appendix A of Ref. 3. For convenience, we quote them:

$$B_{ij}=\frac{1}{(1-x_i)(1-x_j)}\int_0^1 d\alpha(\{(2+\tfrac{1}{2}x_ix_j)d(x_j,x_i)-2x_ix_j+x_b[x_i\alpha+x_j(1-\alpha)]\}\ln|d(x_j,x_i)|$$
$$+\{(2+\tfrac{1}{2}x_ix_j)d(1,1)-2x_ix_j+x_b[x_i\alpha+x_j(1-\alpha)]\}\ln|d(1,1)|$$
$$-\{(2+\tfrac{1}{2}x_ix_j)d(x_j,1)-2x_ix_j+x_b[x_i\alpha+x_j(1-\alpha)]\}\ln|d(x_j,1)|$$
$$-\{(2+\tfrac{1}{2}x_ix_j)d(1,x_i)-2x_ix_j+x_b[x_i\alpha+x_j(1-\alpha)]\}\ln|d(1,x_i)|),$$

$$C_{ij}=\frac{1}{(1-x_i)(1-x_j)}x_b(4+x_ix_j)\int_0^1 d\alpha\,\alpha(1-\alpha)\ln\left|\frac{d(x_j,x_i)d(1,1)}{d(x_j,1)d(1,x_i)}\right|,\quad d(x_j,x_i)=x_j(1-\alpha)+x_i\alpha-x_b\alpha(1-\alpha).$$

Thus, U_2 can be computed numerically for the different values of m_t.

According to Ginsparg, Glashow, and Wise,[4] the total decay width of the B meson is

$$\Gamma_B=\Gamma(b\to c)+\Gamma(b\to u), \tag{13}$$

where

$$\Gamma(b\to c)=(G_F^2m_b^5/192\pi^3)3.3\,|V_{cb}|^2, \tag{14}$$

$$\Gamma(b\to u)=(G_F^2m_b^5/192\pi^3)5.3\,|V_{ub}|^2, \tag{15}$$

and where we have taken $m_c\cong 1.4$, $m_b\cong 4.6$, and the numbers 3.3 and 5.3 in Eqs. (14) and (15) are due to phase-space and QCD corrections. If we use the b lifetime $\tau_b=1.11\times 10^{-12}$ sec, then we deduce

$$|V_{cb}|^2\cong 3.95\times 10^{-3}. \tag{16}$$

Using Eqs. (14) and (15), we get

$$\Gamma(b\to u)/\Gamma(b\to c)\cong 0.0854s_3^2/|V_{cb}|^2. \tag{17}$$

If we use[5]

$$\Gamma(b\to u)/\Gamma(b\to c)<4\%, \tag{18}$$

according to Eqs. (16) and (17) we have

$$s_3<0.043. \tag{19}$$

If we use the safer limit[6]

$$\Gamma(b\to u)/\Gamma(b\to c)<8\%, \tag{20}$$

we have

$$s_3<0.061. \tag{21}$$

Because

$$|V_{cb}|^2\cong s_3^2+s_2^2+2s_3s_2c_\delta, \tag{22}$$

the bounds on the values of s_2,s_3 depend on the phase δ. If we assume $s_\delta\cong 1$, $c_\delta\cong 0$, as done by many authors in the literature according to the *CP*-nonconservation data in the K-$\bar{K}$ system, then we get the following solution for the Kobayashi-Maskawa parameters:

$$s_1\cong 0.231,\quad s_2\cong s_1^2,\quad s_3\cong \tfrac{1}{2}s_2,\quad s_\delta\cong 1,\quad c_\delta\cong 0. \tag{23}$$

Under the assumption of $s_\delta\cong 1$, $c_\delta\cong 0$, this solution is not sensitive to the different bounds of Eqs. (18) and (20). Using

$$m_B\cong 5.28\text{ GeV},\quad \eta_2^{(B)}\cong 0.85,$$
$$B=\tfrac{1}{3},\ 1,\text{ or }\tfrac{3}{2},\quad f_B=160\text{ or }200\text{ MeV}, \tag{24}$$

we can extract the top-quark mass m_t from Eqs. (6) and (7). The results are presented in Table I, case (a). From Table I, case (a), we see that the top-quark mass is larger than 76 GeV. Of course, we should not take any individual value of m_t too seriously because of the uncertainties of B, f_B, and the mixing angles. But one thing is certain: The large mixing of Eq. (1) implies a large top-quark mass.

