Valence p-n Interactions and the Development of Collectivity in Heavy Nuclei

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A universal, physically intuitive description of collectivity in heavy nuclei is obtained in terms of the parameter $P=N_pN_n/(N_p+N_n)$ which gives the average number of interactions of each valence nucleon with those of the other type. *Regardless* of mass region, deformation sets in when $P=P_{crit}\approx 4$–5. This, in turn, gives a rule that medium-mass and heavy nuclei with effective N_p or $N_n \leq 4$ cannot be deformed. An interpretation in terms of the monopole and quadrupole p-n interactions is discussed.

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In recent papers,[1,2] a scheme was introduced in which nuclear systematics are parametrized by the valence-nucleon product N_pN_n instead of the usual N, Z, or A. In the N_pN_n scheme an observable in a given phase transitional region falls on a single smooth curve. Moreover, curves for different regions have nearly identical structure.

Figure 1 illustrates these results for six transitional regions between $A\approx 100$ and the actinides. It is important to note that, here and in the other plots below, one must account for important subshell closures in the counting of the valence-nucleon numbers N_p and N_n. This has been discussed at length in Refs. 1 and 2. Specifically, in the $A=100$ (150) mass regions, the proton shells are defined by $Z=38$–50 ($Z=50$–64) for $N<60$ ($N<90$) and by the normal $Z=28$–50 and 50–82 magic numbers otherwise. In the $A=190$ region a neutron gap at $N=114$ is used for $Z=78,80$.

In Fig. 1, all of the curves but one (the actinides) are nearly parallel. The only significant differences between

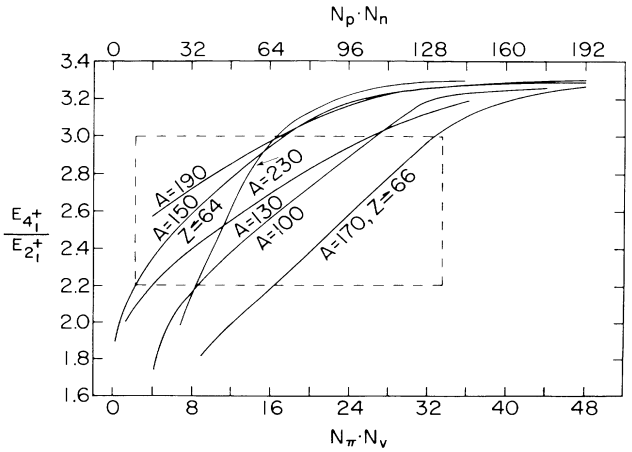

FIG. 1. Collection of smooth curves drawn through N_pN_n plots of $E_{4_1^+}/E_{2_1^+}$ for six transitional regions in heavy nuclei, based on Refs. 2 and 3.

regions are in the varying horizontal displacements. These have been qualitatively related[2] to the differing relative locations of the most crucial, highly overlapping proton and neutron orbits in each pair of proton and neutron shells. These orbits have large p-n interactions and it is thought[1-6] that their filling is the essential driving force toward deformation. Indeed this has been demonstrated quantitatively by Federman and Pittel[4-5] for the $A=100$ region where the valence space is sufficiently small to permit detailed calculations.

The purpose of the present Letter is to discuss a related parameter

$$P=N_pN_n/(N_p+N_n),$$

which may be dubbed a nucleonic promiscuity factor since it can be viewed as the *average number of interactions of each valence nucleon with those of the other type.* It is also clearly related to the relative integrated strengths of the p-n and like-nucleon-pairing interaction. If the data are now plotted in terms of P instead of N_pN_n, then one obtains a nearly universal description of collectivity in heavy nuclei in terms of a quantity with a rather direct physical interpretation. Moreover, it will be shown that, in such a plot, the curves for different regions now coalesce to a much greater extent. This result entails a modification of the above-mentioned interpretation of Fig. 1 which maintains the same basic physics but provides a more detailed understanding of the operation of the p-n interaction.

