Searching for *CP* Nonconservation in Jet Physics

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At future high-energy accelerators, the traditional methods of studying *CP* nonconservation will no longer be useful. We present potential new ways to find *CP* nonconservation at high-energy e^+e^- and $p\bar{p}$ colliders which use jet variables in inclusive or semi-inclusive processes. These make use of asymmetries formed with beam and jet momenta. Signals are estimated in the standard model and appear to be small, but the tests could be sensitive to *CP* nonconservation in new interactions which become visible at higher energies.

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CP nonconservation has been observed only in the decays of neutral kaons, and considerable theoretical and experimental effort has been exerted to try to observe this phenomena in other systems. In most methods proposed for the study of *CP* nonconservation, one reconstructs specific final states in the weak decays of a particle.[1] These techniques require good particle identification and are most compatible with the style of experiment at the lower-energy accelerators. However, much of the effort in particle physics at present is devoted to the development and use of high-energy colliders where the above experiments are not possible. The purpose of this paper is to present some techniques for the study of *CP* nonconservation at e^+e^- and $p\bar{p}$ colliders which use jetlike observables suitable for the detectors used at these machines.

The basic problem is that particle identification is almost nonexistent in most collider detectors, and reconstruction of decay modes is difficult. Particle versus antiparticle identification, which one might think was crucial for any tests involving charge conjugation, is generally not accomplished. What is well measured is the overall flow of energy and momentum summed over all particles, i.e., the jet distribution. Occasionally some of the heavy flavored particles may be isolated by some of their decay characteristics, such as transverse momentum or hard leptons. The task we will set ourselves is to find *CP*-odd observables using only jet variables for inclusive reactions, or possibly allowing the study of semi-inclusive events in which a heavy particle is identified, but in which the full final state is not reconstructed. Under these conditions, we will see that our tests of *CP* can only be accomplished with an initial state that transforms into itself under *CP*. Thus our tests can be applied at e^+e^- and $p\bar{p}$ colliders, but not at pp or ep machines.

Let us start on the most familiar ground by considering a semi-inclusive process, say $e^+e^- \to t\bar{t}+X$. Let us define a plane by the directions of the t quarks in the final state, i.e., $\mathbf{p}_t \times \mathbf{p}_{\bar{t}}$, and consider the correlation of this with the beam direction (that of the e^-) $\mathbf{p}$. All work in this paper is implicitly described in the center-of-mass system. If we define an asymmetry by

$$A_{t\bar{t}} = \frac{N(\mathbf{p}_- \cdot \mathbf{p}_t \times \mathbf{p}_{\bar{t}} > 0) - N(\mathbf{p}_- \cdot \mathbf{p}_t \times \mathbf{p}_{\bar{t}} < 0)}{N_{\text{tot}}}, \tag{1}$$

we obtain a *CP*-odd observable. To prove this, consider the transition probability summed over all possible final states

$$|T|^2 = \sum_X |\langle e^+(p_+) e^-(p_-) | T | t(p_t) \bar{t}(p_{\bar{t}}) X\rangle|^2. \tag{2}$$

If we apply *CP* to the initial state

$$PC|e^-(\mathbf{p}_-)e^+(\mathbf{p}_+)\rangle = Pe^{i\phi}|e^+(\mathbf{p}_-)e^-(\mathbf{p}_+)\rangle = -e^{i\phi}|e^+(-\mathbf{p}_-)e^-(-\mathbf{p}_+)\rangle = e^{i\phi}|e^-(\mathbf{p}_-)e^+(\mathbf{p}_+)\rangle, \tag{3}$$

where ϕ is an arbitrary phase related to the definition of the C operation, we see that in the c.m. system ($\mathbf{p}_- = -\mathbf{p}_+$) it returns to the same state. The final state does not transform exactly into itself, but becomes

$$PC|t(\mathbf{p}_t)\bar{t}(\mathbf{p}_{\bar{t}})X\rangle = |t(-\mathbf{p}_{\bar{t}})\bar{t}(-\mathbf{p}_t)\bar{X}\rangle, \tag{4}$$

