Effects of Z^0 Mixing with an E_6 Gauge Boson at e^+e^- Colliders

David London
Department of Physics, University of British Columbia, Vancouver, British Columbia, Canada V6T 2A6

and

G. Bélanger and J. N. Ng
Theory Group, TRIUMF, Vancouver, British Columbia, Canada V6T 2A3
(Received 9 September 1986)

We consider the effects of Z^0-Z' mixing, where the Z' is an E_6 gauge boson, on processes which will be measured at e^+e^- colliders. Using constraints from current data, we find that significant mixing may be allowed. This mixing leads to measurable changes in the Z^0 partial widths and $\sigma(e^+e^- \to \mu^+\mu^-)$. In one case, we also find that a Z' lighter than the Z^0 is still allowed. We discuss possibilities for its detection.

PACS numbers: 13.10.+q, 13.38.+c, 14.80.Er

The superstring theory with an $E_8 \otimes E_8'$ gauge group in ten dimensions may lead, after compactification, to a four-dimensional E_6 unified theory.[1] Two phenomenologically interesting features of E_6 superstring-inspired models are the possible existence of an extra neutral gauge boson (Z') below the teraelectronvolt scale,[2] and of many new fermions.[3] With the high-precision measurements that can be achieved in e^+e^- colliders (such as the Stanford Linear Collider and CERN's LEP) at the Z^0 pole, the effects of the Z' may be observed. If there is mixing between the two neutral bosons, the mass of the standard Z^0 will be lowered[4] and its couplings to fermions will be changed. Many authors[5] have shown that, although the standard model fits all low-energy neutral-current data, as well as the W and Z^0 mass measurements, other models with one extra Z are consistent with the data, for $M_{Z'}$ well below 1 TeV and small mixing between the two neutral gauge bosons. It has also been shown that the measurements of asymmetries at the Z^0 pole could provide evidence of deviations from the standard model.[6] Here we will see that measurable changes in the cross section to μ pairs, in the Z^0 width, and in the branching ratios to fermions could also be observed.

Under E_6, the fermions belong to a **27** representation. In the **27**, there are two new neutral singlets, denoted N and n. The N is a member of the **16** representation of SO(10), while the n is an SO(10) singlet. These neutral singlets are possibly the only new fermions in the **27** representation of E_6 which are light enough to be produced at energies below 100 GeV.[7] We study the effects of these two light neutral singlets on the Z^0 width and cross section to μ pairs.

After briefly reviewing the notation, we will derive the constraints on the parameters of the model from neutral-current data, the W and Z mass measurements, and the cross section for $p\bar{p} \to Z \to e^+e^-$. The results for the largest possible deviations from the standard model for the Z^0 width and the cross section to μ pairs will then be presented. Finally, we will discuss the possibility of directly observing a light Z'.

Consider the breakdown of E_6: $E_6 \to \mathrm{SO}(10) \otimes \mathrm{U}(1)_\psi \to \mathrm{SU}(5) \otimes \mathrm{U}(1)_\chi \otimes \mathrm{U}(1)_\psi$. An extra low-energy Z in E_6 must be a linear combination of Z_χ and Z_ψ:

$$Z'(\theta) = Z_\psi \cos\theta + Z_\chi \sin\theta. \tag{1}$$

In the absence of mixing, the Lagrangean for the Z's is

$$-L_{\mathrm{NC}} = g_Z Z_{1\mu}^0 J_Z^\mu + g_{Z'} Z_{2\mu}^0 J_{Z'}^\mu, \tag{2}$$

where Z_1^0 and Z_2^0 are the unmixed Z's, and

$$J_Z^\mu = \sum \bar{\Psi}(I_{3L} - Q_{\mathrm{em}} \sin^2\theta_W)\gamma^\mu \Psi,$$
$$J_{Z'}^\mu = \sum \bar{\Psi} Q(\theta) \gamma^\mu \Psi, \tag{3}$$

