Sideband Control in a Millimeter-Wave Free-Electron Laser

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The frequency offset of the sideband instability in a free-electron laser (FEL) should depend on $(1-v_\parallel/v_g)^{-1}$, where $v_\parallel$ is the average longitudinal velocity of the electrons and v_g is the group velocity of the electromagnetic waves. We have tested the $v_\parallel/v_g$ dependence of the sideband shift in a 2-mm, Raman-regime FEL oscillator. A change of $v_\parallel/v_g$ from 0.93 to 0.98, accomplished by an increase in the undulator period, resulted in the measured sideband shift increasing from 6% to 40%, in approximate agreement with theory.

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Free-electron lasers (FEL's), especially in the microwave (3 cm–1 mm) range, have been demonstrated to be prodigious sources of power. Microwave FEL experiments at Columbia University,[1] Massachusetts Institute of Technology,[2] the Naval Research Laboratory,[3,4] and Lawrence Livermore National Laboratory and Lawrence Berkeley Laboratory[5,6] have reported peak output powers ranging from 2 MW to 1 GW. Only a few studies have been made of the spectral quality of this power, sometimes initiated from noise, and sometimes as an amplification of an input signal.

Many applications of microwave FEL's require a signal with a well-defined frequency; i.e., considerable temporal coherence. The sideband instability threatens this coherence, and is a subject of much FEL work, both experimental[1,7] and theoretical.[8]

The physical origin of the sideband instability is the slippage between the light pulse and the electron pulse in an FEL. The slippage is customarily considered to occur because the light travels at c, while the electrons travel at some $v_\parallel < c$. The slippage couples different longitudinal slices of the electron beam, and can lead to growing modulations in the light intensity, coupled to synchrotron oscillations of electrons in buckets.

A microwave FEL must operate in a waveguide because the wavelength of the electromagnetic radiation is long and diffraction is very strong. But a waveguide provides the possibility of controlling the *group* velocity of the radiation, or the velocity at which modulations propagate. It is perfectly possible to arrange for the microwave group velocity to equal the parallel electron velocity. In that circumstance, the slippage vanishes, modulations of the radiation do not propagate forward with respect to the electron pulse, and, in principle, the sideband instability should be completely stabilized.[9]

For a waveguide we may write

$$\omega^2/c^2 = k^2 + k_\perp^2, \tag{1}$$

where $k_\perp$ is the transverse wave number. The group velocity is

$$v_g \equiv d\omega/dk = kc^2/\omega = c[1+k_\perp^2/k^2]^{-1/2}. \tag{2}$$

For large γ, the parallel velocity of the electrons is

$$v_\parallel = c[1-(1+\gamma^2\beta_\perp^2)/2\gamma^2], \tag{3}$$

where $\beta_\perp$ is the normalized transverse velocity of the electrons due to the wiggler magnetic field $B_\perp$ and longitudinal guide field B_z, if any. Equating v_g and $v_\parallel$ we obtain the condition for sideband suppression:

$$\gamma \simeq (1+\gamma^2\beta_\perp^2)^{1/2}(k/k_\perp). \tag{4}$$

It is easy to estimate the frequency, in the laboratory, of the sideband as a function of $v_\parallel$ and v_g. In the Compton regime, the sidebands approximately satisfy the relation

$$[(k\pm\Delta k)+k_w]z-(\omega\pm\Delta\omega)t \simeq k_{\text{synch}}z, \tag{5}$$

where

$$k_{\text{synch}}^2 = \frac{2}{\gamma^2}\left(\frac{eE_s}{mc^2}\right)\left(\frac{eB_\perp}{mc^2}\right) \quad \text{(cgs units)}. \tag{6}$$

Here $\Delta\omega$ and Δk are the shift from the fundamental of the sideband frequency and wave number, respectively, k_w is the wiggler wave number, E_s is the signal electric field strength, and k_{synch} is the wave number of the synchrotron oscillations of electrons in the ponderomotive potential or buckets. Writing $z = v_\parallel t$, using the resonance condition for the FEL

$$\omega = (k + k_w) v_\parallel, \tag{7}$$

and recognizing that $\Delta\omega = v_g \Delta k$, one finds that

$$\Delta\omega \simeq k_{\rm synch} c/(1 - v_\parallel/v_g). \tag{8}$$

Equation (8) has been derived for the Compton regime, but the result $\Delta\omega \propto (1 - v_\parallel/v_g)^{-1}$ is very general, and is rigorously valid (following directly from the Manley-Rowe relations) in both the Compton and the Raman regimes, as long as $\Delta\omega/\omega$ is small. When $\Delta\omega$ becomes a significant fraction of ω, the scaling is approximate. It is Eq. (8) which we study experimentally. As v_g approaches $v_\parallel$ the sideband frequencies should get further from the main frequency.