where $\bar{X}$ is the *CP* conjugate of X. Making use of the fact that one sums over all possible final states with

$$\sum_X = \sum_{\bar{X}},$$

we can see that all terms which are odd in $\mathbf{p}_- \cdot \mathbf{p}_t \times \mathbf{p}_{\bar{t}}$ must vanish if *CP* is conserved,

$$|T|^2 = a + b\mathbf{p}_- \cdot \mathbf{p}_t \times \mathbf{p}_{\bar{t}} \xrightarrow{CP} a + b\mathbf{p}_- \cdot (-\mathbf{p}_{\bar{t}}) \times (-\mathbf{p}_t) = a - b\mathbf{p}_- \cdot \mathbf{p}_t \times \mathbf{p}_{\bar{t}}, \tag{5}$$

as the triple product is odd under *CP*. Thus a nonzero average of $A_{t\bar{t}}$ in Eq. (1) would be an indicator of *CP* violation. It is interesting that triple-product correlations have been extensively discussed as tests of *T* invariance in the past. However, in that case they were flawed indicators as final-state interactions could generate nonzero values for triple products which were naively *T* odd. Here, because of the *CP* property of the initial state, we can use *CP* to probe the triple product and the resulting asymmetry represents true *CP* nonconservation which cannot be faked by final-state interactions.

The most extreme situation is the totally inclusive case where we use only jet information. Here we can imagine that the events are analyzed into jets and the jets have been ordered either by their overall total energy or by their spread in $p_\perp$ ("fatness"). These ordered events (say $E_1 > E_2 > E_3$ or $p_{\perp 1} > p_{\perp 2} > p_{\perp 3}$) can be used to form a *CP*-odd triple product

$$J \equiv \mathbf{p}_1 \cdot \mathbf{p}_2 \times \mathbf{p}_3, \tag{6}$$

and a signal of *CP* nonconservation would be a nonvanishing value of

$$A_J = [N(J>0) - N(J<0)]/N_{\text{tot}}. \tag{7}$$

In this case, *C* would leave the jet variables unchanged because we are summing over particles and antiparticles in defining the jet. However, parity reverses all momenta, and hence *CP* takes $J \to -J$.

To be more systematic, consider the initial state characterized by the beam direction **p** and perhaps some total polarization **S** perpendicular to the beam direction. Both of these quantities return to their original values under the *CP* operation. In the final state, the only characteristic of the jet which is vectorial is the momentum which goes into minus itself under *CP*. To form *CP*-odd triple products from three jet variables one uses Eq. (6) above for any ordering scheme to distintuish the jets. With two jet variables there is no *CP*-odd triple product, but with one we can use asymmetries formed as in Eq. (7) with

$$\tilde{J} = \mathbf{p}_- \times \mathbf{S} \cdot \mathbf{p}_{\text{jet}}, \tag{8}$$

where the jet is chosen by any "*CP*-blind" ordering scheme, e.g., $\mathbf{p}_{\text{jet}}$ could denote the fastest jet. These are the only classes of pure jet triple products.

In semi-inclusive processes, if a determination of the particle versus antiparticle character of the reconstructed particle cannot be made, the particle is only labeled by its momentum, and it enters *CP* tests in the same way as a jet, indicated above. However, if it can be identified, perhaps by the charge of a lepton or hadron in the final state, then a wider class of tests are possible. Denoting the heavy flavored particle by F ($F = D, F, B, B_s, T, T^*, T_s$), the following products are *CP* odd and can be used to form asymmetries to test *CP*:

$$\begin{aligned} &\mathbf{p}_- \times \mathbf{p}_{\text{jet}} \cdot (\mathbf{p}_F - \mathbf{p}_{\bar{F}}), \quad \mathbf{S} \times \mathbf{p}_{\text{jet}} \cdot (\mathbf{p}_F - \mathbf{p}_{\bar{F}}), \\ &\mathbf{p}_- \times \mathbf{S} \cdot (\mathbf{p}_F + \mathbf{p}_{\bar{F}}), \quad \mathbf{p}_{\text{jet }1} \times \mathbf{p}_{\text{jet }2} \cdot (\mathbf{p}_F + \mathbf{p}_{\bar{F}}), \\ &\mathbf{p}_- \cdot \mathbf{p}_F \times \mathbf{p}_{\bar{F}}, \quad \mathbf{S} \cdot \mathbf{p}_F \times \mathbf{p}_{\bar{F}}. \end{aligned} \tag{9}$$

