Nonperturbative Length Scale in High-Temperature QCD

E. Manousakis and J. Polonyi[(a)]
Center for Theoretical Physics, Laboratory for Nuclear Science, Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 8 December 1986)

The string tension of spacelike Wilson loops is computed in SU(3) lattice gauge theory in the high-temperature deconfined phase. Its physical value is extracted following the continuum limit. An upper bound for the length scale where nonperturbative fluctuations become dominant is computed. The value of the string tension and the nonperturbative length scale are very close to their zero-temperature counterparts.

PACS numbers: 12.38.Gc, 11.15.Ha

The investigations of strong interactions at high temperature were initiated by the observation that the effective coupling constant $g_{\text{eff}}^2(T)$ becomes small at high temperature. In fact, the temperature T plays a similar role to the momentum scale in the standard manipulations with the renormalization-group equations. It was hoped that QCD becomes a weak-coupling theory at high enough temperature and we shall have better chances to identify the elementary constituents of the strong interactions. It was encouraging too when analytical[1] and numerical[2] evidence was found for the existence of the deconfined phase at high temperature. It was soon realized that the infrared modes ($\mathbf{p}\rightarrow 0$) spoil the weak-coupling expansion even at arbitrarily high temperature.[3] The real expansion parameter in summing up finite-temperature Feynman graphs is g^2T/m_m, where m_m is the mass gap in the chromomagnetic propagator, which is at most $m_m = c^{-1}g^2T$. Thus one finds an expansion parameter c whose actual value is inaccessible in the weak-coupling expansion.

There were attempts to estimate a mass scale related to the magnetic field by means of numerical simulations.[4] But in order to make statements about the high-temperature weak-coupling expansion we need control over finite-size effects and the verification of the proper continuum behavior on the lattice. In addition, one has to be careful in referring to (lattice) weak-coupling graphs when $g_{\text{eff}}^2(T) > g_{\text{eff}}^2(T/N_0) = g_{\text{lattice}}^2 \sim 1$ (N_0 is the lattice size in the timelike direction). Furthermore, it is not obvious how the mass scale associated with the Z_n magnetic fluxes[4] is related to m_m. The latter controls the infrared behavior of small fluctuations of the magnetic field.

Special care is needed when interpreting the numerical results for the internal-energy density at high temperature.[5] The not uncommon view that we have evidence for a gas of weakly interacting gluons at high temperature is not necessarily correct because of finite-size effects occurring in the calculations done so far. All that one can say is that the results are compatible with having eight light, weakly interacting quasiparticles in quarkless QCD. We believe that this conclusion survives the thermodynamic limit and that the quasiparticles are plane waves of gluons locally (asymptotic freedom) and become distorted beyond a certain length scale.[6] The aim of this Letter is to estimate this length scale which signals the onset of nonperturbative effects. Such a calculation can be done numerically without our making assumptions about the details of the dynamics of the quasiparticles. The discussion of this dynamics and how these distorted gluon waves form a weakly interacting gas requires a nonperturbative framework[7,8] and is beyond the scope of the present work.

First it is worthwhile to refer to an exact result of high-temperature lattice gauge theory. For a finite value of the lattice spacing and sufficiently high temperature the spacelike Wilson loops follow area behavior.[9] The proof is a formal elaboration on the point that high temperature corresponds to strong coupling in spacelike directions. In fact, when we control the temperature by introducing an anisotropic lattice, $\xi = a_t/a_s \neq 1$, in order to separate dependences on time and spacelike extents of the system, the lattice action becomes

$$S = \frac{6}{g^2}\left[\xi \sum_{\Box} \Box_{\text{sp-sp}} + \xi^{-1} \sum_{\Box} \Box_{\text{sp-t}}\right],$$

where $\Box_{\text{sp-sp}}$ and $\Box_{\text{sp-t}}$ denote contributions to the action from spacelike and timelike plaquettes, respectively. At high temperature $\xi \ll 1$ and the spacelike links behave as in a strong-coupling model, i.e., the leading contribution to the spacelike Wilson loop is obtained by tiling up the area of the loop with spacelike plaquettes. Unfortunately such simple considerations can tell us nothing about the cutoff dependence of the spacelike string tension and thus about continuum QCD.