Figure 2 shows the $E_{4_1^+}/E_{2_1^+}$ ratio plotted against P for all six transitional regions in heavy nuclei. This ratio is a convenient indicator of structure since its absolute value is directly meaningful. It is <2.0 for nuclei near a closed shell, ≈ 2.0–2.2 for a vibrational nucleus, and goes to ≈ 3.33 for a deformed symmetric rotor. Other observables (such as $E_{2_1^+}$) can also indicate structural shapes within a given region but their asymptotic values change from region to region ($E_{2_1^+}$ varies roughly as $A^{-5/3}$) and so it is less easy to compare different regions. Nevertheless, plots of $E_{2_1^+}$ or $E_{2_1^+}/A^{-5/3}$ do reflect the

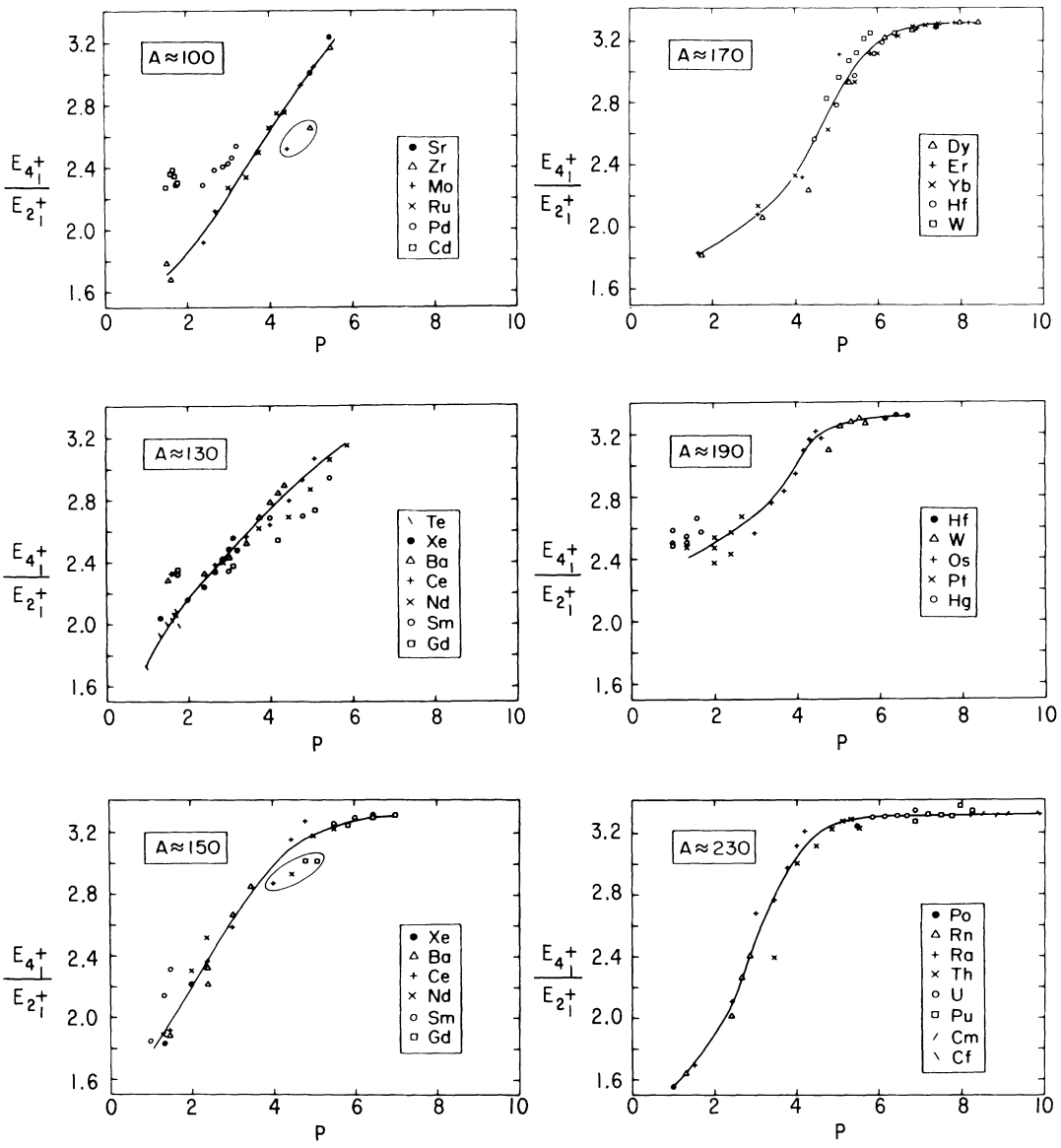

FIG. 2. P plots for six mass regions.

same physics discussed below.

To compare the different regions in Fig. 2, smooth curves have been drawn through the points. While there is some structure in the systematics within each region for low $E_{4_1^+}/E_{2_1^+}$ values (compare, for example, Pd and Cd nuclei with Zr, Sr, and Mo near $A \approx 100$), this is unimportant for the present discussion which is concerned with the onset of deformation and configuration mixing typified by $E_{4_1^+}/E_{2_1^+}$ values near or above 3.0. The curves from each region are well defined for such values of $E_{4_1^+}/E_{2_1^+}$.