In the first four of these, the use of $(\mathbf{p}_F \mp \mathbf{p}_{\bar{F}})$ does not imply that both F and $\bar{F}$ need to be observed, but rather that when a single F or $\bar{F}$ is observed it enters the triple product with a $\mp$ sign. In the last two, both F and $\bar{F}$ need to be observed so that these are clearly less useful. There can in principle also be *CP*-odd signals with only two factors of the momentum, such as $\mathbf{p}_{\text{jet}} \cdot (\mathbf{p}_F - \mathbf{p}_{\bar{F}})$. However, because this is even under the "naive" time transformation $t \to -t$ (i.e., ignoring the antiunitary nature of the full time reversal operation T), it will be generated only proportional to some final-state phase shift. These final-state interference effects are small in jet physics, and we would expect such scalar-product signals to be dynamically suppressed.

Let us give an example to show how these signals could be generated. Consider $e^+e^- \to t\bar{t}$ on or near the Z^0 peak. The t would have a dominant decay via the weak transition $t \to bc\bar{s}$. The flavors cannot be identified directly, but it has been proposed that the different flavors could have identifiably different p distributions in their jets. Then t jets would be fattest, with $p_\perp \sim m_t$, b jets more narrow ($p_\perp \sim m_b$), and c jets still more narrow. This suggests that an algorithm can be devised which will be able to roughly assign the identity of the heavy quark jet based on the fatness or p of the jet. Of course this algorithm cannot distinguish t from $\bar{t}$, b from $\bar{b}$, etc. We will present our calculation at the quark level and then discuss the effects of hadronization, jet misidentifications, etc. An explicit calculation will be described below, but first we describe the basic physics which produces the asymmetry.

CP nonconservation can manifest itself in the decay distribution for the $t \to bc\bar{s}$ transition. In the t rest frame there will be a correlation of the form $\beta \mathbf{S}_t \cdot \mathbf{p}_b \times \mathbf{p}_c$, where β depends on the model of *CP* nonconservation, and $\mathbf{S}_t$ is the spin of the t quark. When produced by a Z^0, a t quark will be polarized primarily left handed by the weak interaction neutral current, while the $\bar{t}$ will be primarily right handed. Since the helicity axis becomes, in the t rest frame, minus the direction that the $\bar{t}$ is moving, $\mathbf{S}_t = -P\,\hat{\mathbf{p}}_{\bar{t}}$ where P is the polarization. The triple product then becomes $K = -\beta P(\hat{\mathbf{p}}_{\bar{t}} \times \mathbf{p}_b \cdot \mathbf{p}_c)$ and the integrated numbers are invariant under the boost back to the center-of-mass frame, as $\mathbf{p}_{\bar{t}} \times \mathbf{p}_b$ defines a normal to a plane which includes the boost axis. This plane is not changed by the boost, and the integrated numbers count the number of charmed quarks above versus below the plane, which is boost invariant. In this e^+e^- c.m. frame then this mechanism describes a three-jet asymmetry of the form $J = \mathbf{p}_1 \times \mathbf{p}_2 \cdot \mathbf{p}_3$ where the jets are ordered by their fatness (roughly $1{:}2{:}3 = \bar{t}{:}b{:}c$),

$$A_{3J} = [N(J>0) - N(J<0)]/N_{\text{tot}} = [N(K>0) - N(K<0)]\alpha\beta P/N_{\text{tot}}. \quad (10)$$

This example also lets one see how final-state interactions cancel out. The asymmetry in t decay also could be generated by final-state interactions. If we denote the final-state phase shift by δ, the correlation would have strength $(\sin\delta+\beta)P$ in t decay and $-(\sin\delta-\beta)P$ in $\bar{t}$ decay. The overall sign change in going from t to $\bar{t}$ decay comes from the reversal of the polatization ($L \to R$) while the change in the sign of β comes from the complex conjugation of the CP-odd phases in the weak transition matrix element. We see that the final-state interaction strength cancels between t and $\bar{t}$, but the true CP-odd signals add.

