Manifestation of the Berry Phase in Diabolic Pair Transfer in Rotating Nuclei

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A new manifestation of the Berry phase is presented in the theory of fast-rotating superfluid nuclei: Diabolic pair transfer, i.e., oscillation of the pair-transfer matrix elements as a function of the angular velocity, is shown to be the direct consequence of the Berry phase, giving a nontrivial contribution at the diabolical points of cranked Hartree-Fock-Bogoliubov spectra.

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Considering a general Hamiltonian $H(R)$, which depends on several parameters $R=(X,Y,\dots)$, Berry discovered recently[1] that a quantum system described as an eigenstate of this Hamiltonian acquires a topological phase factor $\exp\{i\gamma(C)\}$ when transported adiabatically around a path C in parameter space. This phase is of special importance in cases where the path includes a so-called diabolical point, i.e., a point in parameter space where two eigenenergy surfaces $E_1(R)$ and $E_2(R)$ with the same symmetries of the Hamiltonian touch each other. These points are exceptional points in the sense that they violate the no-crossing rule of von Neumann and Wigner.[2]

Recently the manifestation of this phase has appeared in many diverse areas of physics (cf. Chiao and Wu[3] and references given therein). As Berry himself has pointed out, this phase factor is quite universal and appears for any system described by a Hermitian operator, quantum mechanical or not. In this Letter we present an example in nuclear physics. We show that the recently predicted diabolic pair transfer[4] in rotating superfluid nuclei is a direct manifestation of a Berry phase.

It has been already realized in Bengston, Hamamoto, and Mottelson[5] and Frisk and Szymanski[6] that the cranked-shell-model Hamiltonian[7]

$$H(\lambda,\omega)=\begin{pmatrix}\varepsilon_K-\lambda-\omega j_x & \Delta\\ \Delta & -(\varepsilon_K-\lambda-\omega j_x)\end{pmatrix} \qquad (1)$$

has in its spectrum several diabolical points. Its eigenfunctions are many-body wave functions of the Hartree-Fock-Bogoliubov (HFB) type. They describe rotating superfluid nuclei and depend on two parameters: the chemical potential λ and the angular velocity ω. These parameters are determined by the particle number A and the angular momentum I and are closely connected to deformations and pairing correlations, the most most important degrees of freedoms of nuclear collective motion. Many of the interesting features of high-spin physics depend upon only one of these parameters. It is the special property of the effect discussed in this paper that it is connected intimately with both of these parameters. That is to say the physics which is described in the following can only be understood in the (λ,ω) plane.

The diabolical points of the Hamiltonian (1) correspond to alignment processes of particles in the intruder orbits. They indicate sharp level crossings between certain configurations, as for instance between the ground-state band and an aligning two-quasiparticle band or between other multi-quasiparticle configurations. In the rare-earth region the intruder orbit is the neutron $1i_{13/2}$ orbit. We therefore use a model of particles moving in a deformed and superfluid single-j shell with $j=\frac{13}{2}$, with $\varepsilon_K=\kappa[3K^2/j(j+1)-1]$. κ is proportional to the deformation and in the following all energies are measured in units of κ. In particular we use $\Delta/\kappa=0.45$.

In Fig. 1 we show the diabolical points in the (λ,ω) plane. The pattern of these points is very regular, as is shown in a schematic way in Fig. 2. This can be understood[8] in terms of vanishing spatial overlap integrals between rotating single-particle orbits with different signatures. In Fig. 1 we also give the lines of constant average particle number and we find an interesting bunching of these lines at the diabolical points. This means that in the neighborhood of these points we have considerable change in the particle number. Following lines of constant angular velocity, the particle number has a behav-

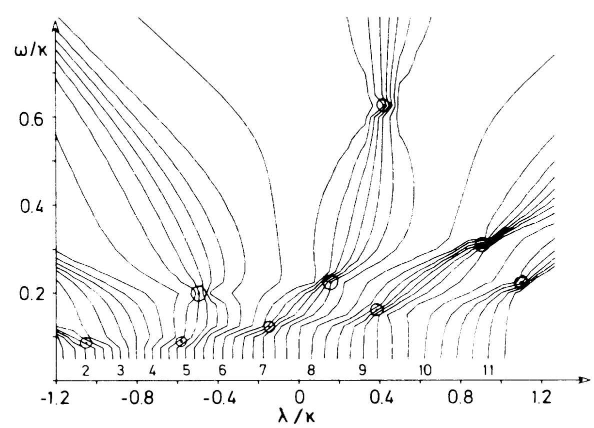

FIG. 1. The diabolical points (indicated by circles) in the (λ,ω) plane for the $j=\frac{13}{2}$ model. Lines correspond to lines of constant average particle number, indicated by the numbers 1,2, . . . , above the abscissa.

