Color Magnetism and the Helicity-Zero ($\gamma_v N \rightarrow \Delta$) Transition Amplitude

Michele Bourdeau and Nimai C. Mukhopadhyay
Department of Physics, Rensselaer Polytechnic Institute, Troy, New York 12180
(Received 10 November 1986)

We compute longitudinal and scalar multipoles in the $\gamma_v N \rightarrow \Delta$ transition in nonrelativistic quark shell models as a function of the virtual-photon mass squared. We show that the gauge relation between these multipoles for arbitrary virtual-photon mass squared is violated in the truncated quark-shell-model basis. The computed scalar multipole, insensitive to this truncation effect, should provide an accurate test of quark models. Existing experiments support the role of color-magnetic tensor force in hadronic structure.

PACS numbers: 13.60.Rj, 12.40.Aa, 13.75.Cs, 14.20.−c

Current theoretical methods to apply quantum chromodynamics (QCD) directly in describing low-energy properties of hadrons are still in their infancy. At present, we have a variety of models that mimic quark confinement and may incorporate QCD-motivated interaction among hadronic constituents. Among these, the quark shell model (QSM) has been quite successful in explaining a large body of low-energy properties of hadrons.[1] As in the nuclear shell model, the baryon Hamiltonian in the QSM is of the form $H = \sum_i H_i^{\text{sp}} + \sum_{i<j} H_{ij}$; in the QSM, the single-particle term H_i^{sp} consists of quark mass and kinetic energy, while the two-body interaction H_{ij} contains a harmonic confining term, and a hyperfine color-magnetic interaction with contact and tensor terms arising from the one-gluon exchange between a pair of valence quarks.[2]

The focus of our paper is the bearing of the hyperfine tensor force on the electromagnetic process $\gamma_v N \leftrightarrow \Delta$, where γ_v is a real or virtual photon, N is the nucleon, and Δ is the $J^\pi = \frac{3}{2}^+$, $T = \frac{3}{2}$, 1232-MeV isobar. So far, other authors[3-7] have dealt with the real-photon transition in the QSM. The virtual-photon case has been ignored theoretically, and poorly investigated experimentally. Our concern here will be the sensitivity of the longitudinal and scalar quadrupole transition amplitudes, L_{1^+} and S_{1^+}, respectively, in the standard notation,[8] to the hyperfine tensor interaction. An important result of this paper is to show that the relation

$$S_{1^+} = (k/k_0) L_{1^+}, \tag{1}$$

where k_0 and k are energy and momentum of the virtual photon, a direct consequence[8] of the gauge invariance of the electromagnetic interaction (or, equivalently, of the conserved electromagnetic current), fails badly in the QSM with a truncated basis. In nuclear many-body problems, an analogous phenomenon has intrigued researchers for a long time.[9] Our is the first direct demonstration[10] of this failure in many-quark physics. Finally, we discuss its theoretical and experimental implications.

The theoretical interest of the relevant hadron structure problem can be easily stated: In the simplest versions[11] of the SU(6) and $\text{SU}(6)_W$ quark models, the nucleons and Δ are states of orbital angular momentum (L) zero. Thus, the $\gamma_v N \leftrightarrow \Delta$ transition is given by the magnetic dipole amplitude (M_{1^+}) alone. The electric quadrupole amplitude (E_{1^+}), and, for virtual photon, S_{1^+} (L_{1^+}) multipole amplitude must vanish in this case. Departures from this theorem directly probe the amplitudes for the SU(6)-forbidden $L=2$ ("deformed") states in the multiquark wave functions of N and Δ. In the QSM, these arise from the color-magnetic two-body tensor force, as a result of one-gluon exchange between quarks.

In the present paper, we shall use two versions of the QSM for the nucleon and delta. The *first model* is due to Isgur and Karl (IK),[1] and was also used by Gershtein and Dzhikiya (GD).[4] The nucleon and delta wave functions are given in this approach as

$$|N\rangle_A = a_S |N\,^2S_S\rangle + a_S' |N\,^2S_S'\rangle + a_M |N\,^2S_M\rangle + a_D |N\,^4D_M\rangle + a_P |N\,^2P_A\rangle, \tag{2a}$$

$$|\Delta\rangle_A = b_S |\Delta\,^4S_S\rangle + b_S' |\Delta\,^4S_S'\rangle + b_D |\Delta\,^4D_S\rangle + b_D' |\Delta\,^2D_M\rangle, \tag{2b}$$

with the usual notation,[1-3] determined by the diagonalization of H. The *second model,* suggested by Glashow,[12] and exploited by Vento, Baym, and Jackson[3] (VBJ), is a special case of the above. Its nucleon and isobar wave functions are

$$|N\rangle_B = (1-\gamma)^{1/2} |N\,^2S_S\rangle + \gamma^{1/2} |N\,^4D_M\rangle, \quad |\Delta\rangle_B = (1-3\beta)^{1/2} |\Delta\,^4S_S\rangle + (2\beta)^{1/2} |\Delta\,^4D\rangle_S - \beta^{1/2} |\Delta\,^2D_M\rangle.$$