The equations that describe sideband growth[10,11] are modifications of the standard, time-independent FEL equations,[12,13] with derivatives with respect to longitudinal distance z replaced by convective derivatives corresponding to the particle motion (in the $2N$-particle equations, where $v_\parallel$ enters) and to the propagation of signal modulations (in the field equations, where v_g enters). The solution of these coupled equations demonstrates that the peak growth rate of the sideband instability is *independent* of $v_\parallel/v_g$. This conclusion cannot be valid as $v_\parallel \to v_g$ because Eq. (8) predicts that the lower sideband moves beyond waveguide cutoff; this must have a significant effect upon the sideband growth rate. The difficulty arises because the modified FEL equations, with convective derivatives, consider a slowly varying amplitude and phase of the central frequency with fixed group velocity. This approximation is not adequate to describe sidebands which are far removed (in $\Delta\omega/\omega$ or in $\Delta k/k$) from the central frequency.

The experiment was done with the Columbia University free-electron laser, and the reader is referred to Ref. 1 for further experimental details and a schematic of the laser. An electron beam was extracted from an apertured, cold-cathode diode immersed in a strong (7–10 kG) magnetic field. A typical value of accelerator voltage was $\sim$700 kV, and a beam current $\sim$200 A was injected down the axis of a 6-mm-diam drift tube which also serves as a cylindrical waveguide. The undulator was a bifilar helical winding energized by a small capacitor bank timed with the accelerator pulse. The undulator consisted of a uniform section 50 cm in length between two zones (7.3 cm each) of adiabatically changing field. Two mirrors were used for feedback; the upstream mirror was a polished annular disk with reflectivity equal to 65% and the output mirror was a surface of a quartz window, with net reflectivity $\sim$5–10%. After a certain start time, the FEL oscillates at a power level $\sim$2–4 MW (determined calorimetrically). The spectrum was observed with a grating spectrometer (resolving power equal to 100)[14] when the power reached saturation. To obtain each spectrum, roughly 100 shots must be taken under circumstances of constant and reproducible accelerator voltage (2% variation was acceptable), and data at each wavelength were averaged. To speed the data acquisition, two calibrated Schottky-barrier diode detectors were used in the spectrometer.

The range of wavelength studied was $\sim$2–4 mm. The laser operates in the regime of stimulated Raman backscattering, as the beam current density is high ($\sim$1 kA/cm^2) and the pump field is "weak" ($B_\perp \sim 600$–800 G); the quiver velocity of the electrons, with the guiding field taken into account, is about 10% of the speed of light. The electron orbits are "stable type I." A 3D theory[15] has been used to predict successfully not only the wavelength of the radiation but also the small-signal coefficient of exponential signal growth ($\sim$0.1 cm^{-1}). The EM wave in the drift tube is propagated in the TE_{11} mode, appreciably above the cutoff wavelength of 1 cm.

A study of the time-dependent power at the wavelength of the FEL fundamental and its sideband showed the following features. After an interval of a few radiation bounce times ($2L_c/c \sim 10$ nsec each), the power was observed to grow to detectable levels: This defines the start time of the device. Roughly one bounce time later the fundamental saturates, and fluctuates in strength at megawatt levels. This is followed, usually within one bounce time, by the appearance of a strong sideband signal. This indicates that the sideband growth rate is comparable to the growth rate of the FEL signal.

The experiment was done in two stages, with the use

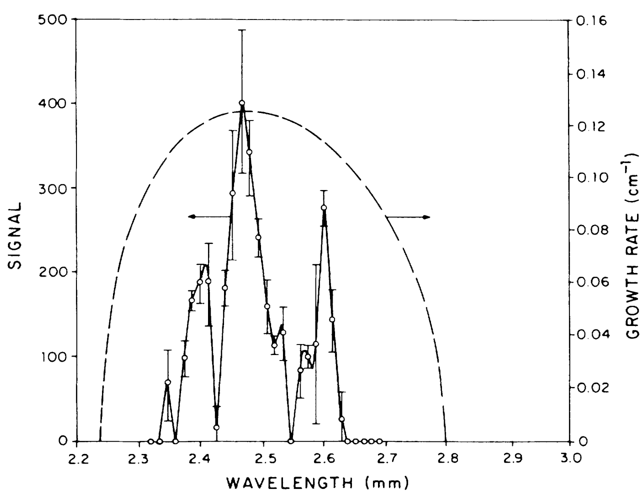

FIG. 1. Spectrum of FEL radiation, obtained under the following conditions: $V = 700$ kV; $B = 760$ G, $I = 200$ A, $\lambda_w = 1.45$ cm. The theory curve (dashed line) is the small-signal growth rate calculated according to Ref. 15. The solid line is drawn through the data points with a spline fit. The spectrum is obtained under conditions of saturated power. Guiding field $= 9.5$ kG.