Consider the Higgs-boson model of CP nonconservation in which charged Higgs bosons mediate a CP-nonconserving interaction.[2] Because Higgs-boson couplings are proportional to quark masses, the CP-odd signals should grow at high energy. This makes it likely that Higgs-boson models will provide the most sizable potential source of CP nonconservation in jet asymmetries. It we study $t \to bc\bar{s}$ decay in this model we find an effective weak Hamiltonian of the form

$$H_w = (G_F/\sqrt{2})V_{tb}V_{cs}^*\{\bar{c}\gamma_\mu(1+\gamma_5)s\,\bar{b}\gamma^\mu(1+\gamma_5)t + \eta\,\bar{c}[m_c(1+\gamma_5) - m_s(1-\gamma_5)]s\,\bar{b}[m_b(1+\gamma_5) - m_t(1-\gamma_5)]t\}, \quad (11)$$

where η contains CP nonconservation in a set of Kobayashi-Maskawa-type angles for the Higgs-boson coupling,

$$\eta = \sum_i m_{H_i}^{-2} \times \text{angles}. \quad (12)$$

Estimates from the kaon sector[2] yield $\text{Im}\eta \approx 0.03\ \text{GeV}^{-2}$ if the Higgs-boson angle factors are assumed to be similar in size in kaon and t decays. The interference of the Higgs boson and W contributions yields an $\mathbf{S}\cdot\mathbf{p}_1\times\mathbf{p}_2$ correlation in the decay rate of the form

$$|M|^2 = G_F^2|V_{tb}|^2|V_{cs}|^2 2\,\text{Im}\eta\, m_t \varepsilon_{\mu\nu\alpha\beta}p_t^\mu S_t^\nu p_b^\alpha(m_c^2 p_c^\beta + m_s^2 p_s^\beta). \quad (13)$$

At this stage we calculate the rate for the full process, i.e.,

$$\sigma = \sum_S \sigma[e^+e^- \to \bar{t}t(s)]\Gamma(t \to bc\bar{s})/\Gamma_{\text{tot}}(t), \quad (14)$$

and form the asymmetry

$$A_{\text{bare}} = [N(J_B>0) - N(J_B<0)]/N_{\text{tot}}, \quad (15)$$

with $J_B = \mathbf{p}_{\bar{t}}\times\mathbf{p}_b\cdot\mathbf{p}_c$ which, as noted above, is boost invariant along the $t\bar{t}$ line of motion. The integrals over the phase space were performed numerically and we find

$$A_{\text{bare}} = 0.04 \quad (16)$$

for the value of $\text{Im}\eta$ estimated above.

This free-quark asymmetry is still not the experimentally measurable jet asymmetry, and several aspects of physics will modify it. Most obvious is the fact that the ordering of jets by how fat they are could improperly identify the different quarks. This could be dealt with in a straightforward way using jet Monte Carlo simulations. More interesting but more unknown is the question of how much of the helicity information for the t quark survives the fragmentation process. Recall that it was a spin asymmetry which produced the signal, and the desired asymmetry would vanish if the t quark ended up with no net helicity. Indeed, such would be the case if we were discussing b decay. The fragmentation process in the case of the b quark would end up almost always with the b quark in a B^0, B^-, or B_s^0 meson, which are spin zero and hence cannot have any net spin direction for the b quark. The only exception to this would be if a b-flavored baryon were produced, in which case some net b helicity could remain. The choice of t-quark decay was motivated by this problem, as in the case of T mesons the spin-one T^* is expected[3] to decay *weakly,* not by electromagnetic or strong processes. This occurs for quite simple reasons. First, the weak decay rate grows by the fifth power of the quark mass and rapidly becomes sizable. Second, the T^*-T mass splitting decreases as an inverse power of the quark mass, so that the P-wave phase space in $T^* \to T+\gamma$ rapidly suppresses this mode. For our purposes, the existence of an almost stable T^* implies that the t quark can maintain its helicity around one half of the time.