where the sum is over all fermions, and $Q(\theta)$ are the $Z'(\theta)$ charges of the fermions.[8] Letting ϕ denote the mixing angle between Z_1^0 and Z_2^0, the Lagrangean now reads

$$-L_{\mathrm{NC}} = g_Z Z_\mu^0 [J_Z^\mu \cos\phi + (g_{Z'}/g_Z) J_{Z'}^\mu \sin\phi] + g_Z Z_\mu' [-J_Z^\mu \sin\phi + (g_{Z'}/g_Z) J_{Z'}^\mu \cos\phi]. \tag{4}$$

As can be seen from the above expression, the fermionic charges of the Z^0 and the Z' have been altered as a result of the mixing. Measurable effects may be seen if the changes in the Z^0 couplings are large enough.

The effective Lagrangean for neutral-current processes can thus be written in terms of five parameters. We use $x \equiv \sin^2\theta_W$, the mixing angle between the two E_6 gauge bosons (θ), the ratio of coupling constants ($g_{Z'}/g_Z$), the Z^0-Z' mixing angle (ϕ), and $M_{Z'}$. The mass of the Z^0 is related to the other parameters since

$$\tan^2\phi = \left(\frac{\rho_1 - 1}{1 - \rho_2}\right)\frac{\rho_2}{\rho_1}, \quad \rho_{1(2)} \equiv \frac{M_W^2}{M_{Z(Z')}^2 \cos^2\theta_W}. \tag{5}$$

TABLE I. Z^0 width and $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ for the maximum mixings allowed by the present data, for various values of θ. Three generations of standard fermions are lighter than $M_Z/2$, with $m_t = 40$ GeV.

θ (deg)	ϕ (rad)	$M_{Z'}$ (GeV)	M_Z (GeV)	Γ (GeV)	$\Delta\Gamma$ (MeV)	σ (nb)	$\Delta\sigma$ (nb)
0	0.33	100	91.7	2.41	160	3.39	−1.40
	−0.16	164	89.8	2.42	93	1.51	0.57
10	0.52	98	90.9	2.12	426	3.88	−1.86
	−0.16	169	89.7	2.41	94	1.64	0.45
20	0.14	159	90.8	2.53	5	2.33	−0.30
	−0.17	164	89.6	2.40	106	1.75	0.35
40	0.10	196	91.0	2.56	−8	2.04	−0.02
	−0.17	164	89.6	2.41	101	2.06	0.03
50	0.08	245	90.7	2.55	−10	1.98	0.06
	−0.13	169	90.7	2.47	68	2.14	−0.10
90	0.07	328	90.3	2.53	−5	1.81	0.25
	−0.06	327	90.7	2.52	16	2.29	−0.25
170	0.15	161	90.6	2.46	70	1.47	0.57
	−0.14	154	90.9	2.53	14	2.68	−0.65

In the following, we will assume that $(g_{Z'}/g_Z)^2 = \frac{5}{3}\sin^2\theta_W$ (this corresponds to the largest value obtainable from the renormalization-group analysis[4]). Thus, for each θ, there will be three parameters to be fitted, x, ϕ, and ρ_2. From Eq. (5), it is clear that the largest deviations from the standard model will occur when ϕ is large. We will therefore find the largest mixing angle ϕ allowed by present data.

For each θ, we find the best fit to the neutral-current data (with use of the same data as Ref. 8) and to the UA2 measurements of the W and Z masses.[9] Since the parameter x depends strongly on M_W, the best-fit value of x is always very close to 0.225, for every θ. Next we find the region in ρ_2-ϕ parameter space corresponding to a 90% confidence level. For a two-parameter fit, the 90% confidence level corresponds to $\Delta\chi^2 = 4.61$.[10] After obtaining the largest ϕ and the corresponding ρ_1 and ρ_2 (i.e., M_Z and $M_{Z'}$) allowed by the data, we check that the Z' is heavy enough, or that its couplings to ordinary fermions are small enough, that the Z' would not have been seen at CERN.