Here the parameter γ is fixed at 0.22, by our fitting it to the nucleon axial-vector coupling constant. We adjust β to reproduce the resonant $E2$ amplitude, extracted at the photon point, by the phenomenological analysis[13] of the multipole data, obtaining $\beta \approx 0.35$. The second model is obviously crude but is useful, along with the first model, to demonstrate the sensitivity of the estimated longitudinal scalar multipole amplitude, in a specific QSM, to the truncation of the valence quark model space.

To compute the longitudinal and scalar quadrupole amplitude in the transition $\gamma_v N \rightarrow \Delta$, where γ_v is a virtual photon, we start with the nonrelativistic transition Hamiltonian,[14] written in terms of quark coordinates.[4] The photon four-momentum is taken to be K_μ $(k_0;0,0,k)$, with $K^2=k_0^2-k^2$ being the virtual-photon mass squared. For photon helicity $\lambda_\gamma=0$, we can define the longitudinal and scalar quadrupole transition and amplitudes[14]:

$$L_{1^+}=\frac{1}{\sqrt{2}}\langle N,M_J=\tfrac{1}{2}\,|-\sum_{i=1}^{3}\frac{e_i}{2m_i}(P_z^i e^{-ikz_i}+e^{-ikz_i}P_z^i)\,|\,\Delta,M_J=\tfrac{1}{2}\rangle, \tag{3}$$

$$S_{1^+}=\frac{1}{\sqrt{2}}\langle N,M_J=\tfrac{1}{2}\,|-\sum_{i=1}^{3}e_i e^{-ikz_i}\,|\,\Delta,M_J=\tfrac{1}{2}\rangle, \tag{4}$$

where L_{1^+} is related to the matrix element of j_z (j is the hadron current) and S_{1^+} to the matrix element[15] of j_0. L_{1^+} and S_{1^+} are related by current conservation [Eq. (1)]. We have *two* ways of calculating the longitudinal quadrupole transition amplitude L_{1^+}: one by the "current method," in which we use directly Eq. (3), and the other by the "charge method," in which we use Eq. (4) to get S_{1^+}, then the identity (1) to get L_{1^+}. *In an exact calculation in which the quark basis is not truncated, these will give identical results.* We shall show below that these *do not* give identical results in our chosen quark shell models.

We now need the transition operators for calculating M_{1^+}, E_{1^+}, L_{1^+}, and S_{1^+} in the QSM. These have already been given for M_{1^+} and E_{1^+} in Ref. 4, where now, for virtual photons, k is given by $K^2=k_0^2-k^2$ and the normalization factor in Ref. 4 due to the real photon

TABLE I. The longitudinal quadrupole amplitude L_{1^+} in the $\gamma_v N \rightarrow \Delta$ transition in current and charge approaches.

