Singularity in the Kapitza Resistance between Gold and Superfluid ^{4}He near T_λ

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We report experimental results for the thermal boundary resistance R_K between gold and superfluid ^{4}He which were obtained by the use of thermometry with 3-nK resolution. The data imply that R_K is singular at the superfluid transition temperature T_λ. Comparison with theory suggests that the singularity results from a hydrodynamic effect proposed by Landau, and that it is associated with the vanishing of the superfluid and normal-fluid currents at the boundaries.

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Heat transport in superfluid ^{4}He occurs by counterflow of the normal-fluid and superfluid currents, and at sufficiently small heat fluxes does not induce any temperature gradient in the bulk liquid.[1] Therefore the temperature difference across a conductivity cell will result only from effects associated with the solid-liquid boundaries and the finite conductivity of the solid end plates of the cell. The thermal resistance R_K associated with the boundaries is the Kapitza resistance,[2] and is generally attributed to a discontinuity of the temperature at the solid-liquid interface. We report the experimental observation of a singular contribution to R_K near the superfluid transition temperature T_λ. A hydrodynamic model proposed by Landau[3] fits our data well.[4] According to Landau, both the superfluid and the normal-fluid currents vanish at the surface, and thus the heat transport very near the surface is by thermal conduction rather than by superfluid counterflow. For nonvanishing heat flux a temperature gradient will then exist in a boundary layer close to the solid surface, even though the bulk fluid is isothermal. For temperatures within 10^{-3} K of T_λ this effect manifests itself as a singularity in R_K. A theoretical prediction for the nature of this singularity has been made recently by Ferrell.[5] This theory, as well as our data, shows that this process makes only a negligible contribution to R_K well below T_λ; but near T_λ it can contribute about 10% of the total resistance. Such a boundary layer exists for other superfluids, i.e., for ^{3}He and for superconductors.[6]

We note that the singularity in R_K was not resolved in previous work which used conventional germanium thermometry.[7,8] Its detection required the extremely high-resolution thermometers which have been developed only recently.[9,10] A brief report of our results has already been presented.[11]

The cryostat used in this work is shown schematically in Fig. 1(a). The vacuum can, which was submerged in a liquid helium bath, contained four stages. Below the refrigerator stage and the isothermal stage was a copper can which shielded the cell against radiation from the thermally uncontrolled bath. This shield was typically operated 0.1 K above T_λ. We used two different cells, labeled J and K. Cell J is shown schematically in Fig. 1(b), and cell K in Fig. 1(a). Both cells consisted of two parallel plates of circular cross section separated by a region of superfluid helium bounded on the sides by a thin stainless-steel wall. The dimensions of the cells are shown in Table I. The surfaces of both cells were prepared by lapping and polishing of oxygen-free high-conductivity copper to a mirror surface. These surfaces were then chemically plated with a layer of gold.

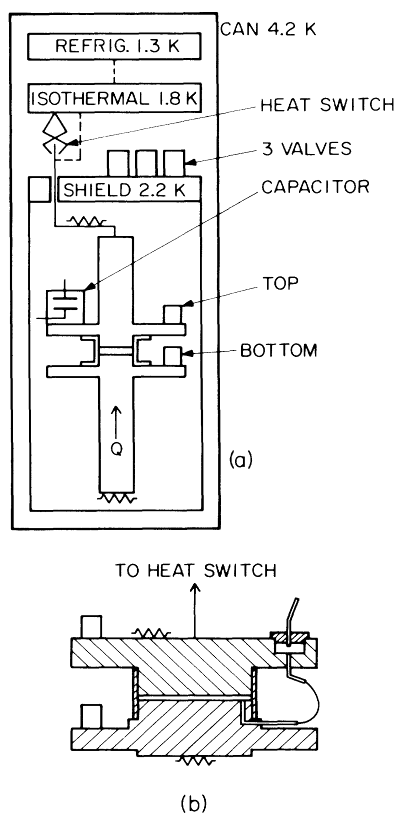

FIG. 1. (a) Schematic diagram of the cryostat showing cell K installed. (b) Cell J, shown in cross section.

TABLE I. Room-temperature dimensions of the cells.

Cell	Diameter (cm)	Area (cm^2)	Height (cm)	D/H	Wall thickness (cm)
J	4.9124 ± 0.0013	18.953 ± 0.010	0.0975 ± 0.0025	50.4	0.0107 ± 0.0003
K	2.534 ± 0.0013	5.043 ± 0.005	0.2878 ± 0.0008	8.80	0.0114 ± 0.0018

