Naturally Small Dirac Neutrino Masses in Superstring Theories

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We show that a $Z_2 \otimes Z_3$ symmetry leads to the radiative generation of naturally small Dirac neutrino masses in a class of superstring theories. This model realizes in a simple and consistent way a recent suggestion by Masiero, Nanopoulos, and Sanda.

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Superstring theories[1] are a serious candidate for a unified theory of all fundamental interactions which predicts the dimensionality of space-time and highly restricts the choice of gauge group. Although there are still many open problems in superstring theories, attempts have been made at extracting some physical implications of these theories for four-dimensional physics at accessible energy scales.[2] The most attractive superstring scenario is based on the $E_8 \otimes E_8'$ heterotic string in ten dimensions leading upon compactification to an observable E_6 grand unified theory (GUT) coupled to $N=1$ supergravity. In order for this superstring-inspired E_6 GUT to be realistic, it has to satisfy various phenomenological constraints, such as preventing fast proton decay and avoiding too large neutrino masses. It is well known that a very attractive way of having naturally small neutrino masses is through the so-called see-saw mechanism.[3] Unfortunately this mechanism, at least in its simplest form, cannot be implemented in superstring-inspired E_6 GUT's, the reason being that the appropriate Higgs representation (**351** in the case of E_6) is not present in order to give a large mass to the right-handed neutrino.[4]

In this Letter, we show how the smallness of neutrino masses can be understood within this class of superstring gauge theories. Our specific model is inspired on a recent suggestion[5] of Masiero, Nanopoulos, and Sanda (MNS) of having naturally small Dirac neutrino masses generated through radiative corrections in a theory of where neutrinos are massless in tree approximation. We will first show that in order to implement the MNS proposal through the introduction of a discrete symmetry, one has to choose a symmetry which distinguishes the various generations. Then we construct an explicit model where a simple $Z_2 \otimes Z_3$ symmetry avoids tree-approximation neutrino masses, while allowing for the generation of naturally small neutrino masses through radiative corrections. The discrete symmetry also prevents fast proton decay, allows for realistic quark and lepton masses, and avoids tree-level flavor-changing neutral currents in the Higgs sector.

The matter fields transform as the fundamental representation of E_6, and so we start by writing the most general cubic superpotential arising from the coupling of three 27-plets of E_6:

$$\begin{aligned} &W = W_0 + W_1 + W_2,\\ &W_0 = \lambda_1 EQu^c + \lambda_2 Qd^c\bar{E} + \lambda_3 L\bar{E}e^c + \lambda_4 E\bar{E}S + \lambda_5 DD^cS,\\ &W_1 = \lambda_6 De^cu^c + \lambda_7 D^cLQ + \lambda_8 Dd^c\nu^c, \qquad (1)\\ &W_2 = \lambda_9 DQQ + \lambda_{10} D^cu^cd^c,\\ &W_3 = \lambda_{11} EL\nu^c. \end{aligned}$$

In order to establish the notation we give the SO(10) content of the states contained in the 27-plet:

$$\begin{aligned} &[\mathbf{16}] = [Q \equiv \binom{u}{d}, u^c, e^c, L \equiv \binom{\nu}{e}, d^c, \nu^c],\\ &[\mathbf{10}] = \left[D, E \equiv \begin{pmatrix} E^+ \\ E^0 \end{pmatrix}, \bar{D}, \bar{E} \equiv \begin{pmatrix} E^0 \\ E^- \end{pmatrix}\right], \qquad (2)\\ &[\mathbf{1}] = S. \end{aligned}$$

The Yukawa couplings λ_1 are tensors in generation space and do not obey[6] in general the E_6 Clebsch-Gordan relations.