	L_{1^+} by the current approach in units of $\sqrt{6}\left(\frac{\pi}{K_0}\right)^{1/2}\frac{e}{m}k\exp\left(-\frac{k^2}{6\alpha^2}\right)$ (GeV$^{-1/2}$)	L_{1^+} by the charge approach in units of $-\frac{1}{6}\left(\frac{2}{15}\right)^{1/2}e\left(\frac{\pi}{K_0}\right)^{1/2}\left(\frac{k}{\alpha}\right)^2\times\frac{k_0}{k}\exp\left(-\frac{k^2}{6\alpha^2}\right)$ (GeV$^{-1/2}$)
$\Delta\,^2D_M \rightarrow N\,^2S_s$	$\frac{1}{9\sqrt{5}}\left(-1+\frac{k^2}{12\alpha^2}\right)$	1
$\Delta\,^4S_S \rightarrow N\,^4D_M$	$-\frac{1}{9\sqrt{5}}\left(1+\frac{k^2}{12\alpha^2}\right)$	-1
$\Delta\,^2D_M \rightarrow N\,^2S_S'$	$\frac{1}{54\sqrt{15}}\left(\frac{k^2}{\alpha^2}-\frac{k^4}{12\alpha^4}\right)$	$\frac{2}{\sqrt{3}}\left(1-\frac{k^2}{12\alpha^2}\right)$
$\Delta\,^2D_M \rightarrow N\,^2S_M$	$-\frac{1}{648\sqrt{30}}\frac{k^4}{\alpha^4}$	$-\frac{\sqrt{2}}{12\sqrt{3}}\frac{k^2}{\alpha^2}$
$\Delta\,^4S_S' \rightarrow N\,^4D_M$	$-\frac{1}{54\sqrt{15}}\left(\frac{k^2}{\alpha^2}-\frac{k^4}{12\alpha^4}\right)$	$-\frac{2}{\sqrt{5}}\left(1-\frac{k^2}{12\alpha^2}\right)$
$\Delta\,^4D_S \rightarrow N\,^4D_M$	$\frac{\sqrt{2}}{1080\sqrt{3}}\left(\frac{7k^2}{\alpha^2}-\frac{k^4}{3\alpha^4}\right)$	$\frac{7}{\sqrt{30}}\left(1-\frac{k^2}{21\alpha^2}\right)$
$\Delta\,^2D_M \rightarrow N\,^2P_A$	$\frac{1}{36\sqrt{30}}\frac{k^2}{\alpha^2}$	$\left(\frac{3}{2}\right)^{1/2}$

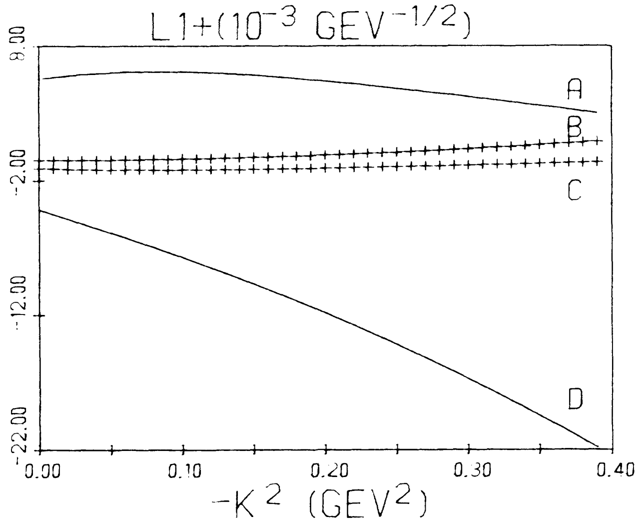

FIG. 1. Comparison of L_{1^+} obtained by charge (A,C) and current approaches (B,D) for the IK-GD (B,C) and VBJ (A,D) models.

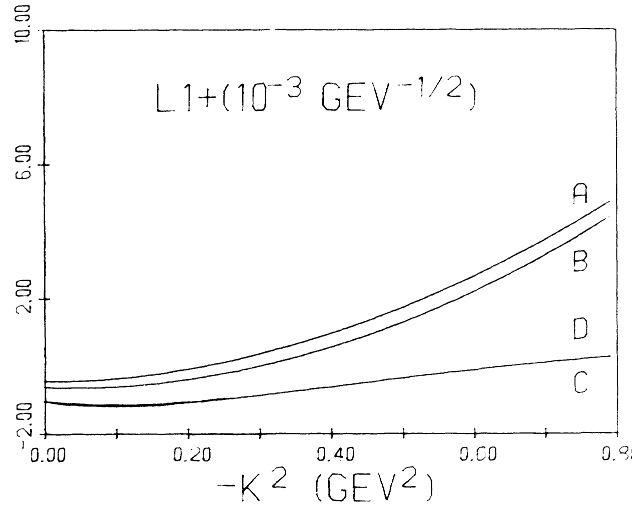

FIG. 2. Comparison of L_{1^+} obtained by charge (D,C) and current approaches (A,B) in the IK and GD models, where A and C include the state $|N\,^2P_A\rangle$ and B and D do not. Curves C and D are coincident in the shown scale.

field is replaced by $(2\pi/K_0)^{1/2}$, where $K_0=(M_\Delta^2-M_N^2)/(2M_\Delta)$ (equal to $|\mathbf{k}|$, for real photons). For the helicity-0 amplitudes, the quark transition operators are

$$\mathcal{L}_{1^+}=(3e_3/m)(\pi/K_0)^{1/2}[(\tfrac{2}{3})^{1/2}p_{\lambda_z}+\tfrac{1}{2}k]\exp[ik\lambda_z(\tfrac{2}{3})^{1/2}],\quad \mathcal{S}_{1^+}=3e_3(\pi/K_0)^{1/2}\exp[ik\lambda_z(\tfrac{2}{3})^{1/2}], \tag{5}$$

which follow from (3) and (4). The factor $(\pi/K_0)^{1/2}$ in (5) in definitions of $\mathcal{L}_{1^+}$ and $\mathcal{S}_{1^+}$ is to normalize L_{1^+} and S_{1^+} according to Ref. 4, and to satisfy the condition[16] $L_{1^+}/E_{1^+}\to 1$, as $|\mathbf{k}|\to 0$.