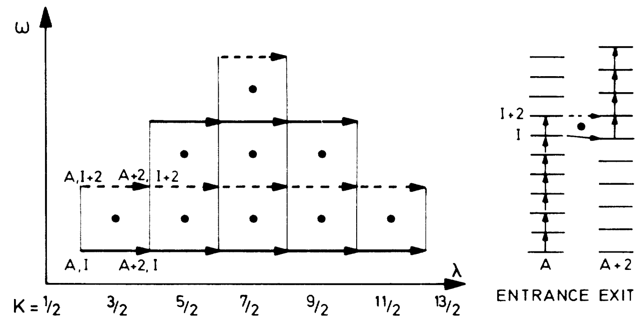

FIG. 2. Schematic representation of the pattern of diabolical points in the (λ,ω) plane. Full horizontal arrows indicate pair-transfer matrix elements with positive sign and dashed arrows indicate those with negative sign. Below the abscissa we indicate the K quantum numbers in the deformed $j=\frac{13}{2}$ shell. On the right-hand side experimental trajectories are shown, which correspond to two different paths around a diabolical point which yield destructive interference.

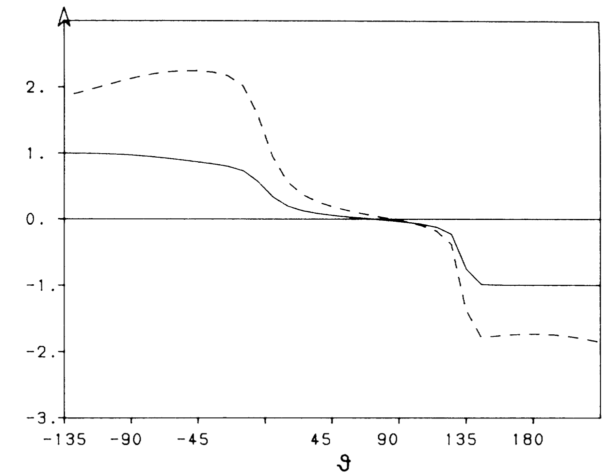

FIG. 3. The norm overlap $\langle\Phi(\theta_0)\,|\,\Phi(\theta)\rangle$ (unbroken line) and the pair-transfer overlap $\langle\Phi(\theta_0)\,|\,S^\dagger\,|\,\Phi(\theta)\rangle$ (dashed line) for an elliptic path $\lambda=\lambda_0+0.2\cos(\theta)$, $\omega=\omega_0+0.01\sin(\theta)$ around the diabolical point $\lambda_0=-0.94$, $\omega_0=0.089$. θ is the angle in the (λ,ω) plane.

ior similar to a step function at these points, i.e., we have a very small "effective" pairing in these regions. However, this does not mean that here we have pairing collapse as predicted by Mottelson and Valatin[9]; in fact these calculations have been carried out with a constant gap parameter Δ and, apart from the diabolical regions, the particle number is a smooth function of the chemical potential λ.

We now study the influence of the Berry phase at a specific diabolical point. We have to choose a closed path in the (λ,ω) plane around this point. To understand the following considerations we have to remind ourselves what the physical meaning of such a path is: Changes in λ at constant ω mean changes in the particle number at constant angular momentum, i.e., transitions from the nucleus A with spin I to the nucleus $A\pm 2$ with the same spin I. They are connected with the transfer of a pair of nucleons coupled to angular momentum zero. Changes in ω for constant λ indicate changes of the angular momentum within the same nucleus. This is made clear schematically in Fig. 2.