of different undulators. These were identical in all respects, except that the period (λ_w) in one case was 1.45 cm (corresponding to the experimental situation of Ref. 1) while the other undulator had a period of 1.85 cm. In Fig. 1 we show a spectrum obtained with the $\lambda_w = 1.45$-cm undulator [another spectrum, at weaker $B_\perp$, is shown as Fig. 1(c) in Ref. 1]. In Fig. 1 the spectrum shows both upper and lower sidebands; at stronger pump field, the long-wavelength sideband grows to a power level comparable with that of the fundamental. The linewidth of the fundamental, under conditions where the sideband power is not large,[1] is about 1%. On the other hand, the width of the linear growth spectrum is theoretically found to be $\sim$5–10%, and so appreciable line narrowing has occurred from the quasioptical resonator ($Q \sim 1000$). From Eq. (8), the fractional sideband shift is proportional to the ratio N_{synch}/N, where N, the number of undulator periods, is 35. The synchrotron period, L_{synch}, defines $N_{\text{synch}} = L/L_{\text{synch}}$, where L is the undulator length. The synchrotron period can be calculated once the power level in the resonator is known; this gives $N_{\text{synch}} \sim 1$. With $V = 700$ kV, at the given $B_\perp$, $v_\parallel/c = 0.90$ and we compute $v_\parallel/v_g = 0.93$ from the waveguide properties; the predicted (Eq. 8) sideband shift is 6%, in good agreement with the measured value.

The experiment was then "repeated" at $V = 780$ kV, with the same hardware, but with the 1.45-cm period undulator replaced by the 1.85-cm undulator. This has the effect of increasing the fundamental wavelength in which case the ratio $v_\parallel/v_g$ becomes a very important factor in increasing the sideband shift. In order to keep the Group I orbits away from magnetoresonance, we reduced the guiding field from 9.5 to 7.3 kG. In Fig. 2 the spectrum clearly shows the FEL line at 2.75 mm and the lower sideband at 4 mm; the most striking feature is the increase of the sideband shift to $\sim$40% (the upper sideband fell outside the range of the spectrometer grating). Computing $v_\parallel/c = 0.91$, noting that now $N = 27$, and taking $N_{\text{synch}} = 1$ (the laser power is nearly the same), we find, from Eq. (8), $v_\parallel/v_g = 0.98$. As the cutoff wavelength has not changed, we can calculate what FEL wavelength would be required to give this group velocity: the result is 3 mm, which is close to the actual FEL fundamental. The spectrum of Fig. 2 shows a dramatic increase of the sideband shift due to a small change in group velocity, in accord with the result in Eq. (8).

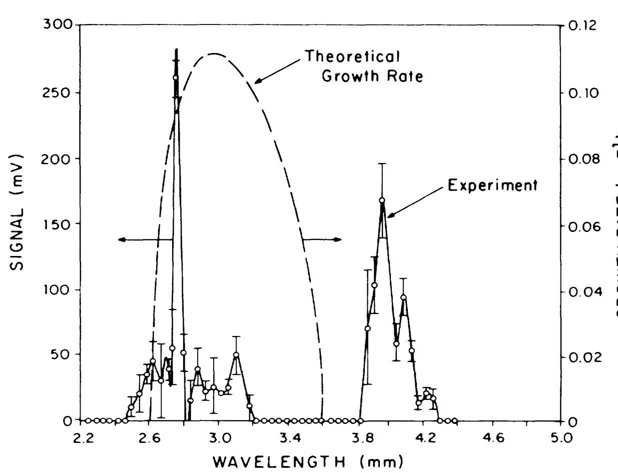

FIG. 2. Data obtained for the same FEL as in Fig. 1, but under the following conditions: $V = 780$ kV, $B = 690$ G, $\lambda_w = 1.85$ cm, and guiding field $= 7.3$ kG.

Calculations of the small-signal growth rate have been made for the conditions appropriate for Figs. 1 and 2, with the use of a 3D theory[15]; these are shown as dotted lines in the figures (the peak growth rate is kept about the same). It is interesting to note that the sideband falls outside the zone of unstable growth for the FEL in the case of the 1.85-cm undulator. Although the effect of the guiding field is accounted for in the small-signal theory, it is not accounted for by the theory in this paper, nor has it been used in the waveguide calculations. The waveguide is filled with electrons through which a right-handed circularly polarized EM wave is propagated, but the effect of the guiding field on the group velocity is negligible because the invariant plasma frequency $[ne^2/\pi\gamma m]^{1/2}$ is $\sim$2 GHz and the cyclotron frequency $eB_z/2\pi\gamma mc$ is $\sim$9 GHz, whereas the wave frequency is 75–100 GHz.

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