At the $p\bar{p}$ collider, a search for extra Z bosons was done. The UA1 and UA2 groups obtained an upper limit of 3 pb for $\sigma(q\bar{q} \rightarrow Z')\Gamma(Z' \rightarrow e^+e^-)$.[10] For a Z' with the same coupling as the Z^0, this limit corresponds to $M_{Z'}$ greater than 186 GeV. For the Z' in E_6, however, the couplings to ordinary matter are always smaller than for the standard Z^0, so that lighter Z''s are allowed. We calculate the cross section for production of the Z' at the CERN $p\bar{p}$ collider by use of the Eichten-Hinchliffe-Lane-Quigg quark distribution functions.[11] We then find the maximum mixing angles ϕ, both positive and negative, which are consistent with the 90% confidence limit from neutral currents, and with the UA1-UA2 limit on Z' production. For θ in the range 50°-160°, the neutral-current data are more restrictive than the $p\bar{p} \rightarrow e^+e^-$ limit. For other θ's, where a lower mass Z' is possible, the $p\bar{p}$ limit reduces the allowed parameter space. The results for the maximum ϕ and for $M_{Z'}$ and M_Z are summarized in the first columns of Table I for a few typical values of θ. For θ between 50° and 160°, the maximum mixing angle $|\phi| \approx 0.07$, and the result at $\theta = 90°$ is representative of these cases. Note that we need only consider θ as running from 0° to 180°,

TABLE II. Same as Table I, but with three generations of n included.

θ (deg)	ϕ (rad)	$M_{Z'}$ (GeV)	M_Z (GeV)	Γ (GeV)	$\Delta\Gamma$ (MeV)	σ (nb)	$\Delta\sigma$ (nb)
0	0.33	100	91.6	2.46	106	3.24	−1.25
	−0.21	131	90.4	2.41	115	1.33	0.73
10	0.56	97.6	90.5	2.17	355	3.60	−1.55
	−0.24	131	89.9	2.36	146	1.40	0.68
20	0.22	118	91.1	2.53	24	2.45	−0.43
	−0.25	131	89.5	2.33	168	1.56	0.54
40	0.10	196	91.0	2.56	−11	2.04	−0.02
	−0.20	140	90.1	2.39	124	2.01	0.05
50	0.08	245	90.7	2.55	−11	1.98	0.05
	−0.20	140	90.1	2.40	122	2.23	−0.16

