Scaling of the Excess Area of Interfaces

In the recent Monte Carlo results of Mon,[1] the excess area density $A_{MC}(L)$ of rough solid-on-solid interfaces in $d=2$ and 3 bulk dimensions measured on a length scale L was found to vary as

$$A_{MC}(L) \sim L^{-x(d)}, \qquad (1)$$

with $x(2)=0.58\pm 0.06$ and $x(3)=1.25\pm 0.05$. Here I argue that for $L\gg 1$ and $A_{MC}(L)\gg 1$ the exact exponents[2] are $x(2)=\frac{1}{2}$ and $x(3)=1$, and that for $A_{MC}(L)\ll 1$, $A_{MC}(L)$ decreases with increasing L faster than the power law (1).

In general dimension d, the excess area density, is given by $\tilde{A}(l)=(d-1)\langle |h_0-h_1|\rangle$. Here the h_i are height variables defined on a $d-1$ dimensional hypercubic lattice with lattice constant l. The subscripts 0 and 1 denote the neighboring sites.

Now consider the interface defined by a subset of the original height variables on sites that form a hypercubic lattice with lattice constant L. The excess area density $\tilde{A}(L)$ of this interface is given by

$$\tilde{A}(L)=(d-1)L^{-1}\langle |h_0-h_L|\rangle. \qquad (2)$$

Sites 0 and L are separated by L original lattice constants but are neighboring sites on the coarse-grained lattice. For rough interfaces

$$\langle |h_0-h_L|\rangle \sim \langle (h_0-h_L)^2\rangle^{1/2} \sim \begin{cases} L^{(3-d)/2}, & d<3, \\ (\ln L)^{1/2}, & d=3, \end{cases} \qquad (3)$$

for $L\gg 1$, as follows from the capillary-wave picture.[3,4] Combining Eqs. (2) and (3) gives

$$\tilde{A}(L) \sim \begin{cases} L^{-(d-1)/2}, & d<3, \\ L^{-1}(\ln L)^{1/2}, & d=3. \end{cases} \qquad (4)$$

In Ref. 1 $A_{MC}(L)$ was not determined from the original height variables like $\tilde{A}(L)$ in Eq. (2) but from coarse-grained heights restricted to integer multiples of L. There are two distinct regimes in which $A_{MC}(L)$ and $\tilde{A}(L)$ can be simply compared.

In regime I $\langle |h_0-h_L|\rangle \gg L$ or $\tilde{A}(L)\gg 1$. For typical interface configurations $|h_0-h_L|$ is so large in comparison with L that the excess area density is insensitive to the coarse graining of the heights, and $A_{MC}(L)\cong\tilde{A}(L)$. Much of the Monte Carlo data in Figs. 1 and 2 of Ref. 1 satisfies $A_{MC}(L)\gg 1$, $L\gg 1$. Thus, it is appropriate to compare these data with Eq. (4), or equivalently, with Eq. (1) where $x(d)=\frac{1}{2}(d-1)$.

In regime II $\langle |h_0-h_L|\rangle \ll L$ or $\tilde{A}(L)\ll 1$. Since $A_{MC}(L)$, unlike $\tilde{A}(L)$, is exclusively determined by fluctuations with $|h_0-h_L|$ of the order of L or larger and such fluctuations are rare in regime II, $A_{MC}(L)\ll\tilde{A}(L)$. In the large-L limit $A_{MC}(L)$ approaches zero more rapidly[5] than a power law. The downward deviation with increasing L of the data in Figs. 1 and 2 of Ref. 1 from the straight-line fits has a natural explanation in terms of the two regimes $A_{MC}(L)\gg 1$ and $A_{MC}(L)\ll 1$.

I thank T. Ala-Nissila, A. J. Bray, D. Forster, D. Stauffer, and P. Z. Wong for helpful discussions.

Theodore W. Burkhardt
Department of Physics
Temple University
Philadelphia, Pennsylvania 19122

Received 4 September 1986
PACS numbers: 68.35.Fx, 64.60.Ak, 68.35.Bs, 73.40.−c

[1]K. K. Mon, Phys. Rev. Lett. **57**, 866, 1963(E) (1986).

[2]After completion of this work I learned that similar results have been obtained by P.-z. Wong, J. Howard, and J.-S. Lin, Phys. Rev. Lett. **57**, 637 (1986).

[3]F. P. Buff, R. A. Lovett, and F. H. Stillinger, Jr., Phys. Rev. Lett. **15**, 621 (1965).

[4]D. Jasnow, Rep. Prog. Phys. **47**, 1059 (1984), and references therein.

[5]In $d=2$, from the exact distribution of the variable h_0-h_L {see P.-z. Wong and A. J. Bray, preceding Comment [Phys. Rev. Lett. **59**, 1057 (1987)]}, it follows that

$$A_{MC}(L) \sim L^{-1/2}\exp[-L\sinh^2(K/2)]$$

in the limit $L\to\infty$ with fixed interface stiffness K (Ref. 1). Since interface fluctuations are weaker in higher dimensions, for $d>2$ one also expects $A_{MC}(L)$ to tend to zero faster than a power law in the limit $L\to\infty$.