A more systematic way to describe nonperturbative features of thermal Green's functions of the continuum theory is to consider the effective theory of the gluon fields at a fixed time slice $t=t_0$,

$$\exp\{-S_{\text{eff}}[\tilde{A}_\mu(\mathbf{x})]\}=\int D[A_\mu(\mathbf{x},t)]\exp\{-S[A_\mu(\mathbf{x},t)]\}\prod_{\mathbf{x}}\delta(A_\mu(\mathbf{x},t_0)-\tilde{A}_\mu(\mathbf{x})).$$

In fact, the spacelike Wilson loops are given in terms of $A_\mu(\mathbf{x},t_0)$ only and are thus governed by $S_{\text{eff}}[\tilde{A}_\mu(\mathbf{x})]$. It is known that $S_{\text{eff}}[\tilde{A}_\mu(\mathbf{x})]$ describes a three-dimensional Yang-Mills-Higgs system with $\tilde{A}_0(\mathbf{x})$ as the matter field in the adjoint representation. In addition at sufficiently high temperature, where the static modes dominate the path integral, S_{eff} is of the form

$$S_{\text{eff}}=-(1/4g^2T)\tilde{F}_{ij}\tilde{F}_{ij}-D_i\tilde{A}_0^\dagger D_i\tilde{A}_0-V(\tilde{A}_0),\qquad i,j=1,2,3.$$

$\tilde{F}_{ij}$ is the antisymmetric tensor of the spatial field components. The coupling constants of the manifestly gauge-invariant local potential $V(\tilde{A}_0)$ are functions of g^2T. It is known that the three-dimensional Yang-Mills theory is confining.[10] Moreover, all the nonperturbative dynamics of high-temperature QCD come from this effective theory.

A Monte Carlo calculation was carried out to determine the actual value of the string tension σ_s of spatial Wilson loops and to study its continuum limit. We obtained an upper bound for the length scale where nonperturbative effects dominate thermal Green's functions as a by-product of the calculation in the following way: Consider $V(b,c)=-\ln[W(b,c)]$, where $W(b,c)$ is the expectation value of a spacelike Wilson loop with sizes b and c. The $\ln[W(b,c)]$ selects the connected parts of the correlation function. It involves the contributions of modes with length scale $\xi<b$ or c. In order to select the contributions of modes with $\xi\sim b$ we form $Z(b,c)=\partial V(b,c)/\partial b$. Confinement of the three-dimensional theory guarantees that the nonperturbative tail $Z(b,c)=\sigma_s c$, as b tends to infinity. We define the length scale ξ_0, that at which $Z(\xi_0,c)=2\sigma_s c$. It is clear that for $b>\xi_0$ the nonperturbative contributions dominate in Z. This discussion is similar to the one in the case of the force acting between static charges. In that case, one considers timelike Wilson loops and takes the derivative with respect to the spatial separation in order to obtain the force. It is obvious that the force is the relevant quantity to look into as opposed to the potential. This way of defining ξ_0 is rather arbitrary, but the important point is to use the same definition when we compare zero and finite temperature.

In the numerical part, the spacelike Wilson loop $W(b,c)$ was replaced by the correlation function of the Polyakov lines wrapping around the periodic lattice in spatial direction. We define

$$W_{\mu j}(b,L)=\langle\mathrm{tr}\,\Omega_\mu(0)\mathrm{tr}\,\Omega_\mu^\dagger(b\mathbf{e}_j)\rangle,$$

$$\Omega_\mu(n_{j\neq\mu})=\prod_{n_\mu=1}^{N_\mu}U_\mu(n_{j\neq\mu},n_\mu),$$

$$L=aN_\mu,\quad \mu=0,1,2,3,\quad j=1,2,3,\quad \mu\neq j.$$

The difference between the Wilson loops and $W_{\mu j}$ is obvious when the z direction is interpreted as the Euclidean time. In that case W includes the contribution of the color singlet and octet states of the static quark-antiquark system. The Wilson loop corresponds to the singlet case only. Consequently W_{ij} and the Wilson loop agree when both follow area behavior and $L>b$.

We may define a static "potential" from the spacelike correlation function as $V_s^{(N_0)}(b)=-[\ln W_{ij}(b,L)]/L+\text{const}$. The V_s computed on a $10^3\times6$ lattice at $\beta\equiv6/g^2=6$ is shown in Fig. 1 (squares). The physical static potential $V_t^{(N_0)}(b)=-[\ln W_{0j}(b,c)]/c+\text{const}$, measured (by timelike Wilson loops) on confined (zero temperature) lattice at the same value of β, taken from D'Hoker,[10] is plotted as well (dashed line). The constants in the previous expressions are chosen such that $\frac{1}{2}[V_s(2a)+V_s(4a)]=\frac{1}{2}[V_t(2a)+V_t(4a)]=0$. The physical static potential at finite temperature can be obtained from the correlation function of the timelike Polyakov lines as measured on our $10^3\times6$ lattice. This potential, $V_t^{(N_0)}(b)=-[\ln W_{0j}(b,\beta)]/\beta+\text{const}$, is also plotted in Fig. 1 (dash-dotted line). Since at $T=0$, $W_{ij}=W_{0j}$, the obvious lesson of this calculation is that the correlation function of the spacelike components of the gauge field at the present $T\sim2T_{\text{dec}}$ is very close to its zero-temperature confined counterpart. The act of deconfinement appearing in the behavior of the timelike