Notice that the values of $s_2,s_3,s_\delta,c_\delta$ in Eq. (23) are just those extensively used in the literature. They have large uncertainties. Although s_δ cannot be too small, c_δ could be negative. For instance, taking $\delta\cong 143°$, then

$$s_\delta\cong 0.6,\quad c_\delta\cong -0.8. \tag{25}$$

Under the bounds of Eqs. (18), (19), (16), and (22) we can set

$$s_3\cong 0.04,\quad s_2\cong 0.09. \tag{26}$$

In that case, the estimated values of m_t are listed in Table I, case (b). Similarly, under the bounds of Eqs. (20), (21), (16), and (22) we can set

$$s_3\cong 0.06,\quad s_2\cong 0.10. \tag{27}$$

The corresponding m_t values are presented in Table I, case (c). In the latter case, because the upper bound of $\Gamma(b\to u)/\Gamma(b\to c)$ is relaxed compared with the former one, we get a smaller $m_t\cong 46$–60 GeV for the most likely value of $B\cong 1$. In some marginal case m_t might be even smaller.

The other implication of the large B_d^0-$\bar{B}_d^0$ mixing is the large CP-nonconservation effects for nonleptonic $B_d^0,\bar{B}_d^0$ decays. But the calculated same-sign dilepton asymmetry

$$a=\frac{N^{++}-N^{--}}{N^{++}+N^{--}}\cong -\frac{\mathrm{Im}(M_{12}^*\Gamma_{12})}{|M_{12}|^2}\lesssim 10^{-3} \quad (28)$$

for $m_t \gtrsim 70$ GeV is very small. Actually, the larger m_t is, the smaller the same-sign dilepton asymmetry. For the nonleptonic decays, things are different. Larger mixing gives larger CP nonconservation. In fact, following Ref. 2 and Du, Dunietz, and Wu,[7] the time-integrated asymmetry is

$$C_f=\frac{\Gamma(B_{d,\mathrm{phys}}^0\to f)-\Gamma(\bar{B}_{d,\mathrm{phys}}^0\to \bar{f})}{\Gamma(B_{d,\mathrm{phys}}^0\to f)+\Gamma(\bar{B}_{d,\mathrm{phys}}^0\to \bar{f})}$$
$$\cong -\frac{2z\,\mathrm{Im}\lambda}{2+z^2+z^2|x|^2}, \quad (29)$$

where f is the decay final state, $x=\bar{A}(\bar{B}^0\to \bar{f})/A(B^0\to f)$, and $\lambda=\lambda_t^* x/\lambda_t$. The number of B_d^0-$\bar{B}_d^0$ pairs needed for testing this asymmetry is, for three-standard-deviation signature,

$$N_{b\bar{b}}=9/C_f^2 B(f+\bar{f})\epsilon, \quad (30)$$

where ϵ is the detection efficiency of the final state f, and

$$B(f+\bar{f})=B(B_{d,\mathrm{phys}}^0\to f)+B(\bar{B}_{d,\mathrm{phys}}^0\to \bar{f})$$
$$\cong B(B_d^0\to f)\frac{2+z^2+z^2|x|^2}{1+z^2} \quad (31)$$

is the combined branching ratio. All possible two-body decay channels are recalculated by use of the same method as Ref. 7. The results are shown in Table II. Note that the values of $|x|^2$, $\mathrm{Im}\lambda$, and ϵ are taken from Tables II and III of Ref. 7, respectively. As for the CP-nonconservation phase δ, we use $\delta=45°$ for $\bar{b}\to \bar{u}u\bar{d}$, $\bar{u}u\bar{s}$ in order to enhance the corresponding asymmetry. We take $\delta\cong 90°$ in all other cases. We must emphasize that in Table II we only give those final states into which both B_d^0 and $\bar{B}_d^0$ can decay. In this case the amplitude interference will enhance the asymmetry, and the theoretical calculation does not involve the computation of the decay amplitudes explicitly, and so it is more reliable.