The five couplings emerging in W_0 are needed in order to give mass to the standard quarks and charged leptons, as well as to the E_6 exotic fermions D, D^c, $E^{\pm}$, E^0, and $\bar{E}^0$. The terms W_1, W_2 cannot both be present in order to avoid rapid proton decay. However, the presence of either one of them alone is consistent with baryon number conservation. In order to avoid tree-approximation neutrino masses the term W_3 should be forbidden. However, calculable and naturally small neutrino masses could still be generated[5] through the diagram of Fig. 1, provided λ_7, λ_8 are both nonvanishing. The vanishing of W_3 may arise for topological reasons or as a result of the existence of discrete symmetries. Let us consider that we impose on the superpotential a discrete symmetry G. We show next that the above scheme of generation of radiative neutrino masses can be implemented in a consistent way, but it requires the symmetry G not to be generation blind. In order to see how this constraint arises, note that for a generation-blind symmetry, each superfield will transform as a one-dimensional representation of G, with equal charges for all generations. If one designates by α_1 the phase acquired by the term with coefficient λ_1

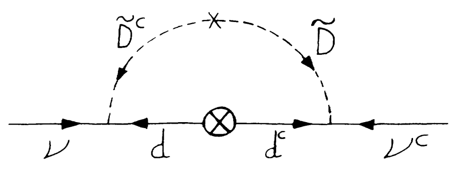

FIG. 1. Diagram contributing to generation of neutrino Dirac mass.

in W, under the action of an element of G [e.g., $\lambda_7 D^c Q \rightarrow \exp(i\alpha_7)\lambda_7 D^c LQ$], then the following relation holds:

$$\alpha_{11} = (\alpha_4 + \alpha_7 + \alpha_8) - (\alpha_5 + \alpha_2). \tag{3}$$

Since λ_7, λ_8 are needed in order for the diagram of Fig. 1 to exist and $\lambda_2, \lambda_4, \lambda_5$ are required in order to generate masses for the down quarks and the exotic E_6 fermions, it follows that a generation-blind symmetry cannot at the same time forbid tree-approximation neutrino Dirac masses and allow for radiatively generated neutrino masses. Needless to say, even with a generation-blind symmetry it is possible to have *exactly massless neutrinos* without running into conflict with any phenomenological constraints.[7] The discrete symmetry is constrained to distinguish the various generations only if one requires naturally small but nonvanishing neutrino masses.

Next we will show that if one allows the various generations to have different charges under the discrete group, then it is indeed possible to find a rather simple symmetry leading to a realistic model where neutrino masses are radiatively generated. The search for an appropriate discrete symmetry is facilitated if one writes the trivial generalization of Eq. (1) to the case where generations are allowed to have different charges. If one introduces Latin subscripts to denote the various families, writing, for example,

$$[\lambda_{11}]_{ijk} E_i L_j \nu_k^c \rightarrow [\lambda_{11}]_{ijk} \exp(i[\alpha_{11}]_{ijk} E_i L_j \nu_k^c),$$

then the generalization of Eq. (3) becomes

$$[\alpha_{11}]_{ijk} = ([\alpha_4]_{ilm} + [\alpha_7]_{njp} + [\alpha_8]_{qrk}) - ([\alpha_5]_{qnm} + [\alpha_2]_{prl}). \tag{4}$$

The meaning of Eq. (4) is straightforward. If one wants to avoid all tree-approximation neutrino masses (i.e., $[\lambda_{11}]_{ijk} = 0$ for any i,j,k), then there should be no choice of l,m,n,p,q,r for which all the couplings corresponding to the right-hand side of Eq. (4) are allowed. It turns out that there is a simple $Z_2 \otimes Z_3$ symmetry which satisfies this criterion and leads both to realistic fermion masses and radiatively generated neutrino masses:

$$\begin{aligned}
&Z_2\colon\ [Qu^c d^c DD^c]_i \rightarrow -[Qu^c d^c DD^c]_i,\\
&Z_3\colon\ [Qd^c L\nu^c DD^c]_i \rightarrow e^{i(2\pi/3)}[Qd^c L\nu^c DD^c]_i,\\
&E_1 \rightarrow e^{-i(2\pi/3)}E_1,\quad E_2 \rightarrow E_2,\quad E_3 \rightarrow E_3,\\
&\bar{E}_1 \rightarrow e^{-i(2\pi/3)}E_1,\quad \bar{E}_2 \rightarrow e^{i(2\pi/3)}\bar{E}_2,\quad E_3 \rightarrow E_3,\\
&S_1 \rightarrow e^{-i(2\pi/3)}S_1,\quad S_2 \rightarrow e^{i(2\pi/3)}S_2,\quad S_3 \rightarrow S_3.
\end{aligned} \tag{5}$$