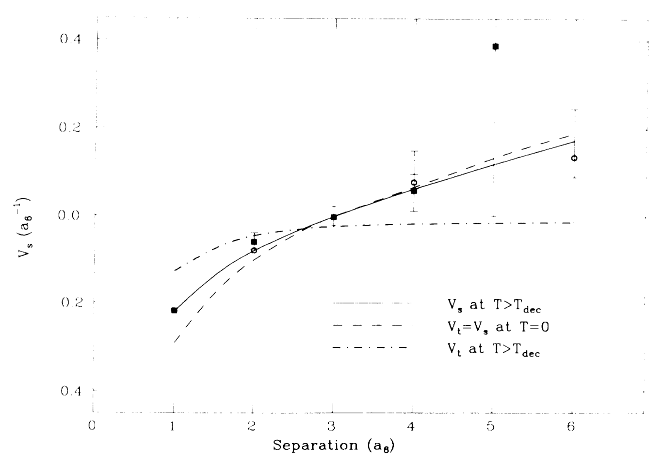

FIG. 1. The correlation function of spacelike Polyakov lines $V_s(b)$ computed on $10^3\times6$ and $6^3\times3$ lattices are shown by squares and circles, respectively. The solid line is obtained by a fit with a function of the form $V(b)=a/b+\sigma_s b+\text{const}$. The dashed line is the V_t at $T=0$, taken from Bowler *et al.* (Ref. 11). The dash-dotted curve is the V_t computed at $10^3\times6$ and $6^3\times3$ at $T>T_{\text{dec}}$. Everything is expressed in units of the lattice spacing a_6 of the $10^3\times6$ lattice.

component of the gauge field does not influence the spatial components in a substantial way.[12]

The lattice spacing a was left unchanged in the previous discussion. In order to make contact with the continuum theory one has to follow the limit $a \to 0$. This program has been carried out quite exhaustively for the physical zero-temperature static potential and the result is consistent with having finite string tension in the continuum limit.[13] The results of Fig. 1 and the standard flux-tube picture of the linearly rising part of the static potential suggest that the slope of the function $V_s(b)$ is insensitive to the dynamics of the shorter timelike direction. Thus it seems likely that the slope of $V_s(b)$ remains finite in the continuum limit.

To verify this conjecture, $V_s(b)$ was calculated on two lattices with lattice spacings a and a' with $a'=2a$, in the following way: The ratio of the lattice spacings $a'/a=2$ is achieved on lattices $N_s^3\times N_t$ and $(2N_s)^3\times 2N_t$ by the tuning of $g^2(a')$ and $g^2(a)$, so that the finite part of the correlation functions $V_t^{(N_t)}(na')$ and $V_t^{(2N_t)}(2na)$ computed at the corresponding g^2 agree. It is advantageous to use $V_t(b)$ to find the values of the coupling constant $g^2(a)$ and $g^2(a')$ which correspond to the same temperature,[14] because it has strong temperature dependence.

Because of limitation of computer power we had to choose lattices of the size $10^3\times 6$ and $6^3\times 3$ in the present calculation. The subtracted potential

$$V_t^{(N_0)}(b) = -[\ln W_{0j}(b,\beta) - 2\ln\langle \mathrm{tr}\,\Omega_0(0)\rangle]/\beta$$

is plotted in Fig. 2. The best matching between the two lattices $10^3\times 6$ and $6^3\times 3$ was found at $6/g^2$ equal to 6 and 5.5615, respectively. So $\Delta\beta$ which realizes the halving of the lattice spacing is 0.44 for $\beta=6$. $\Delta\beta$ has been calculated with the use of much larger lattices and was