From Table II we can see that $N_{b\bar{b}}$ have been reduced by 2 orders of magnitude in comparison with the previous estimation (see Table IV of Ref. 7). Roughly speaking, we now need $\cong 10^6$ B_d^0-$\bar{B}_d^0$ pairs for testing CP-nonconservation effects in two-body nonleptonic decays, and the favorite channels are

$$B_d^0,\bar{B}_d^0\to \pi^+\pi^-, K^+\pi^-, D^+\pi^-, \psi K_S, \phi K_S,$$
$$D^+D^-, \pi^0K_S, \eta K_S, D^-K_S. \quad (32)$$

We can make a discussion for the B_s^0-$\bar{B}_s^0$ system parallel to that for the B_d^0-$\bar{B}_d^0$ case.

Actually, a larger top-quark mass means a larger mixing parameter $z_s\equiv(\Delta m/\gamma)_{B_s}$ according to Eqs. (6) and (7) where $\lambda_t=V_{tb}V_{ts}^*$ instead. For example, if we take $B=1$, $f_{B_s}=0.11$ GeV for the B_s^0 case, then for $m_t=76$,

TABLE II. Two-body hadronic final states, z, C_f, branching ratios, and $N_{b\bar{b}}$ for B_d^0-$\bar{B}_d^0$ decays.

Quark decay	$B_{d,\mathrm{phys}}\to f$	$z=\Delta m/\gamma$	Asymmetry C_f	$B(B_{d,\mathrm{pure}}^0\to f)$	$N_{b\bar{b}}$
$\bar{b}\to \bar{u}u\bar{d}$	$\pi^+\pi^-$			$<10^{-4}$	$>2.0\times 10^5$
$\bar{u}u\bar{s}$	π^0K_S	0.72	0.474	1.8×10^{-6}	3.4×10^7
	K^+K^-			$<10^{-4}$	$>2.0\times 10^5$
	ηK_S			6.1×10^{-7}	2.5×10^8
$\bar{b}\to \bar{u}c\bar{d}$	$D^+\pi^-$			10^{-5}	4.3×10^6
	F^+K^-		−0.059	10^{-5}	4.3×10^7
	ψD^0	0.72		5×10^{-6}	6.1×10^7
	$D^0\pi^0$			5×10^{-6}	8.6×10^6
$\bar{b}\to \bar{c}u\bar{d}$	$D^-\pi^+$			2×10^{-2}	1.9×10^7
	F^-K^+	0.72		2×10^{-2}	1.9×10^8
	$\psi\bar{D}^0$		−0.012	10^{-2}	2.7×10^8
	$\bar{D}^0\pi^0$			10^{-2}	3.8×10^7
$\bar{b}\to \bar{c}c\bar{s}$,	F^+F^-			10^{-4}	3.1×10^9
$\bar{c}c\bar{d}$,	$\psi\phi$	0.72		10^{-5}	4.5×10^7
$\bar{s}$	ψK_S		0.38	5×10^{-4}	1.5×10^6
	ϕK_S			5×10^{-5}	3.8×10^6
	D^+D^-			10^{-3}	3.1×10^6
	π^0K_S			2.5×10^{-5}	3.8×10^6
	ηK_S			10^{-4}	2.4×10^6
$\bar{b}\to \bar{u}c\bar{s}$	D^0K_S	0.72	0.564	10^{-4}	2.5×10^6
$\bar{b}\to \bar{c}u\bar{s}$	$\bar{D}^0K_S$	0.72	0.22	5×10^{-4}	6.5×10^6

TABLE III. Two-body hadronic final states, z, C_f, branching ratios, and $N_{b\bar{b}}$ for B_s^0-$\bar{B}_s^0$ decays.