These six smooth curves as functions of P are collected in Fig. 3, which summarizes the most important results of this study. With the above physical interpretation of P, Fig. 3 immediately gives the average number of valence-nucleon p-n interactions required for *any* degree of collectivity in *each* mass region. Thus, the curves provide a simple, physically intuitive approach to nuclear phase transitions. The figure exhibits the same detailed variations in form evident in Fig. 1, such as the slightly smaller slopes for the $A \approx 190$ and $A \approx 130$ regions and the very steep one for the actinides. It also maintains roughly the same horizontal order of the different curves from the $A = 190$ curve on the left to the $A \approx 170$ curve on the right. However, undoubtedly the most remarkable result is the horizontal compression: The curves are

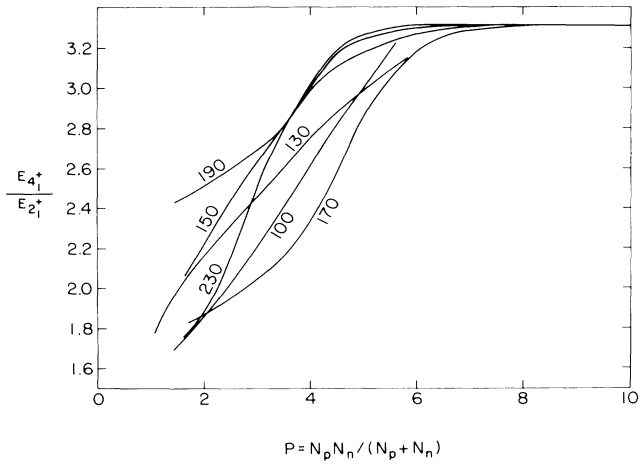

FIG. 3. Collection of the smooth curves of Fig. 2.

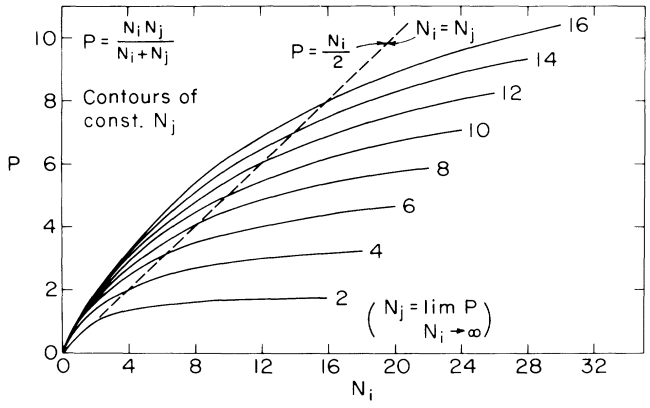

FIG. 4. Plot of P against N_i for N_j constant. The dashed line corresponds to $N_i = N_j$ (i.e., $N_p = N_n$) and is given analytically by $P = N_p/2 = N_n/2$. The curves are labeled by the constant N_j which is also the limiting value for P as $N_i \to \infty$.

