

**Mon Replies:** The analytic results<sup>1,2</sup> technically need not be exact for arbitrary dimension, comparisons with the simulations<sup>3</sup> are erroneous, and the bound ( $D \leq d$ ) does not apply for solid-on-solid lattice models.

(1) The analytic results<sup>1,2</sup> consider a subset of the original height variables on sites that form a regular lattice with lattice constant  $L$ . The "analytic" excess area density is  $A_a(L) \approx L^{-1} \langle |h_L| \rangle_a$ , with  $h_L \equiv h(0) - h(L)$ . With the assumption  $\langle |h_L| \rangle_a \approx aL^{Y_a}$  with  $a > 0$ ,  $A_a(L) \approx L^{-(1-Y_a)}$  and  $D_a(d) = d - Y_a(d)$  follow. The  $Y_a$  were from scaling results<sup>1,2</sup> assuming  $\langle |h_L| \rangle_a \approx \langle (h_L)^2 \rangle_a^{1/2}$ , where  $\langle (h_L)^2 \rangle_a^{1/2} \approx bL^{X_a}$  for large  $L$ , with  $b > 0$ .  $X_a = \frac{1}{2}$  and 0 for  $d=2$  and 3, respectively. This actually obeys a Schwartz inequality,  $\langle |h_L| \rangle_a \leq \langle (h_L)^2 \rangle_a^{1/2}$ ,  $aL^{Y_a} \leq bL^{X_a}$ , and  $0 < a/b \leq L^{X_a - Y_a}$ . Since  $L$  must be large for scaling to hold,  $Y_a < X_a$  or  $Y_a = X_a$ . For  $d=2$ ,  $Y_a = X_a$  follows from exact results,<sup>1,2</sup> but for  $d=3$ , nothing rigorous is known.

(2) The analytic calculations do not include the important feature that the column height differences in the simulations are measured in units of  $L$ . This neglect contributes to the disagreement between the analytic results and the simulations. Consider the distribution  $P_L(|h_L|)$  and note that the analytic results assume

$$\langle |h_L| \rangle_a = \int_0^\infty |h_L| P_L(|h_L|) d|h_L|, \quad (1)$$

but the simulations measure  $|h_L|$  in units of  $L$ . This introduces truncations

$$\langle |h_L| \rangle_{MC} = \sum_{i=1}^{\infty} i \int_{iL}^{(i+1)L} LP_L(|h_L|) d|h_L|. \quad (2)$$

To show the consequences of these two different definitions, Eq. (2) has been evaluated numerically for a Gaussian, which is exact for  $d=2$  in the large- $L$  limit. A width of  $20L^{1/2}$  is used (see Fig. 1). The dots represent Eq. (2) and the dashed line is the predictions of Refs. 1 and 2 with a slope of  $\frac{1}{2}$ , fitted to the data at small  $L$ . The truncation is not expected to be important in the small- $L$  limit and this is confirmed in Fig. 1. Deviations of the results with and without truncations become noticeable above  $L=5$  and increase with  $L$ . Note that for large  $L$  (the limit for which the analytic results are valid), the analytic results without truncations do not even describe the correct trends of the results with truncations.

The data between  $L=7$  to 100 can be roughly fitted

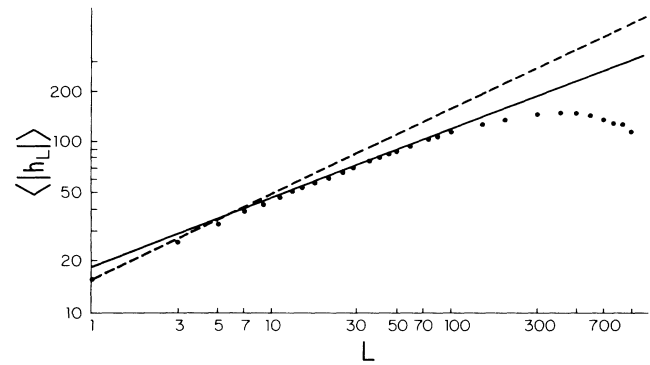


FIG. 1. A comparison of the scaling properties for  $\langle |h_L| \rangle$  between the predictions of Refs. 1 and 2 (dashed line) and numerical solution of Eq. (2) (dots) for  $d=2$  (see text).

with a slope of 0.408 (solid line in Fig. 1), consistent with the Monte Carlo results of  $0.42 \pm 0.06$ . The small  $L$  ( $< 7$ ) data do deviate from scaling, in contrast to the simulations. This may be attributed to the inaccuracy of the Gaussian approximation for small  $L$ . The large- $L$  deviations from scaling are clearly present in both the simulations and Eq. (2), but absent in the predictions of Refs. 1 and 2. This supports the simulations; details will be considered elsewhere.

(3) The bound  $D \leq d$  does not apply here. The simplest reason is that a solid-on-solid lattice model is highly anisotropic.

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<sup>1</sup>P.-z. Wong, J. Howard, and J.-S. Lin, Phys. Rev. Lett. **57**, 637 (1986); P.-z. Wong and A. J. Bray, second preceding Comment [Phys. Rev. Lett. **59**, 1057 (1987)].

<sup>2</sup>T. W. Burkhardt, preceding Comment [Phys. Rev. Lett. **59**, 1058 (1987)].

<sup>3</sup>K. K. Mon, Phys. Rev. Lett. **57**, 866, 1963(E) (1986).