We now come to the calculation of the L_{1^+} multipole for arbitrary K^2 in the transition $\gamma_v N\to\Delta$, in the "current" and "charge" approaches, using the IK-GD and VBJ models. Relevant matrix elements, in the SU(6) basis for the nucleon and delta are given in Table I. Clearly, these are not identical. Thus, the truncated QSM wave functions for the nucleon and Δ severely violate the identity (1).

We plot, in Fig. 1, L_{1^+} as a function of K^2, using current and charge approaches in the IK-GD and VBJ models. Although curves B and C are not too dissimilar in shape, curves A and D for VBJ show large differences. This is indicative of the crudeness of the VBJ model.

Figure 2 shows L_{1^+} up to $K^2=-0.8$ GeV$^2/c^2$ in the IK-GD model. Curves A and C are calculated by use of the wave functions of Gershtein and Dzhikiya[4] (GD), while curves B and D use the coefficients of IK.[5] The only difference between the two is the inclusion of the antisymmetric state $|N\,^2P_A\rangle$ in the nucleon wave function of GD. This changes the other coefficients slightly. We see that the charge approach (curves C and D which overlap in Fig. 2) is not very sensitive to the details of the wave functions, while the current approach (curves A and B) yields noticeable differences. We may conclude from this, and similar calculations in nuclear physics[9] in larger model space, that the charge approach is relatively insensitive[6] to the truncation of the quark model space. This has important experimental implications: The scalar quadrupole transition amplitude S_{1^+} is reliably estimated from the QSM, and the role of the color-magnetic tensor force in the hadronic wave function can now be tested experimentally by this observable. Such an experiment would be of high priority[17] at the planned Continuous Electron Beam Accelerator Facility (CEBAF).

Some remarks concerning a comparison of the QSM prediction of the transition amplitudes for the $\gamma_v N\to\Delta$ process with existing experimental data are in order here. In Fig. 3, we show the ratio S_{1^+}/M_{1^+} as a function of $-K^2$ in the IK-GD model. Up to $-K^2\sim 3.5$ (GeV$/c)^2$, this ratio is predicted to be negative. Experimental results agree with the sign; at $-K^2=3$ (GeV$)/c)^2$ the experiment[18] gives -12% for the ratio, while the theoretical prediction is -6%. Considering the possible nonresonant background contributions in the experimental value, this is at least a qualitative indication of the role of the color tensor force in wave functions of the nucleon and delta. A stronger indication of this comes from the analysis of the multipole data at the photon point. Here, using the relation[9] $E_{1^+}(0)\simeq S_{1^+}(0)$, we get the ratio $E_{1^+}(0)/M_{1^+}(0)\simeq -0.6\%$ in the IK-GD

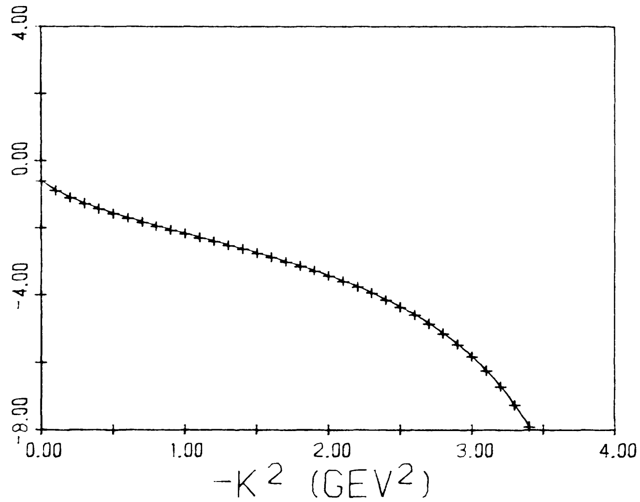

FIG. 3. The ratio S_{1+}/M_{1+} as a function of $-K^2$ in the IK-GD model.

model, while its value in the current approach[1] is -0.3%, and that inferred from the experiment[13] is between -0.5% and -1.5%. The correct sign of this ratio and correct order of magnitude are firm indications of the importance of color tensor force in many-quark physics.