nearly coalesced. Each region passes through the transitional value $E_{4_1^+}/E_{2_1^+} \approx 3.0$ around $P \approx 4$–5. Thus, *independent of mass region in heavy nuclei,* the transition to deformation occurs when the number of valence protons and neutrons reaches the point that, *on average, each valence nucleon interacts with* 4–5 *nucleons of the other type.* The resistance to deformation exhibited for $P < 4$ can no longer be maintained when P reaches this critical range although, of course, a degree of softness appears for lower values which, not unexpectedly, increases monotonically with P. It is interesting that the coalescence in Fig. 3 is even greater if one takes account of the $Z = 76$ gap, recently proposed by Mach,[7] who pointed out that when the $1h_{11/2\pi}$ orbit is lowered in energy by an attractive p-n interaction with the $1h_{9/2\nu}$ orbit (obliterating the $Z = 64$ gap) another gap at $Z = 76$ between the $2d_{3/2}$-$3s_{1/2\pi}$ and $1h_{11/2\pi}$ orbits is simultaneously created. This reduces N_p values for nuclei like Dy, Er, and Yb, and thus shifts the curve for most of the $A \approx 170$ region to the left. Finally, it is interesting that the value, $P_{\text{crit}} \approx 4$–5, is reasonable if one recalls that the like-nucleon-pairing interaction is ≈ 1 MeV and that typical values for p-n interaction strengths are a couple of hundred kiloelectronvolts; thus, $P_{\text{crit}} \approx 4$–5 corresponds roughly to the point where the integrated p-n interaction strength begins to dominate.

The results raise a question concerning the earlier interpretation[2] of Fig. 1. The coalescence in Fig. 3 seems to suggest that the most important factor is simply the total number of valence p-n interactions (per valence nucleon) and that the orbit dependence of the p-n interaction is not so important. This issue has also been recently raised by Mach.[7] In fact, the two viewpoints can be reconciled by our noting that the p-n interaction is complex and that Ref. 2 did not adequately distinguish between its monopole and quadrupole components.[8,9] As Mach[7] and Heyde[9] have stressed, the monopole part essentially acts to shift the shell-model single-particle energies of one type of nucleon as a function of the number of the other type. These shifts create and destroy shell and subshell gaps[4,6–9] and therefore determine the actual number of valence nucleons on which the (less orbit-dependent) quadrupole component acts. For example, the $Z = 64$ gap between the $1g_{7/2}$-$2d_{5/2}$ and $1h_{11/2}$ proton orbits disappears when the $1h_{9/2}$ neutron orbit begins filling near $N = 90$ since the large monopole component of the $1h_{11/2\pi}$-$1h_{9/2\nu}$ interaction effectively contributes a negative component to these single-particle energies. This lowers the $1h_{11/2}$ orbit and obliterates the energy gap. Thus the effective number of valence nucleons is increased, and the integrated strength of the long-range quadrupole part of the p-n interaction increases commensurately. Similar gap-obliteration mechanisms occur in the $A = 100$, 130, and 190 regions and, to a lesser extent, near $A = 170$. Thus the horizontal displacements in Fig. 1 do in fact reflect the filling of crucial orbits because their occupation determines the effective number of valence nucleons which in turn determines where the curves lie along the abscissa. Remaining differences in the shapes and positioning of different curves probably reflect the weak orbital dependence of the quadrupole component of the p-n interaction and perhaps different A dependence of the p-n and like-nucleon-pairing interactions.