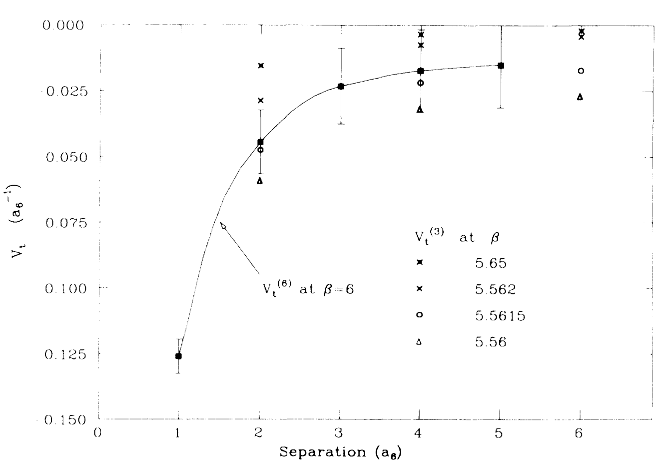

FIG. 2. The correlation function of timelike Polyakov lines $V_t(b)$ computed on $10^3\times 6$ at $\beta=6$ and $6^3\times 3$ lattices at various β values. The matching condition is satisfied at $\beta=5.5615$ for the $6^3\times 3$ lattice.

found around 0.3[14] or 0.35.[11,15] This discrepancy is related to finite-size effects. In fact, the spatial size of the two lattices was almost the same in our case. So our $\Delta\beta$ corresponds to the theory placed in a finite quantization box with size $L\sim 1$ fm which tends to increase $\Delta\beta$. The spatial correlation function $V_s^{(3)}(b)$ is plotted in Fig. 1 (circles). The constant term was determined on the smaller lattice by requiring $\frac{1}{2}[V_s^{(3)}(a)+V_s^{(3)}(2a)]=0$. The stability of the potential $V_s(b)$, as far as this change $a\to 2a$ is concerned, is supported by this result.

The static potential can be approximated by the function $V(b)=-\alpha/b+\sigma b+c$. In the case of the correlation function of spacelike Polyakov lines we find $\alpha=0.184\pm 0.02$, $\sqrt{\sigma}=0.22\pm 0.03$. The corresponding zero-temperature values are $\alpha=\pi/12$ and $\sqrt{\sigma}=0.22 \pm 0.02$.[11]

The numerical results presented above support the picture that spacelike Wilson loops follow area behavior even in the high-temperature deconfined phase. Moreover it was found that at the temperature considered, $T\sim 2T_{\rm dec}$, the value of the string tension of the spacelike Wilson loop is very close to those of the zero-temperature theory. Considering the path integral as the partition function of a four-dimensional classical system, one can say that the compactification of one (time) direction does not influence the asymptotic behavior of the correlation function of the gauge-field components of the other (spacelike) directions. We find this behavior surprising, since the system undergoes a phase transition at $T=T_{\rm dec}$ which changes the dynamics of the time component of the gauge field completely.

The physical string tension at finite temperature becomes zero at $T=T_{\rm dec}$.[16] It remains to be seen how the spacelike string tension behaves in the vicinity of $T_{\rm dec}$. Spacelike Wilson loops are controlled by the three-dimensional Yang-Mills-Higgs system of the static modes at sufficiently high temperature and by the complete four-dimensional theory at low T. There is no obvious reason to believe that the spacelike string theory would be even approximately the same for $T<T_{\rm dec}$ and $T>T_{\rm dec}$. Thus the temperature dependence of the spacelike string tension is a proper test of our understanding of the confinement mechanism. As an example one may consider the suggestion that chromomagnetic monopoles are present in QCD.[6] This scenario involves the condensate at $T<T_{\rm dec}$ and the gas of localized monopoles for $T>T_{\rm dec}$. Although no self-consistent analytical approximation is known to sum up the contributions of such objects in the path integral, it seems plausible that the spacelike string tension originates ultimately from the condensate by dual Meissner effect[17] at low temperature[18] and from the fluctuations around localized monopoles[19] at high temperature. The monopole density becomes zero at the transition temperature and increases as a singular function of $T-T_{\rm dec}$. If correct, this description should explain why the spacelike string tension depends weakly on the monopole density and happens to be similar in both cases.

