Hadron Production near Threshold in Photon-Photon Collisions

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A dual picture between perturbative QCD and resonance production is exploited to describe nondiffractive meson pair production near threshold in $\gamma\gamma$ annihilation.

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In quantum chromodynamics, hadron production in $\gamma\gamma$ annihilation is initiated by the coupling of the incident photons to either one or two quark pairs. At high momentum transfer, both inclusive and exclusive reactions can be computed in terms of underlying elementary quark- and gluon-scattering subprocesses.[1] A large body of data from the SLAC and DESY storage rings PEP and PETRA gives good support to the validity of this perturbative QCD description. At low momentum transfer, the processes which convert the quarks into hadrons are generally nonperturbative, coherent, complex, and thus, at this point, not calculable.

There are, however, exceptional low-momentum processes where perturbative QCD predictions should be reliable, even at the threshold energies, if Coulombic rescattering corrections are properly taken into account; e.g., those rare $\gamma\gamma$ annihilation processes in which only hadrons containing heavy quarks are produced. As an example, we illustrate the leading-order minimally connected QCD diagrams for heavy-meson pair production in Fig. 1. Since the exchanged-gluon momenta and the effective renormalization scale are of order of the heavy-quark mass, a perturbative QCD analysis can be justified. Unfortunately, the predicted rate is small, and tests of this basic feature of QCD appear remote.

Nevertheless, we shall argue here that given the validity of a dual picture between the perturbative amplitudes and suitably averaged resonant contributions, one can obtain reasonable estimates of relatively-light-meson pair production $\gamma\gamma$ cross sections even near threshold.

Let us recall the situation for heavy-quark pair production in e^+e^- annihilation. In this case the cross section is well under control in QCD not only at high energies but also near threshold where only a minimal number of hadrons can be produced. Close to threshold, the familiar one-gluon-exchange vertex correction (α_s/π at high energies) transforms into a nonrelativistic Coulombic scattering correction $4\pi\alpha_s/3v_{\text{rel}}$.[2] In fact this QCD Sommerfeld correction appears to describe resonance production very well[3] if properly smeared over level spacings.[4] The dynamical reason for the validity of this dual picture is the Coulombic form of the heavy-quark potential with a strength given by the running strong coupling constant at the heavy-quark momentum scale ($\approx \alpha_s m_Q$).

While the dual picture appears to be well understood for heavy quarks, it turns out to be applicable empirically even for light quarks—at least approximately.[5] In this note we shall apply this extended duality picture to a class of $\gamma\gamma$ reactions which have recently attracted experimental attention. Typical examples are $\gamma\gamma \to \rho^+\rho^-$, $K^{*+}K^{*-}$, and $K^{*0}\bar{K}^{*0}$ in which the vector mesons cannot be produced diffractively,[6] and the quarks created by the two photons must be interchanged to form the final-state resonances. The various mechanisms are again described by the generic diagrams shown in Fig. 1. Before discussing these examples in detail, we first develop the

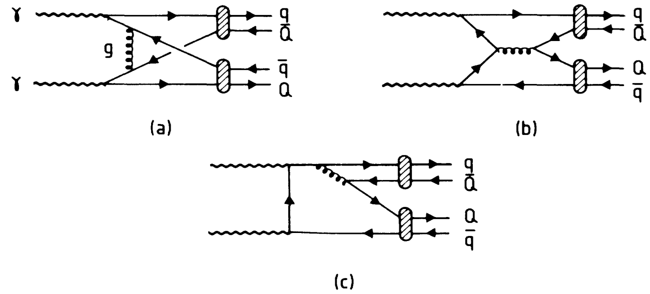

FIG. 1. Mechanisms for the nondiffractive production of color-neutral $q\bar{q}$ compounds in $\gamma\gamma$ annihilation. (a) Constituent interchange, (b) formation, and (c) bremsstrahlung.

basic formalism for the production of two heavy-quark pairs to which perturbative QCD—including final-state corrections—should safely be applicable in analogy with the results for e^+e^- annihilation.