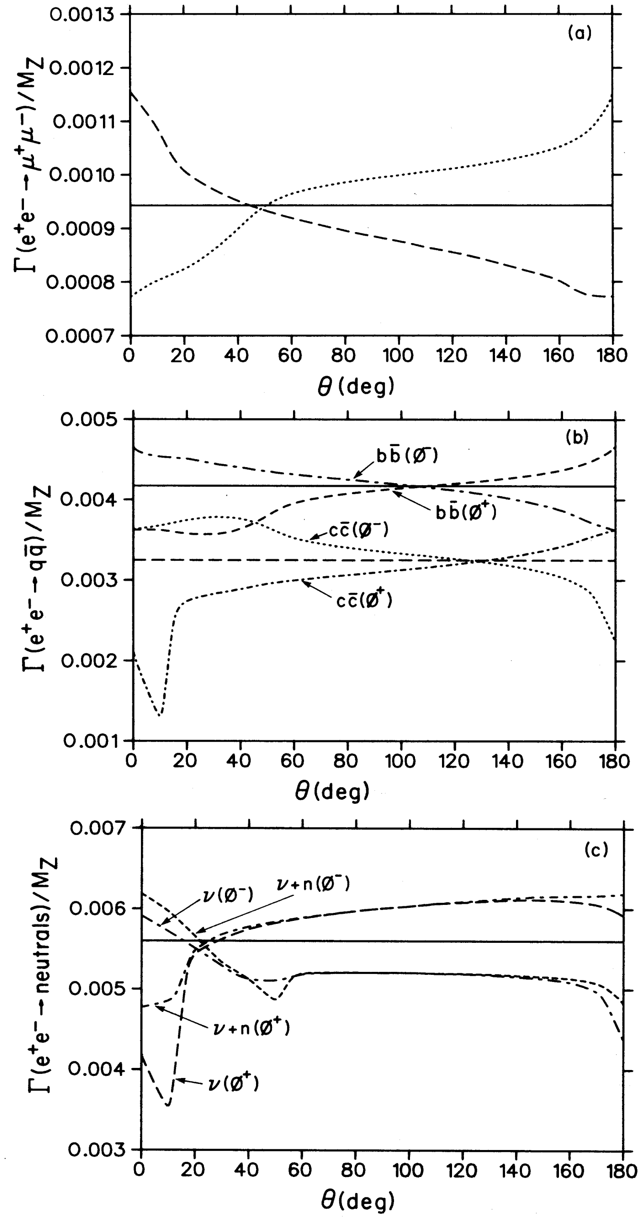

FIG. 1. Ratio of $\Gamma(Z^0 \to f\bar{f})$ to M_Z as a function of θ: (a) $Z^0 \to \mu^+\mu^-$ for ϕ^+ (maximum positive mixing) (dotted line) and ϕ^- (maximum negative mixing) (dashed line), (b) $Z^0 \to c\bar{c}$ for ϕ^+ (dash-dotted line) and ϕ^- (dotted line) and $Z^0 \to b\bar{b}$ for ϕ^+ (dashed line) and ϕ^- (dash-dotted line), (c) $Z^0 \to$ three generations of ν's for ϕ^+ (dash-dotted line) and ϕ^- (dotted line). The solid lines correspond to the standard model.

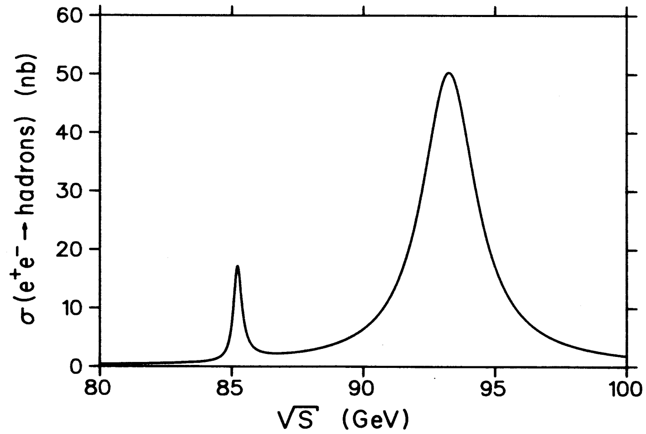

FIG. 2. Cross section for $e^+e^- \to$ hadrons vs center-of-mass energy, $\sqrt{s}$, for $\theta = 10°$, $\phi = 0.3$, and $M_{Z'} = 85.2$ GeV.

since the results for θ between 180° and 360° are the same, but with $\phi \leftrightarrow -\phi$.

