Determination of the Real Part of the Isospin-Even Forward-Scattering Amplitude of Pion-Nucleon Scattering at 55 MeV as a Test of Low-Energy Quantum Chromodynamics

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The real part of the isospin-even forward-scattering amplitude of pion-nucleon scattering has been determined at a pion energy of $T_\pi = 55$ MeV by measurement of the elastic scattering of positive and negative pions on protons within the Coulomb-nuclear interference region. The value confirms the prediction of the Karlsruhe-Helsinki phase-shift analysis for that energy. These phases have been used to determine the σ term of pion-nucleon scattering by means of dispersion relations, resulting in a value for σ which is in contradiction with chiral perturbation theory of QCD.

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Strong interactions are believed to be described by quantum chromodynamics (QCD). At low energies, predictions can be made within the framework of chiral perturbation theory, which treats the current quark masses m_q as small perturbations. The chiral limit itself is defined by the setting of the masses of the light quarks equal to zero: $m_u = m_d = m_s = 0$. All the well-known low-energy (soft-pion) theorems, in the past derived from current algebra and PCAC (partial conservation of axial-vector current), are preserved in chiral perturbation theory.

One of the most important testing grounds of current algebra and PCAC, and consequently of chiral perturbation theory, is low-energy pion-nucleon scattering. A basic number is the pion-nucleon σ term in $\pi^\pm p$ scattering,[1]

$$\sigma = (\hat{m}/2m_N)\langle p\,|\,\bar{u}u + \bar{d}d\,|\,p\rangle, \quad \hat{m} = (m_u + m_d)/2 \qquad (1)$$

(m_N being the nucleon mass), which is a measure of the size of explicit chiral-symmetry breaking of QCD due to the quark-mass term in the QCD Lagrangean. The σ term has been evaluated by Gasser and Leutwyler[1] from the baryon mass spectrum to be

$$\sigma = \frac{35 \pm 5\ \text{MeV}}{1-y}, \quad \text{where } y = \frac{2\langle p\,|\,\bar{s}s\,|\,p\rangle}{\langle p\,|\,\bar{u}u + \bar{d}d\,|\,p\rangle}. \qquad (2)$$

The value of y determines the value of the nucleon mass in the chiral limit. The value $y = 0$ corresponds to $m_N \simeq 0.87$ GeV, and $y = 0.2$ leads to $m_N \simeq 0.67$ GeV. On the other hand the σ term is related to the isospin-even πN scattering amplitude D^+ evaluated at the (unphysical) Cheng-Dashen point $t = 2m_\pi^2$, $\nu = (s-u)/4m_N = 0$.

Let Σ be the πN amplitude at the Cheng-Dashen point,

$$\Sigma = (f_\pi^2/2)[D^+(t = 2m_\pi^2, \nu = 0) - g^2/m_N], \qquad (3)$$

where g is the πN coupling constant, $f_\pi = 0.132$ GeV the pion decay constant, and s, t, and u the usual Mandelstam variables. Then chiral perturbation theory relates the two quantities Σ and σ by $\Sigma = \sigma \pm 5$ MeV.[1,2] It is therefore possible to determine the σ term from experimental pion-nucleon scattering data by means of dispersion relations,[3,4] extrapolating from the physical region of the ν-t plane to the Cheng-Dashen point. One of the authors obtained in a recent dispersion analysis[3] $\Sigma = 65 \pm 8$ MeV and thus $\sigma = 60 \pm 8$ MeV. This experimental value is almost a factor of 2 bigger than the value obtained from Eq. (2) for $y = 0$, and y would have to be of the order of 0.4 if $\langle p\,|\,\bar{s}s\,|\,p\rangle \neq 0$ is made responsible for the discrepancy. The nucleon mass in the chiral limit would then be completely different from what it is in this world.

This discrepancy between the σ-term values evaluated from the baryon spectrum and from πN scattering is not understood and is regarded as a serious problem which hints either at theoretical deficiencies or at an insufficient πN data basis. Unfortunately, only a few measurements of πp scattering exist below 100 MeV, which are even partly contradictory.[5] Low-energy πp data ($T_\pi \lesssim 100$ MeV) are, however, of particular significance to the dispersion analysis to obtain the amplitude Σ at the Cheng-Dashen point. More definitely, it has been shown[6,7] that the knowledge of the amplitude D^+ at threshold, and of the t dependence of D^+ [determined, e.g., by the P-wave scattering lengths a_{1+}^+ and a_{1-}^+ plus an integral over the averaged total $\pi^\pm p$ cross section $\sigma^+ = \frac{1}{2}(\sigma_{\pi^+p}^{\text{tot}} + \sigma_{\pi^-p}^{\text{tot}})$], is sufficient to pin down the value of Σ.

The goal of our measurement is the determination of the real part of the isospin-even D^+ amplitude as a function of t, and the subsequent extrapolation of $\text{Re}D^+(t)$ to $t=0$ at an energy closer to the threshold than ever before. The quantity $\text{Re}D^+(t=0)$ has been determined in Ref. 3 as a second subtraction constant in a twice-subtracted forward dispersion relation together with Σ, and, hence, a measurement of $\text{Re}D^+(t=0)$ can be regarded as an independent test of this analysis and of the value obtained for Σ. In addition, the knowledge of $\text{Re}D^+(t=0)$ at low energies is an important constraint for future phase-shift analyses.