We can conveniently describe the production of two heavy-quark pairs of type q and $Q\neq q$ in $\gamma\gamma$ collisions,

$$\gamma\gamma \rightarrow q\bar{q}Q\bar{Q}, \tag{1}$$

in terms of photon helicity amplitudes. We choose the flight axis of the γ's in the c.m. frame as the quantization axis of the quark spins. The following expressions can be derived for the helicity amplitudes and spin-averaged matrix element squared, corresponding to the three mechanisms shown in Figs. 1(a)–1(c). (a) Constituent interchange:

$$\tau(++)=\tau(--)=-\frac{8g_s^2}{3}\frac{e_qe_Q}{m_qm_Q}(-)^{s_1}(-)^{s_3}\delta_{s_1n_2}\delta_{s_3n_4}, \tag{2a}$$

$$\tau(+-)=\tau(-+)=0, \tag{2b}$$

$$\tfrac{1}{4}\sum|\tau|^2=\frac{128}{9}\frac{g_s^4e_q^2e_Q^2}{m_q^2m_Q^2} \tag{2c}$$

(b) Formation:

$$\tau(++)=\tau(--)=0, \tag{3a}$$

$$\tau(\pm\mp)=-\frac{4g_s^2}{3}\frac{e_q^2+e_Q^2}{m_qm_Q}[1\mp(-)^{s_1}][1\mp(-)^{s_3}]\delta_{s_1s_2}\delta_{s_3s_4}, \tag{3b}$$

$$\tfrac{1}{4}\sum|\tau|^2=\frac{128}{9}\frac{g_s^4(e_q^2+e_Q^2)^2}{m_q^2m_Q^2}. \tag{3c}$$

(c) Bremsstrahlung:

$$\tau(++)=\tau(--)=\frac{4g_s^2}{3}\frac{m_qe_q^2+m_Qe_Q^2}{m_qm_Q(m_q+m_Q)}\{[1-(-)^{s_1+s_3}]\delta_{s_1s_2}\delta_{s_3s_4}-\delta_{s_1n_2}\delta_{s_3n_4}\}, \tag{4a}$$

$$\tau(+-)=\tau(-+)=0, \tag{4b}$$

$$\tfrac{1}{4}\sum|\tau|^2=\frac{32}{3}\frac{g_s^4(m_qe_q^2+m_Qe_Q^2)^2}{m_q^2m_Q^2(m_q+m_Q)^2}. \tag{4c}$$

The masses and charges of the quarks are denoted by $m_{q,Q}$ and $e_{q,Q}$ respectively; g_s is the QCD coupling constant. The photon helicities are specified by the arguments of the helicity amplitudes. The quark-spin indices are denoted by $s_i=1,2$ and $n_i=2,1$ for spin = up, down. The lable $i=1,\ldots,4$ corresponds to $q,\bar{q},Q,\bar{Q}$, respectively.

It has been assumed in the derivation of the formulas that $(q\bar{Q})$ and $(\bar{q}Q)$ each form color singlets, resulting in a universal color factor $\frac{4}{3}$. As a consequence, the squared amplitudes of the three mechanisms summed over quark spins do not interfere with each other at threshold. Quarks and antiquarks in color singlet states attract each other, so that the production amplitude gets enhanced by the Coulombic rescattering corrections. In the octet state, by contrast, they repel each other and the production is de-enhanced.

We thus apply the Coulomb correction factor,[2]

$$C=8\pi\alpha_s m_{\mathrm{red}}/3\,|p_{\mathrm{rel}}|, \tag{5}$$

to each quark pair in the final state, as indicated by the dashed blobs in Fig. 1. Here $m_{\mathrm{red}}=m_qm_Q/(m_q+m_Q)$ is the reduced quark mass and p_{rel} is the relative momentum of the quark-antiquark pair in its rest frame. The final-state correction factor is singular at zero relative momentum, strongly enhancing the threshold region $m(Q\bar{q})\approx m_Q+m_q$. Since this singularity cancels part of the overall phase-space suppression, we find fairly large threshold $\gamma\gamma$ cross sections.

By far the largest contribution is due to the formation mechanism. For equal quark charges, for example, and $m_Q\gg m_q$ or $m_Q=m_q$ the cross sections at threshold corresponding to constituent interchange, formation, and bremsstrahlung are in the ratio

$$\sigma_a:\sigma_b:\sigma_c=\tfrac{1}{4}:1:\tfrac{3}{16}, \tag{6}$$

and similarly for other cases. Notice that the formation mechanism produces pairs of spin-one mesons only. Final states composed of charge $\frac{2}{3}$ quarks are of course much more frequently created in $\gamma\gamma$ reactions than those of the down-type quarks.

There is a different rescattering mechanism that should contribute to four-quark final states in γ-γ collisions: Two quarks attract each other in a di-quark $\mathbf{3}^*$ state; the di-quark and di-antiquark may then form a $qq\bar{q}\bar{q}$ resonance. Such states would be expected to decay preferentially into many-particle final states since the color-neutral $q\bar{q}$ pairs are by assumption absent in the amplitude. We shall leave this very interesting problem aside in this paper.