We characterize the path around the diabolical point by a path parameter θ. By the diagonalization of the Hamiltonian (1) the wave function at each point on this path $(\lambda(\theta),\omega(\theta))$ is determined only up to an arbitrary phase. As stated in the work of Simon[10] we use a natural Hermitian connection to determine the relative phase between two neighboring HFB functions: $\langle\Phi(\theta)\,|\,\Phi(\theta+\varepsilon)\rangle=1+O(\varepsilon^2)$. Since we use real matrix elements Berry tells us that our wave function $|\Phi(\theta)\rangle$ acquires a phase -1, if we follow this path around the diabolical point. In Fig. 3 we have as an example chosen a closed path around the left-most diabolical point in Fig. 1. We clearly see the change in phase close to $\theta=60°$.

We now investigate the influence of the Berry phase on the transfer matrix element

$$P(I)=\langle A+2,I\,|\,S^\dagger\,|\,A,I\rangle$$
$$=\langle\Phi(\lambda(A+2),\omega(I))\,|\,S^\dagger\,|\,\Phi(\lambda(A),\omega(I))\rangle \quad (2)$$

of a pair of particles coupled to angular momentum zero $[S^\dagger=(c^\dagger c^\dagger)_{L=0}]$. For this purpose we study the behavior of off-diagonal matrix elements $\langle\Phi(\theta_0)\,|\,S^\dagger\,|\,\Phi(\theta)\rangle$ as a function of θ along a path around a diabolical point. As we see in Fig. 3, this matrix element behaves very similarly to the overlap $\langle\Phi(\theta_0)\,|\,\Phi(\theta)\rangle$. In particular it shows the same change of phase. We can easily understand this fact by considering only the close neighborhood of the diabolical point, where we have to a good approximation a two-level problem (as discussed by Berry in Ref. 1). In our case these two levels are the ground-state band and the band of an aligned pair of quasiparticles (usually called s band). The diagonalization of the Hamiltonian (1) is then reduced to a 2×2 problem

$$H(Z,X)=\begin{bmatrix} Z & X \\ X & -Z \end{bmatrix}, \quad (3)$$

where the parameters Z and X are closely connected to $-\lambda$ and $-\text{const}\times\omega$. On a closed circle with infinitesimal radius the normalized eigenfunctions of (3) can be expressed by an angle θ', with $\tan(\theta')=X/Z$:

$$|\Phi(\theta')\rangle=\begin{bmatrix} \cos(\theta'/2) \\ \sin(\theta'/2) \end{bmatrix}, \quad (4)$$

which gives $\langle\Phi(\theta_0)\,|\,\Phi(\theta')\rangle=\cos[(\theta'-\theta_0)/2]$ and the phase -1 by going around the circle form $\theta'=\theta_0$ to $\theta'=\theta_0+2\pi$. We can consider the matrix elements of the pair-transfer operator $S^\dagger$ to be roughly constant in this 2×2 representation and find the same phase change in the matrix element $\langle\Phi(\theta_0)|S^\dagger|\Phi(\theta')\rangle$. In Fig. 3 we present the norm overlap and the pair-transfer overlap for the full matrix (1). In this case both quantities behave rather similarly; in particular, both show the phase factor -1 on the closed path. This similar behavior can be understood as usual by the fact that $S^\dagger$ is only a generalized one-particle operator and can change only a small part of the many-body wave function $|\Phi\rangle$, an argument which is often used in the literature.

We now consider the pair-transfer matrix element $P(I)$ in Eq. (2) as a function of the angular momentum. At $I=0$ we choose the relative phases of the wave functions in such a way that this matrix element $P(I=0)$ is positive. With increasing angular momentum we then have two paths corresponding to the two sets of wave functions with particle number A and $A+2$. On each of these paths we again choose the phases of the wave functions by a continuous connection $[\langle\Phi(\omega)|\Phi(\omega+\varepsilon)\rangle = 1 + O(\varepsilon^2)]$. From Fig. 2 it becomes clear for this choice of phases that, starting at angular momentum zero, the matrix element $P(I)$ stays positive as long as the transfer happens at an angular velocity smaller than the first diabolical point. In the diabolical region the phase changes, i.e., the pair-transfer matrix element goes through zero, otherwise we could choose a path enclosing the diabolical point without the phase change predicted by Berry. Following these arguments we conclude that with increasing angular momentum, the pair-transfer matrix element $P(I)$ always changes sign at the diabolical regions. We thus have a very natural explanation for the recently predicted effect of diabolic pair transfer,[4] namely the oscillating behavior of this matrix element shown in Fig. 4.