Failure of the relation (1) in the QSM indicates that the currently available estimates[18] of the electrogmagnetic transition amplitudes for hadron excitation, computed in the QSM, should be reexamined.

In summary, the longitudinal and scalar quadrupole amplitudes in the $\gamma_v N \rightarrow \Delta$ transition are strongly sensitive to the deforming d-state admixtures introduced by the color-magnetic tensor interaction between valence quark pairs, predicted by QCD. We have shown that the relation (1) between these multipoles, derived from gauge invariance, fails badly in the truncated quark basis. However, the scalar quadrupole amplitude S_{1+} is insensitive to the truncation effects, and is reliably estimated in the quark-shell model. Available experiments are strongly indicative of a nonzero value of this amplitude, supporting theoretical predictions of the QSM. Experimentally, it would be interesting to be able to study carefully the helicity zero amplitude as a function of the virtual-photon mass squared. This should be an important goal at CEBAF. Finally, truncation effects, demonstrated here, should be of concern in any approximate model for hadrons.

We thank R. Davidson for many useful discussions. One of us (N.C.M.) thanks Professor N. Isgur and Professor G. Karl for valuable discussions on their model. Our research has been supported in part by the U. S. Department of Energy under Contract No. DE-AC02-83ER40114-A003.

[1]See, for example, N. Isgur and G. Karl, Phys. Rev. D **18**, 4187 (1978); R. Koniuk and N. Isgur, Phys. Rev. D **21**, 1868 (1980).

[2]A. DeRújula, H. Georgi, and S. L. Glashow, Phys. Rev. D **12**, 147 (1975).

[3]V. Vento, G. Baym, and A. D. Jackson, Phys. Lett. **102B**, 97 (1981); V. Vento and J. Navarro, Phys. Lett. **141B**, 28 (1984).

[4]S. S. Gershtein and G. V. Dzhikiya, Yad. Fiz. **34**, 1566 (1981) [Sov. J. Nucl. Phys. **34**, 870 (1981)].

[5]N. Isgur, G. Karl, and R. Koniuk, Phys. Rev. D **25**, 2394 (1982).

[6]D. Drechsel and M. M. Giannini, Phys. Lett. **143B**, 329 (1984).

[7]M. Weyrauch and H. J. Weber, Phys. Lett. **171B**, 13 (1986).

[8]F. A. Berends, A. Donnachie, and D. L. Weaver, Nucl. Phys. **B4**, 1 (1967).

[9]A. J. F. Seigert, Phys. Rev. **52**, 787 (1937); R. G. Sachs and N. Austern, Phys. Rev. **81**, 705 (1951); J. L. Friar and S. Fallieros, Phys. Lett. **114B**, 403 (1982); J. L. Friar and W. C. Haxton, Phys. Rev. C **31**, 2027 (1985).

[10]First hint of this failure was obtained by Drechsel and Giannini, Ref. 6, who found that the Siegert theorem (Ref. 9), for *real photons in the long-wavelength limit,* was not fulfilled in the QSM. Some minor errors in Ref. 6 do not negate this conclusion.

[11]G. Becchi and G. Morpurgo, Phys. Lett. **17**, 352 (1965); H. Harari and H. J. Lipkin, Phys. Rev. B **140**, 1617 (1965).

[12]S. L. Glashow, Physica (Amsterdam) **96A**, 27 (1979).

[13]R. Davidson, N. C. Mukhopadhyay, and R. Wittman, Phys. Rev. Lett. **56**, 804 (1986); R. Davidson, private communications.

[14]A. Donnachie, in *High Energy Physics,* edited by E. H. S. Burhop (Academic, New York, 1972), Vol. 5, p. 1; H. F. Jones and M. D. Scadron, Ann. Phys. (N.Y.) **81**, 1 (1973); R. S. Willey, Nucl. Phys. **40**, 529 (1963).

[15]J. D. Bjorken and J. D. Walecka, Ann. Phys. (N.Y.) **38**, 35 (1966).

[16]E. Amaldi, S. Fubini, and G. Furlan, *Pion-electroproduction* (Springer-Verlag, Berlin, 1979), Vol. 83.

[17]N. C. Mukhopadhyay, invited talk at the Continuous Electron Beam Accelerator Facility 1986 Workshop, to be published.

[18]M. Roos *et al.* (Particle Data Group), Phys. Lett. **111B**, 1 (1982), and **170B**, 259 (1986).