The actual form of $V_s(b)$ allows us to give an upper bound for the length scale ξ_0 characterizing the appearance of nonperturbative contributions in thermostatic correlation functions. We find $\xi_0 \sim 2.0a \sim 0.20$ fm at $T \sim 2T_{\rm dec}$ as opposed to $\xi_0 \sim 2.3a \sim 0.23$ fm at $T=0$. Although the string tension appears to be almost the same, the short-distance behavior differs at the two values of the temperature studied. The coefficient of the $1/b$ term in $V_s(b)$ is consistent with the value obtained from the two transverse fluctuations of the flux tube at $T=0$, $\alpha=\pi/12$.[20] At $T \sim 2T_{\rm dec}$ this coefficient becomes smaller and lies between the values corresponding to two or one transverse mode, $\pi/24 < \alpha < \pi/12$. In general, α should approach $\pi/24$ as the temperature increases. Since the correlation function V_s is less steep at short distances for high temperature, $\xi_0(2T_{\rm dec}) < \xi_0(0)$. But $\xi_0(T)$ is an upper estimate for the nonperturbative length scale only. The applicability of the perturbation expansion must be even more restricted in the vicinity of the critical temperature. ξ_0 defined by $V_t(b)$ diverges in this region since the physical string tension vanishes at $T=T_{\rm dec}$. The upper bound ξ_0, based on the generation of a linear term in static potentials, is clearly inadequate in this case. One needs an insight into the dynamics at $T \sim T_{\rm dec}$ to select another nonperturbatively generated term for our purpose. What one can say at the present stage is that it does not help to increase the temperature if we intend to eliminate the nonperturbative aspects of the strong interactions. Any description of the dynamics of high-temperature QCD, which involves length scales larger than 0.2 fm, must rely substantially on nonperturbative effects.

We thank J. Smit for a useful discussion. The numerical calculations were done at the VAX system at the Laboratory for Nuclear Science of Massachusetts Institute of Technology and the Cray-XMP machine of the Lawrence Livermore National Laboratory. This work was supported in part by funds provided by the U.S. Department of Energy under Contract No. DE-AC02-76ER03069.

[a]On leave of absence from the Central Research Institute for Physics, Budapest, Hungary.

[1]A. Polyakov, Phys. Lett. **72B**, 477 (1978); L. Susskind, Phys. Rev. D **20**, 2610 (1979).

[2]L. McLerran and B. Svetitsky, Phys. Lett. **98B**, 195 (1981); J. Kuti, J. Polonyi, and K. Szlachanyi, Phys. Lett. **98B**, 199 (1981); J. Engels, F. Karsch, I. Montvay, and H. Satz, Phys. Lett. **101B**, 89 (1981).

[3]A. D. Linde, Phys. Lett. **96B**, 289 (1980).

[4]A. Billoire, G. Lazarides, and Q. Shafi, Phys. Lett. **103B**, 450 (1981); T. DeGrand and D. Toussaint, Phys. Rev. D **25**, 526 (1982).

[5]J. Engels, F. Karsch, I. Montvay, and H. Satz, Nucl. Phys. **B205**, 565 (1982); T. DeGrand and C. DeTar, University of Colorado Report No. COLO-HEP-131, 1986 (to be published).

[6]C. DeTar, Phys. Rev. D **32**, 276 (1985).

[7]J. Polonyi and H. W. Wyld, Massachusetts Institute of Technology Report No. CTP-1452, 1986 (to be published); J. Polonyi, in Quark Matter '86, Proceedings of the Fifth International Conference on Ultrarelativistic Nucleus-Nucleus Collisions, Pacific Grove, California, 1986 (to be published).

[8]D. Gross, R. Pisarski, and L. Yaffe, Rev. Mod. Phys. **53**, 43 (1981); K. J. Dahlem, University of Bonn Report No. HE-85-17, 1985 (to be published).

[9]J. Borg, Nucl. Phys. **B261**, 455 (1985).

[10]E. D'Hoker, Nucl. Phys. **B180**, 341 (1981).

[11]K. C. Bowler, F. Gutbrod, P. Hasenfratz, U. Heller, F. Karsch, R. D. Kenway, I. Montvay, G. S. Pawley, J. Smith, and D. J. Wallace, Amsterdam University Report No. ITFA-85-07, 1985 (to be published).

[12]M. Lev, P. Peterson, and H. Satz, unpublished.

[13]See the papers quoted in Ref. 11.

[14]A. D. Kennedy, J. Kuti, S. Meyer, and B. J. Pendleton, Phys. Lett. **155B**, 414 (1985).

[15]A. Hasenratz, P. Hasenfratz, U. Heller, and F. Karsch, Phys. Lett. **143B**, 193 (1984).

[16]T. DeGrand and C. DeTar, Phys. Rev. D **34**, 2469 (1986).

[17]H. B. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973); S. Mandelstam, Phys. Rep. **23C**, 245 (1976); G. 't Hooft, Phys. Scr. **25**, 133 (1982).

[18]J. Polonyi, in *Relativistic Dynamics and Quark Nuclear Physics,* edited by M. B. Johnson and A. Pisklesimer (Wiley, New York, 1986).

[19]A. Polyakov, Nucl. Phys. **B120**, 429 (1977).

[20]M. Luscher, Nucl. Phys. **B180**, 317 (1981).