As pointed out in Ref. 4 this effect is a nuclear analog of the dc Josephson effect in solid-state physics. Obviously the nucleus is a finite system and therefore the number of oscillations is finite. As we see from Fig. 2 it depends on the number of diabolical points between the two paths $\lambda(A),\omega$ and $\lambda(A+2),\omega$. In Fig. 1 and in Fig. 4 we find that we have no such diabolical point and no oscillation if the pair of nucleons is transferred to the neighborhood of the $K=\frac{1}{2}$ or of the $K=\frac{13}{2}$ orbit in the $i_{13/2}$ shell. We have *one* diabolical point and *one* sign change of the pair-transfer matrix element, if the pair is transferred to the $K=\frac{3}{2}$ or to the $K=\frac{11}{2}$ orbit. For the $K=\frac{5}{2}$ and the $K=\frac{9}{2}$ we have two and for the $K=\frac{7}{2}$ orbit we have three oscillations.

From the extended calculations of Refs. 6 and 8 it is evident that the effect is not only restricted to the $i=\frac{13}{2}$ model, but also shows up in fully realistic configuration spaces. In fact, all the arguments given in the present paper apply to the general case, too. It is, therefore, certainly a most interesting question to ask if this new effect can be discovered experimentally. The most direct evidence for the phase change would certainly be a destructive interference between the two trajectories, viz. $(A,I)\to(A+2,I)\to(A+2,I+2)$ and $(A,I)\to(A,I+2)\to(A+2,I+2)$ which enclose a diabolical point, as shown in Fig. 2.

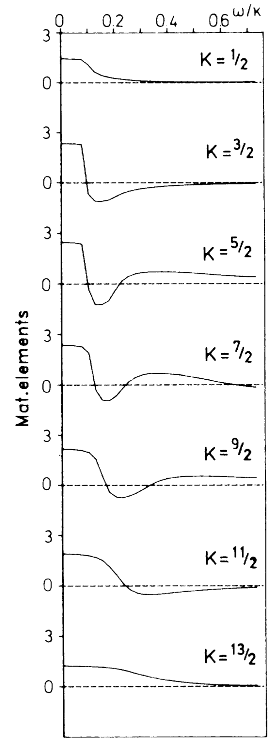

FIG. 4. The pair-transfer matrix elements in Eq. (2) as functions of the angular velocity for different positions of the chemical potential (indicated by the quantum number K). This means that the transferred pair occupies the orbits $\pm K$ with a large probability.

On the right-hand side of Fig. 2 we show, at least schematically, the experiment which would correspond to these two paths. It represents Coulomb excitation using heavy ions in connection with the transfer of a pair of particles. In a classical approximation the transfer takes place at the distance of closest approach, at which half the final angular momentum is transferred. We therefore propose the following: Choose an appropriate initial energy and look only for the largest final angular momenta, such that the two most important reaction contributions come from the paths on two different sides of a diabolical point. In such a case one should be able to study the destructive interference between the wave functions corresponding to these two trajectories.

Since this is probably a very difficult experiment, we would propose only looking at the square of the pair-transfer matrix element: In the diabolic region it goes through zero, which means that one should observe a reduction of this quantity in the region. There are essentially two ways to search for such a reduction: (i) One could investigate the behavior of $P(I)$ for a fixed value of A as a function of the angular momentum I. In this case one crosses the diabolical region in Fig. 2 in a vertical direction. (ii) One also could investigate the behavior of $P(I,A)$ for fixed "diabolical" I in a chain of isotopes, i.e., as a function of A. Now we cross the diabolical regions in a horizontal direction. One should observe a reduction for those nuclei which show a particularly strong backbending.[11]

Since the diabolical points lie at relatively low angular momenta in many nuclei, which can be reached by Coulomb excitation with heavy ions in connection with pair transfer,[12] we hope that this new manifestation of the Berry phase will be seen experimentally in the near future.

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