Further insights into the deformation process and heavy-element systematics can be seen by consideration of the behavior of P as a function of either N_p or N_n with the other nucleon number held constant. Figure 4 shows such curves where, to avoid specifying the proton or neutron case, the constant valence-nucleon number is denoted by N_j and the variable one by N_i. Since, in the limit of large N_i, $P \to N_j$, the maximum value of P can never be larger than the lesser of N_p or N_n. If one accepts the general rule that P must be > 4 in order for

deformation to set in, it immediately follows that *medium-mass and heavy nuclei with fewer than four valence nucleons of one type will never become deformed.* The idea that nuclei near a closed shell do not deform, of course, is not new. The novel aspect of the P factor in this context is that it discloses and quantifies a consistent pattern underlying the observation that nuclei such as Cd and Pd or Te and Xe do not go deformed while elements such as Ru, Ba, and Os do when the valence-neutron number is sufficiently large. Moreover, it relates this codification directly to the effective average number of valence p-n interactions.

An interesting implication from this analysis is that, in deformed nuclei with *apparent* N_p (and therefore P) values <4, such as the very neutron-deficient $_{80}$Hg, $_{78}$Pt isotopes, one must *not be counting* valence protons correctly. For Hg and Pt, of course, this has been confirmed by detailed microscopic calculations[10] which associate the onset of deformation with the descent into the valence space of the $1h_{9/2\pi}$ orbit from above the $Z=82$ gap. Similarly, $_{64}$Gd and $_{62}$Sm cannot become deformed as long as $Z=64$ is a significant subshell closure,[6] but suddenly deform when that gap disappears at $N=90$.

Another interesting point concerns the dashed line in Fig. 4 which corresponds to $N_p=N_n$ for which $P=N_i/2$ $(i=p,n)$ is a straight line. For constant total valence-nucleon number N_p+N_n, this line gives the locus of maximum P values. That is, for a given N_{total}, P is largest, and so is the collectivity, when the valence nucleons are equally distributed over protons and neutrons. Such a distribution also gives the largest N_pN_n value and so the P plots are fully consistent with the idea that collectivity and the onset of deformation increase as functions of N_pN_n.

Finally, since the value of P depends on the proton and neutron subshell structure, it is interesting that it may be possible[11] to measure directly some P values experimentally. It has been shown[12] that $B(M1)$ values to isovector 1^+ states in deformed nuclei are proportional to $(g_\pi-g_\nu)^2P$ where g_π,g_ν are boson g factors. If reliable values for g_π,g_ν can be ascertained, then such $B(M1)$ values would give a quantitative indication of P values or at least of discontinuities in the systematics of P values over sequences of nuclei. Alternatively, in regions where subshell effects make the counting of valence nucleons difficult, empirical values of collective observables, such as $E_{4_1^+}/E_{2_1^+}$, can be used to extract effective P values which could then be used to estimate $B(M1)$ values to these isovector states.

In summary, it has been shown that the parameter P, which is the average number of interactions of each valence nucleon with those of the other type, provides a simple way to obtain a universal interpretation of the development of collectivity and deformation in heavy nuclei. Its use shows that, *regardless of region,* deformation sets in when P reaches 4–5. Since $\lim_{N_p\to\infty}P\to N_n$ and $\lim_{N_n\to\infty}P\to N_p$, heavy nuclei with fewer than four valence nucleons of either type can never become deformed; apparent violations of this rule (Hg,Pt) simply reflect intruder states whose presence alters the effective number of valence nucleons. The results of Fig. 3 point to the crucial, but different, roles[7-9] of the monopole and quadrupole parts of the p-n interaction. The filling of the most highly overlapping proton and neutron orbits is important primarily because the monopole p-n interaction induces shifts in single-particle energies which lead to modifications in subshell gaps and therefore change the valence space in which the quadrupole p-n interaction acts.

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