The isospin-even combination of differential cross sections can be experimentally obtained by the expression

$$\Delta(t) = t[d\sigma_+(t)/d\Omega - d\sigma_-(t)/d\Omega], \qquad (4)$$

which has to be extrapolated to $t=0$ and represents then the Coulomb-nuclear interference term:

$$\lim_{t\to 0} \text{Re}D^+(t) = \frac{\pi s}{2am_N^2\omega}\Delta(0) \qquad (5)$$

[$d\sigma_\pm/d\Omega$ being the differential cross section of $\pi^\pm p$ scattering, $\omega = (p_\pi^2 + m_\pi^2)^{1/2}$ the total laboratory pion energy, and α the Sommerfeld constant]. The quantity $\Delta(t)$ has been used for the extrapolation to $t=0$, because it turned out to depend with high accuracy linearly on t, as long as t is as small as in our measurement.

We have measured angular distributions of the elastic scattering of 55-MeV pions on protons in the region of the Coulomb-nuclear interference (between 7.5° and 27.5°) at the $\pi M3$ channel of SIN. The main components of our experimental setup are liquid-hydrogen (LH_2) targets, six multiwire proportional chambers with single-wire readout (which allow us to trace back the trajectories of the particles in the entrance and exit channels to the interaction point in order to select events from the target region only and to determine the scattering angle precisely), and a range telescope (which consists of twenty plastic scintillator sheets of thicknesses 2, 3, 5, and 10 mm and an area of 160×200 mm^2) to measure the range and the energy loss of the detected particles. The range telescope allows the elimination of the most significant background of muons, arising from pion decay within the target region. The electronics provides control over the number and kind of particles in the entrance and exit channels.

A substantial additional reduction of the muon background is possible by the registration of the decay sequences $\pi^+ \to \mu^+ \nu$ and $\mu^- \to e^- \nu\nu$ following the stop of the parent particle in the range telescope.[8] Positive pions have been identified within the range telescope by measurement of the kinetic energy of 4.2 MeV of their decay muons in addition to the kinetic energy of the stopping π^+. Stopped negative muons originating from the target region have been identified within the range telescope by their electrons (within a 10-μs window), and by their decay curve.

The data analysis includes identification of the incident and scattered particles, determination of the coordinates of the scattering vertex and consequently of the scattering angle (within 1° bins), and subtraction of the background (obtained by empty-target measurements). The error bars of the data contain, besides the statistical errors, those of the angle-dependent backgrounds subtracted.

In order to correct for the modification of the background after emptying of the target, an absorber of LH_2 equivalent mass density has been put at two different positions: With the absorber in front of the target (about 1 m upstream) the background distribution behind the target position (i.e., downstream) is simulated, while with the absorber in front of the range telescope the background in front of the target is approximated. The two LH_2 targets used (40 and 80 mm thick) provide further control of the influence of the energy loss and small-angle scattering of the pions and of varying effective target thickness on the results. The percentage of pions stopped in the range telescope and the efficiency of the identification of $\pi^\pm$ have been measured at 0° many times between various runs with an electronically prepared pure pion beam. This way corrections for absorption in flight and interactions in the range stack are taken into account automatically.

The data have been corrected for pion decay, for Coulomb small-angle scattering in the various wire chambers, in the scintillation detectors, and in the target, and for finite angular resolution. Monte Carlo calculations have been carried out for the latter corrections as well as for the determination of the effective solid angle. The errors of the absolute normalization are between $\pm 7\%$ (at $\theta_{\text{lab}} \approx 8°$) and $\pm 4\%$ ($\theta_{\text{lab}} \gtrsim 20°$). An independent check of the absolute normalization is possible by means of the pure Coulomb scattering, which dominates at very forward angles. It turned out that our π^+p cross sections agree with the Coulomb normalization, while the π^-p cross sections are higher by 10%. We decided to normalize the π^-p data to the Coulomb cross sections.

A description of the experimental procedure and of the data analysis can be found elsewhere,[8] and will be published in detail in a forthcoming paper.