As noted previously, the $\gamma\gamma$ cross sections for pairs of heavy quarks are too small to be readily accessible experimentally. However, by postulating the dual picture, we can at least tentatively extend the analysis to light quarks in order to obtain a semiquantitative understanding of the pair production cross section for meson resonances. The duality picture is used in its local form, approximating a single vector meson and the related π-π or π-K continuum by the quark-antiquark continuum with the appropriately integrated invariant mass. In general one expects duality to become increasingly accurate as the range of invariant mass averaged over is increased.

The parameters of the calculation are defined as follows. (i) We set the u- and d-quark masses to 300 MeV, the strange-quark mass to 500 MeV, and the charmed-quark mass to 1.5 GeV, corresponding to the familiar effective constituent-quark-mass values. The calculation is thus applicable to ρ and K^* production in the light-quark sector. The predicted rates for each resonance also include the corresponding $\pi\pi$ and $K\pi$ continuum of equivalent mass. One cannot expect these calculations to be applicable to $\gamma\gamma \to \pi\pi$ since the dual pion cannot be described in an additive quark picture in QCD, not even approximately. (ii) For the QCD coupling constant we have adopted $\Gamma = 200$ MeV and the argument of α_s is chosen as the geometric mean of all the quark energies involved in the process.

In the analysis, $\alpha_s = g_s^2/4\pi$ was generally small, but in any event its value was frozen to $\alpha_s \leq 1$ where necessary. For the final numerical results the matrix elements for $\gamma\gamma \to \bar{q}q\bar{q}q$ were not approximated by threshold values but were evaluated numerically by use of the results of Kunszt[7] (with straightforward integration over four-particle phase space) so that the calculation should remain meaningful for light-quark resonances with production energies up to $O(1$ GeV) beyond threshold. Beyond this energy the dual representation of the vector mesons by the color-neutral blobs presumably ceases to be valid.

The results of the calculations for nondiffractive vector-meson pair production (plus the corresponding $\pi\pi, K\pi$ continua) are summarized in Fig. 2.

(a) $\gamma\gamma \to (u\bar{d}) + (\bar{u}d) \sim \rho^+\rho^-$: The cross section is shown in Fig. 2(a) where it is compared with the yield of $\gamma\gamma \to (\pi^+\pi^0) + (\pi^-\pi^0)$, the invariant $\pi\pi$ masses restricted to the $\rho^{\pm}$ band, as measured by the JADE collaboration,[8] and presented in the summary of Kolanoski.[9] The theoretical analysis appears compatible with the $\pi\pi$ yield in the ρ band. Note that no attempt has been made to adjust any parameters; they have been fixed on physical grounds before the numerical work was started. Nonresonant 4π final states are also created by mechansims not discussed in this note, such as $\gamma\gamma \to uggg\bar{u} \to (u\bar{d}) + (\bar{d}d) + (\bar{d}d) + (d\bar{u}) \sim \pi^+\pi^0\pi^0\pi^-$. These channels can spill into the $\rho\rho$ region.

(b) $\gamma\gamma \to (s\bar{u}) + (\bar{s}u) \sim K^{*-}K^{*+}$ and $(s\bar{d}) + (\bar{s}d) \sim \bar{K}^{*0}K^{*0}$: As a result of the heavier s-quark mass, we expect $\sigma(K^{*+}K^{*-})$ to be approximately a factor of ~ 3 smaller than $\sigma(\rho^+\rho^-)$. This is indeed born out by the detailed analysis shown in Fig. 2(b). Furthermore, the ratio $\sigma(K^{*0}\bar{K}^{*0})/\sigma(K^{*+}K^{*-})$ is expected to be of the order of $2(e_d/e_u)^4 = \frac{1}{8}$; see Fig. 2(b). The neutral K^* sector has been analyzed by the TPC/Two-Gamma[10]