Quark decay	$B_{s,\text{phys}} \to f$	$z_s = \Delta m/\gamma$	Asymmetry C_f	$B(B^0_{s,\text{pure}} \to f)$	$N_{b\bar{b}}$
$\bar{b} \to \bar{u}u\bar{d}$	$\pi^+\pi^-$			10^{-5}	1.0×10^7
$\bar{u}u\bar{s}$	$\pi^0 K_S$	3.54	0.21	5×10^{-5}	6.2×10^6
	K^+K^-			10^{-5}	1.0×10^7
$\bar{b} \to \bar{u}c\bar{s}$	$D^+\pi^-$			2×10^{-4}	2.1×10^6
	F^+K^-	3.54	0.19	2×10^{-4}	2.1×10^7
	ϕD^0			2×10^{-4}	4.2×10^6
	$D^0\pi^0$			10^{-4}	4.1×10^6
$\bar{b} \to \bar{c}u\bar{s}$	$D^-\pi^+$			10^{-3}	2.1×10^6
	F^-K^+	3.54	0.19	10^{-3}	2.1×10^7
	$\phi\bar{D}^0$			10^{-3}	4.2×10^6
	$\bar{D}^0\pi^0$			5×10^{-4}	4.2×10^6
$\bar{b} \to \bar{c}c\bar{s}$,	$\psi\phi$			3×10^{-3}	1.5×10^8
$\bar{c}c\bar{d}$,	ψK_S	3.54	−0.012	2.7×10^{-5}	2.5×10^{10}
	D^+D^-			5×10^{-3}	6.3×10^8
	F^+F^-			2×10^{-2}	1.6×10^{10}
$\bar{b} \to \bar{u}c\bar{d}$	D^0K_S	3.54	0.012	5.7×10^{-6}	1.9×10^8
$\bar{b} \to \bar{c}u\bar{d}$	$\bar{D}^0K_S$	3.54	−0.0102	10^{-2}	2.6×10^8
$\bar{b} \to \bar{d}$	ϕK_S	3.54	−0.226	2.1×10^{-6}	2.5×10^8

100, and 200 GeV, we obtain $z_s = 3.54$, 5.51, and 15.91, respectively. This large mixing parameter will affect dramatically the CP-nonconservation effects of B_s^0-$\bar{B}_s^0$ nonleptonic decays. To illustrate that, we did the similar recalculation for the two-body hadronic decays only for $m_t = 76$ GeV, $B=1$, $f_{B_s} = 0.11$ GeV. The result is shown in Table III. We see from Table III that the number of B_s^0-$\bar{B}_s^0$ pairs needed for testing CP nonconservation has been reduced by 1 order of magnitude. Roughly we need $\cong 10^6$ B_s^0-$\bar{B}_s^0$ pairs. The favorite channels are

$$D^0\phi, D^+\pi^-, D^0\pi^0, \bar{D}^0\phi, D^-\pi^+, \bar{D}^0\pi^0. \qquad (33)$$

Notice that most of the branching ratios in Tables II and III are estimated by use of Kobayashi-Maskawa matrix elements and so they have large uncertainties. With consideration of the uncertainty of our calculation method, the numbers $N_{b\bar{b}}$ in these tables might differ by an order of magnitude.

In conclusion, the recent result of a large B_d^0-$\bar{B}_d^0$ mixing implies a much larger top-quark mass and larger CP-nonconservation effects in $B_{d,s}^0$-$\bar{B}_{d,s}^0$ nonleptonic decays. But for testing these effects we still need $\cong 10^6$ B^0-$\bar{B}^0$ pairs. Although this number is large, it is not inaccessible in the near future.

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