For these maximum mixing angles, at the Z^0 pole, we then calculate the total width of the Z^0 and $\sigma(e^+e^- \to \mu^+\mu^-)$, assuming that only three generations of standard fermions have masses below $M_Z/2$. We present the results in Table I. The deviations from the standard model are defined as $\Delta\Gamma = \Gamma_{SM} - \Gamma$ and $\Delta\sigma = \sigma_{SM} - \sigma$. To obtain the standard-model values (σ_{SM} and Γ_{SM}) for each case, we use the same value of M_Z as for the mixed case, but assume standard coupling to the fermions with $x = 0.225$. For $\sigma(e^+e^- \to \mu^+\mu^-)$, we find that, for several cases, shifts of several hundred picobarns from the standard result are possible. Thus, the cross section to μ pairs, which will be measured to 50 pb,[12] is clearly a good test of Z^0-Z' mixing. For example, at $\theta = 0°$, very large deviations in the cross section to μ pairs can occur. This is fortunate, since the left-right asymmetry at this angle is somewhat less sensitive to the mixing.[6] The asymmetry depends on $g_L^2 - g_R^2$, while the cross section depends on $g_L^2 + g_R^2$, and at 0°, shifts in the left-handed coupling are the same as shifts in the right-handed coupling. Only if $\theta \approx 40°$–$50°$ will $\Delta\sigma$ be too small to be observable. The reason for this is that, at the pole, $\sigma \propto (g_L^2 + g_R^2)$, and around 45°, the changes in g_L for the muon are almost exactly compensated for by changes in g_R. The effect of mixing on the measurement of the total width is small, but could nevertheless be measurable. The leading correction in the partial width $Z^0 \to f\bar{f}$ is $\sim \phi(I_3^f - Q_{em}^f x) Q^f(\theta)$. For the total width, where we sum over all fermions, this term vanishes, and the correction to the total width is proportional to ϕ^2. For the cases where ϕ is large, we find deviations of about 100 MeV, which can be seen, on the assumption that an accuracy of 50 MeV is achievable.[12]

We also consider the cases where there are three generations of light singlets n or N. The presence of one of these singlets allows an additional decay channel to the Z' and hence reduces the $Z' \to e^+e^-$ branching ratio, which softens the $p\bar{p}$ constraint. Thus, a lighter Z', and hence a larger mixing angle, is allowed. The results for n

are shown in Table II, where we list only the few angles for which there is a difference in the amount of mixing allowed. Comparing Tables I and II, we see that, for angles below 50°, there may be measurable differences between the case where no n's are included and the case where we have included three generations of n. The reason for this is that, for an unmixed Z', n couples most strongly at $\theta = 0°$. Thus, at small θ, where large mixing angles ϕ are allowed, the n coupling to the Z^0 is relatively large. For the other singlet, N, the results remain extremely close to the case where there are no extra neutrals, since N couples strongly only around 90°, where the mixing ϕ is too small.

While the measurement of the total width could yield only small changes from the standard model, the partial widths could exhibit large deviations. Since the partial width is proportional to M_Z for massless fermions, the quantity $\Gamma(Z^0 \rightarrow f\bar{f})/M_Z$ is a function only of the Z^0 couplings to fermions. To compare the maximal deviations from the standard model in the partial widths to fermions, we plot $\Gamma(Z^0 \rightarrow \mu^+\mu^-)/M_Z$ as a function of θ in Fig. 1(a). With an expected precision of 2% on the muon partial width,[12] the cases where the maximum mixing occurs can easily be tested for all θ, with the exception of $\sim 50°$, where, as noted above, the muon coupling is almost standard. For the cases of maximal mixing, large deviations are also expected in the branching ratio to quarks [Fig. 1(b)]. Although the measurements will be less precise, the region where θ is small could be tested. In Fig. 1(c), we plot the partial width to three generations of neutrinos, both with and without three generations of light singlets n. The addition of a decay mode into neutral particles increases the partial width of the Z^0, but the effect of the extra n's may be measurable only for small θ, where large mixings are allowed.

We should emphasize that $\phi = 0$ is always consistent with the data, and in that limit, the standard-model results at the Z^0 pole will be reproduced. For this reason, the E_6 models cannot be ruled out even if all data at the Z^0 pole are consistent with the standard model (although some theoretical reason for the absence of mixing must be given). Certain values of θ can be ruled out only if deviations are observed which are larger than the ones listed in the tables and in the figures.