The above example has the CP nonconservation generated in the final state, and the signal may be diluted by the fragmentation process. It is possible to generate a CP-odd signal at the production vertex in such a way that it is not modified by the subsequent fragmentation. For example, if there is a spinless particle whose interactions violate CP invariance, then interference between ϕ^0 and Z^0 or ϕ^+ and W^+ in

$$(e^+e^-)_{\text{polarized}} \to \begin{Bmatrix} Z^0 \\ \phi^0 \end{Bmatrix} \to t + \bar{t} + \text{gluon},$$

or

$$(p\bar{p})_{\text{polarized}} \to \begin{Bmatrix} W^+ \\ \phi^+ \end{Bmatrix} + X \to t + \bar{b} + \text{gluon} + X,$$

will generate an asymmetry using $\tilde{J} = \mathbf{p}_-\times\mathbf{S}\cdot\mathbf{p}_{\text{jet}}$, where $\mathbf{p}_{\text{jet}}$ is the momentum of the *fastest* jet. The identification of the fastest jet should be relatively independent of the details of fragmentation. In the $p\bar{p}$ case, the relevant matrix element could be of the form

$$M=\frac{g^2}{q^2-M_W^2}J_\mu^W J_\mu^W+i\bar{\eta}\frac{g^2}{q^2-M_\phi^2}\bar{d}u\,\bar{t}\gamma_5 b,$$

where $\bar{\eta}$ measures the strength of the ϕ interaction. The gluon emission is needed in order to have jets of unequal momentum. The asymmetry vanishes as $m_t/E\rightarrow 0$ and is highly dependent on m_t. However, for large values of m_t, working at energies such that $\langle q^2-M_W^2\rangle \approx \langle q^2-M_\phi^2\rangle$, we estimate that the asymmetry can be as large as

$$A_{\tilde{J}}=\frac{N_{3\,\mathrm{jet}}(\tilde{J}>0)-N_{3\,\mathrm{jet}}(\tilde{J}<0)}{N_{3\,\mathrm{jet}}(\tilde{J}>0)+N_{3\,\mathrm{jet}}(\tilde{J}<0)}\sim\frac{\eta}{10}.$$

Signals of this form could then be useful for looking at *CP* nonconservation in the production vertex in a way which should be more independent of the fragmentation process.

In the standard model, most jetlike *CP*-odd observables are likely to be small unless a method can be devised to isolate rare classes of events. The reason is because in the Kobayashi-Maskawa model[4] all *CP* nonconservation is proportional to the combination of angles $s_1^2 s_2 s_3 s_\delta \lesssim 10^{-4}$. If a process proceeds unsuppressed by small angles, then the *CP* nonconservation will be hidden by at least this 10^{-4} factor. To make the *CP* nonconservation more visible in the standard model one can investigate processes in which the *CP*-even signal is also suppressed by small angles. However, these rare events are not in general easy to find in the jet environment. The jet observables are more likely to be useful in the case where there are new interactions which become more important at higher energies. The Higgs-boson models of *CP* nonconservation are an example of interactions in this class. In addition, the decays of new heavy particles (charged Higgs bosons, scalar quarks, ...) may allow classes of events to be isolated (by the characteristic large p released) in which *CP* nonconservation is more visible.

This paper has described some new tools for the study of *CP* nonconservation at colliders. While they do not appear useful in the standard model, they could be sensitive to other forms of *CP* nonconservation. To the extent that they are easy to apply experimentally, they should be used to search for *CP*-odd signals in all classes of events. The importance of finding new forms of *CP* nonconservation is so large that such searches should not be overlooked. In addition, when new particles and/or reactions are discovered in the future, these tests can be applied to those events to help determine if the new interactions conserve *CP*.

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