It can be easily verified that this symmetry forbids all tree-approximation neutrino masses, avoids fast proton decay by forbidding the term W_2, allows for radiative generation of neutrino masses, and leads to realistic quark- and lepton-mass matrices:

$$m_e = \lambda_3 \langle \tilde{\bar{E}}_1 \rangle;\quad m_d = \lambda_2 \langle \tilde{\bar{E}}_2 \rangle;\quad m_u = \lambda_1 \langle \tilde{E}_1 \rangle, \tag{6}$$

where, as before, λ_i stands for matrices in generation space. For definiteness we have assumed three generations, but the extension to a larger number of generations is straightforward. A simple way of checking how this discrete symmetry avoids the constraints implied by Eq. (4) is by noting that $(\lambda_2)_{prl}, (\lambda_5)_{qnm}$ are nonvanishing only if $l = m = 2$. Since $(\lambda_4)_{i22}$ vanishes for all i, it follows that there is no choice of l,m for which all the couplings $(\lambda_4)_{ilm}, (\lambda_5)_{qnm}, (\lambda_2)_{prl}$ are nonvanishing, and therefore the criterion implied by Eq. (4) is satisfied. We would like to point out that the $Z_2 \otimes Z_3$ symmetry automatically solves another potential problem one encounters in superstring-inspired E_6 GUT's. Since there will be three sets of $E, \bar{E}$ states, natural flavor conservation in the Higgs sector will no longer be guaranteed.[8] It is clear from Eq. (5) that in the present model, as a result of the $Z_2 \otimes Z_3$ symmetry, the standard charged leptons and quarks of a given charge couple only to one Higgs field. This in turn guarantees that there will not be tree-level flavor changing neutral currents in the Higgs sector.

A rough upper bound on the neutrino mass, generated by the diagram of Fig. 1, was made[5] by MNS, by analyzing the contribution of $\tilde{D}$ and $\tilde{D}^c$ to rare processes such as $K^+ \rightarrow \pi^+ \nu\bar{\nu}$, μ + nucleus $\rightarrow e^+$ nucleus, and $\mu \rightarrow e\gamma$. For values of $M_{\tilde{D}}$ of 100–200 GeV they derived rough bounds on λ_7, λ_8 which in turn allow for neutrino masses in the range 0.1 to 50 eV. This is only a very rough estimate, since the structure of the matrices λ_7, λ_8 is not known. The important point is that, without fine tuning, one arrives at naturally small but nonvanishing neutrino masses in a range which can be of experimental interest. An important feature of this mechanism for generating small neutrino masses is that it leads to Dirac neutrinos[9] thus forbidding double-β decay.

In conclusion, we have shown that in the class of superstring-inspired GUT's considered here, a family-blind discrete symmetry can lead to exactly massless neutrinos, but cannot allow for the generation of neutrino masses through radiative corrections. We have then shown that if one allows the various generations to have different charges under the discrete symmetry, then a simple $Z_2 \otimes Z_3$ symmetry can lead to naturally small but nonvanishing Dirac neutrino masses, generated through radiative corrections. It is clear that the symmetry $Z_2 \otimes Z_3$ is far from unique, its choice having been dictated by simplicity.

After this work was essentially complete, we have learned that Ma in a recent Comment[10] on Ref. 5 has pointed out the impossibility of realizing the MNS proposal for having naturally small Dirac neutrino masses. This conclusion by Ma stems from the fact that in his analysis, only generation-blind symmetries were considered. In a subsequent Reply[11] to Ma's Comment, MNS have pointed out the need to assign different charges to the various generations.

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