Finally, it is interesting to note that a Z' lighter than the Z^0 is consistent with the data for certain values of θ, not considered by previous authors. For each angle θ, there is a range of values for ϕ where the cross section for $p\bar{p} \rightarrow Z' \rightarrow e^+e^-$ becomes very small, even for $M_{Z'}$ below the Z^0 mass. When θ is around 10°, this range of values for ϕ is also allowed by the neutral-current data. At $\theta = 10°$, for $\phi = 0.3$, the smallest allowed mass for Z' is 85.2 GeV. Such a light Z' could be directly observed. However, the coupling of the electron to Z' is very weak, so that the best signal can be seen in $e^+e^- \rightarrow$ hadrons. In Fig. 2, we see that for $M_{Z'} = 85.2$ GeV and $\phi = 0.3$, the Z' cross section is about one third of the Z^0 cross section, and is unmistakable.

We gratefully thank Ron Balden for his help with the computer program. We would also like to thank Dr. N. G. Deshpande, Dr. L. S. Durkin, and Dr. K. Whisnant for helpful conversations. This work was supported in part by the National Sciences and Engineering Research Council (Canada).

Note added.—After completion of this work, we received a preprint by P. J. Franzini and F. J. Gilman (Phys. Rev. D, to be published), in which they discuss similar effects.

[1]M. B. Green and J. H. Schwarz, Phys. Lett. **149B**, 117 (1984), and **151B**, 21 (1985); D. Gross, J. Harvey, E. Martinec, and R. Rohm, Phys. Rev. Lett. **54**, 502 (1985); P. Candelas, G. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. **B258**, 46 (1985).

[2]M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi, and N. Seiberg, Nucl. Phys. **B259**, 549 (1985).

[3]R. W. Robinett, Phys. Rev. D **33**, 1908 (1986); V. Barger, N. Deshpande, R. J. N. Phillips, and K. Whisnant, Phys. Rev. D **33**, 1912 (1986); J. Rosner, Comm. Nucl. Part. Phys. **15**, 195 (1986); T. G. Rizzo, Iowa State University Report No. IS-J-2037, 1986 (unpublished).

[4]P. Langacker, R. W. Robinett, and J. L. Rosner, Phys. Rev. D **30**, 1470 (1984).

[5]S. M. Barr, Phys. Rev. Lett. **55**, 2778 (1985); E. Cohen, J. Ellis, K. Enqvist, and D. V. Nanopoulos, Phys. Lett. **165B**, 76 (1985); V. Barger, N. G. Deshpande, and K. Whisnant, Phys. Rev. Lett. **56**, 30 (1986); L. S. Durkin and P. Langacker, Phys. Lett. **166B**, 436 (1986); P. J. Franzini, SLAC Report No. SLAC-PUB-3920, 1986 (unpublished); D. London and J. L. Rosner, Phys. Rev. D **34**, 1530 (1986).

[6]G. Bélanger and S. Godfrey, Phys. Rev. D **34**, 1309 (1986); J. P. Ader, S. Narison, and J. C. Wallet, Phys. Lett. **176B**, 215 (1986); V. D. Angelopoulos, J. Ellis, D. V. Nanopoulos, and N. D. Traces, Phys. Lett. **176B**, 203 (1986); M. Cvetič and B. W. Lynn, Phys. Rev. D **35**, 51 (1987).

[7]D. London, G. Bélanger, and J. N. Ng, Phys. Rev. D **34**, 2867 (1986).

[8]D. London and J. L. Rosner, Ref. 5.

[9]G. Altarelli, lectures given at the Twenty-Third International Conference on High Energy Physics, Berkeley, California, 1986 (to be published).

[10]As noted in L. S. Durkin and P. Langacker, Ref. 5, a correct statistical treatment of confidence levels would modify the 90% confidence level from its naive ($\Delta\chi^2 = 4.61$) value. However, the effects of such modifications are small, and we have neglected them.

[11]E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg, Rev. Mod. Phys. **56**, 579 (1984).

[12]G. Altarelli *et al.*, CERN Report No. 86-02, 1986 (unpublished).