Figure 1 shows the angular distributions for the elastic scattering of pions ($\pi^\pm$) on protons together with the predictions of the Karlsruhe-Helsinki (KH) phase-shift analysis[9] (solid lines). The agreement is very good for the differential cross sections as well as for the difference

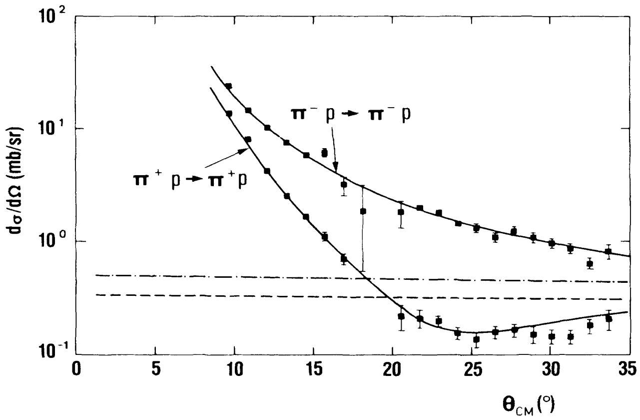

FIG. 1. Elastic $\pi^{\pm}p$ scattering at 55 MeV. The solid lines represent predictions by means of the Karlsruhe-Helsinki phase-shift analysis. The dashed-dotted and dashed lines denote the pure hadronic cross sections for π^+p and π^-p, respectively, as calculated from KH phases. The large error bars around 19° (systematic and statistical errors are drawn) arise from the large muon background near the Jacobian angle.

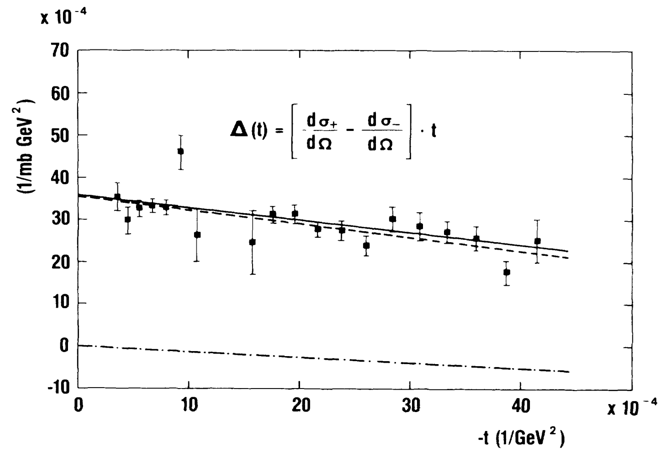

FIG. 2. The difference $\Delta(t)$ as given in Eq. (4) as a function of t. Re$D^+(t=0)$ is obtained by Eq. (5). The extrapolation of $\Delta(t)$ to $t=0$ was obtained by a least-squares fit (dashed straight line). The solid line represents the prediction of the Karlsruhe-Helsinki phase-shift analysis. The dashed-dotted line denotes the difference of the calculated pure hadronic cross sections for π^+p and π^-p scattering multiplied by t.

$\Delta(t)$, which is displayed in Fig. 2. The solid line again represents the prediction of the KH phase-shift analysis.

The KH prediction for $\Delta(t=0)$ as read from Fig. 2 is $\Delta(t=0)=3.6\times10^{-3}$ mb GeV^2, and consequently Re$D^+(t=0$, 55 MeV$)=14.7$ GeV^{-1} (with use of 1 mb GeV^2=2.568) and $\frac{1}{2}f_\pi^2\,$Re$D^+(t=0$, 55 MeV$)=0.128$ GeV. Our measurement for $\Delta(t=0)$ as obtained by a least-squares fit to the data (straight dashed line) results in $\Delta(t=0)=(3.5\pm0.2)\times10^{-3}$ mb GeV^2, Re$D^+(t=0$, 55 MeV$)=14.3\pm0.8$ GeV^{-1}, and $\frac{1}{2}f_\pi^2\,$Re$D^+(t=0$, 55 MeV$)=0.124\pm0.007$ GeV. The agreement of our value for Re$D^+(t=0$, 55 MeV) with the KH-phase prediction implies a value for the isospin-even S-wave scattering length[3,4,9] of $a_{0+}^{+}=(-0.0097\pm0.0017)m_\pi^{-1}$, while the recent determination of the scattering length $a_{0+}^{\pi^- p}$ by measurement of the $2P$-$1S$ x-ray transition energy in pionic hydrogen[10] results in $a_{0+}^{+}=(0.032\pm0.006)\times m_\pi^{-1}$, a value which would cause Σ to be close to that predicted by chiral perturbation theory. The discrepancy is presently unresolved.[5]

Another way to obtain Re$D^+(t=0)$ from our data is to extrapolate the expression

$$A(t)=\frac{d\sigma_+/d\Omega-d\sigma_-/d\Omega}{d\sigma_+/d\Omega+d\sigma_-/d\Omega}\frac{1}{t}$$

to $t=0$, because

$$\lim_{t\to0}\mathrm{Re}D^+(t)=4\pi\alpha A(0)\omega. \qquad (6)$$

$A(0)$ then represents the Coulomb-nuclear interference term normalized to the pure Coulomb-scattering cross section. The idea behind building the ratio $A(t)$ out of the cross sections is a reduction of the systematical errors of the value of Re$D^+(t=0)$. The extrapolation to $t=0$, however, cannot be carried out by a straight line.

We conclude from our results, which have been measured in a kinematical region never explored experimentally before, and which agree with the KH phase-shift analysis, that the value for Σ of Ref. 3 is heavily supported by our investigation. The disagreement between the value for the σ term of Gasser and Leutwyler derived from the baryon spectrum and the value from the dispersion analysis of πN scattering obviously persists.

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