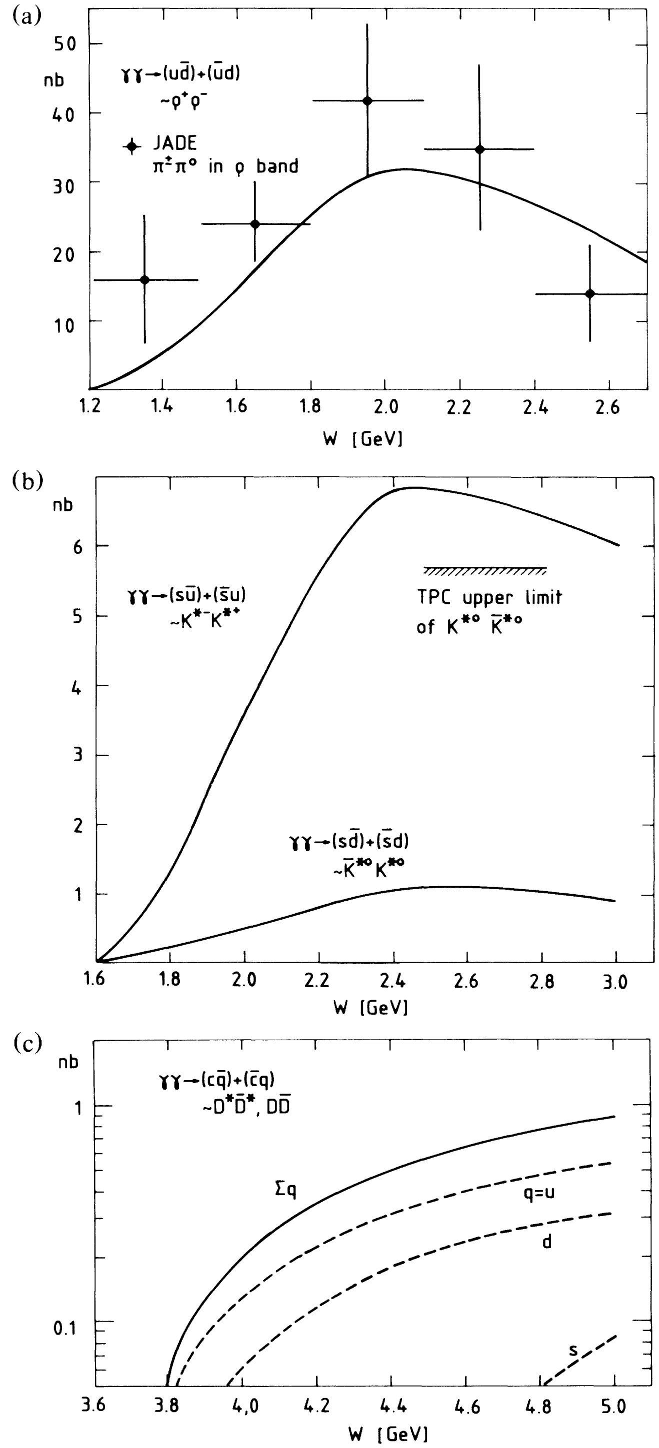

FIG. 2. Production cross sections for $q\bar{q}$ plus $q\bar{q}$ compounds in $\gamma\gamma$ annihilation near threshold. [The data points in (a) are upper limits for the $\rho^+\rho^-$ cross section.]

and Tasso[11] collaborations. An upper limit of 5.7 nb has been reported for $K^{*0}\bar{K}^{*0}$ resonance production in Ref. 10, still much above our theoretical analysis. At the same time, a large ratio $\sigma(K^{*0}K\pi)/\sigma(K^{*0}\bar{K}^{*0}) \gg 1$ is observed. This is nicely compatible with general expectations: Whereas an s or d quark has to be coupled to the photons to produce a $K^{*0}\bar{K}^{*0}$ final state, the formation of a nonresonant $K^+K^{*0}\pi^-$ final state can proceed via $\gamma\gamma \rightarrow ugg\bar{u} \rightarrow (u\bar{s})+(s\bar{d})+(d\bar{u})$ where a u-quark line is attached to both photons, thus resulting in a substantial increase of the cross section.

(c) $\gamma\gamma \rightarrow (c\bar{q})+(\bar{c}q) \sim D\bar{D}, D^*\bar{D}^*$: Adding up all light-quark channels, the cross section is predicted to be of the order of $\sim \frac{1}{2}$ nb in the threshold region [see Fig. 2(c)].

Even though the application of a perturbative QCD analysis in conjunction with duality arguments is a conjecture that can only be justified *a posteriori*, it is an attractive theoretical scheme since it is based soley on fundamental quark-gluon parameters: the quark masses, and the quark-gluon coupling constant. The picture is too crude to expect high accuracy; however, it offers a simple qualitative understanding of the magnitude of the cross sections and their ratios for different final states. We hope that this analysis will be useful to describe the global features of $\gamma\gamma$ experiments now underway at DESY and the upgraded PEP at SLAC. It also would be useful to construct model amplitudes which interpolate from the threshold region analyzed here to the regime of large momentum transfer where the leading power-law predictions[1] of the perturbative QCD factorization analysis become relevant.

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