A ^{3}He vapor-pressure thermometer capable of 3-nK resolution near T_λ was installed on each side of the cell. These thermometers were calibrated against the "top" germanium thermometer of Ref. 7. Three cryogenic valves were located on the shield stage and used to close the cell and thermometer supply lines during data acquisition. All measurements were made at saturated vapor pressure. For this purpose an overflow volume was provided in the top cell end (see Fig. 1) where the liquid-vapor interface was maintained. Cell K had a 6.35-cm length between the cell surfaces and the ends of the copper pieces at which the heat flux was applied or removed. This ensured that the isotherms within the copper would be flat in the regions between the thermometer location and the gold–liquid-helium interface. A copper rod was machined from the same metal stock used to produce the end pieces of cell K, and its thermal conductivity was measured over the range $1.6\ \mathrm{K} \le T \le 4.4$ K. The result could be represented by $\lambda_{\mathrm{Cu}} = 0.162 + 0.973T$ W/cm K, and was used to evaluate the contribution from the copper ends (typically 25%) to the measured temperature differences ΔT for cell K. This contribution is given by $\Delta T_{\mathrm{Cu}} = L_{\mathrm{Cu}} Q/\lambda_{\mathrm{Cu}}$, where L_{Cu} is the total length of the copper sections between the thermometers, and Q is the heat flux through the cell. For cell J a correction was estimated by subtraction of a contribution ΔT_{Cu} which yielded overall agreement with the cell-K data. The required ΔT_{Cu} differed by only 24% from a rough *a priori* estimate based on the geometry of cell J and the λ_{Cu} of the copper stock used in cell K.

Ultrapure ^{4}He with a measured ^{3}He-impurity concentration of 5×10^{-10} was used in cell K. Standard well helium with an estimated ^{3}He-impurity concentration of 2×10^{-7} was used in cell J. The boundary resistance was measured by applying a heat flux Q to the bottom of the cell while maintaining the top thermometer at a constant temperature T_t. The resulting $\Delta T(Q)$ was used[12] to evaluate the boundary resistance

$$R_B = (\Delta T - \Delta T_{\mathrm{Cu}})/2Q. \quad (1)$$

For the temperature corresponding to this measurement we chose the mean temperature $\overline{T} = T_t + \Delta T/2$. Since the superfluid transition temperature at a given level in the liquid was suppressed by the hydrostatic pressure as a result of the liquid above, there existed a variation of T_λ along the vertical axis of the cell. We used T_λ at the middle of the liquid layer to define the reduced temperature $t \equiv 1 - \overline{T}/T_\lambda$.

Shown in Fig. 2 are cell-K measurements of R_B plotted against Q at nine values of $\log_{10} t$. Notice that for $t \le 1.6 \times 10^{-5}$ the data show a dependence on Q. Figure 3 shows the dependence of the same measurements on t at three different values of Q. For $t > 1.6 \times 10^{-5}$, R_B is independent of Q over the range $Q \le 10\ \mu\mathrm{W/cm}^2$. For $t < 1.6 \times 10^{-5}$ the measurements of R_B increase rapidly

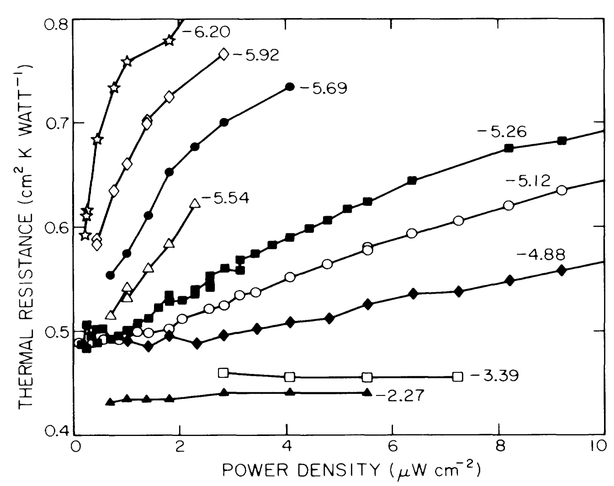

FIG. 2. Cell-K boundary resistance measurements R_B vs Q for the nine values of $\log_{10} t$ given in the figure.

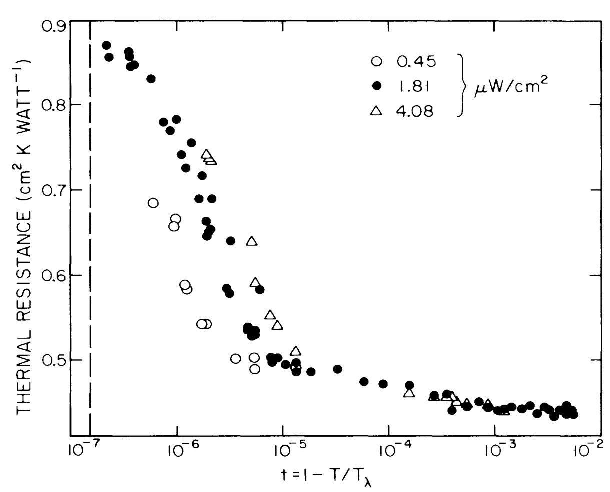

FIG. 3. Cell-K boundary resistance measurements R_B vs t for three values of Q. The dashed line represents the limit of single-phase measurements in the presence of gravity.

with Q. The reduced temperature at which finite-power effects become important is weakly Q dependent. Referring to Fig. 3 we see that when $Q=4.08$ μW/cm^2, finite-power effects persist out to $t=1.3\times10^{-5}$, while at $Q=0.45$ μW/cm^2 these effects are negligible for $t\geq 4\times10^{-6}$. The rise of R_B in the Q-dependent region has also been observed by Dingus, Zhong, and Meyre.[8] The dashed vertical line on the left side of Fig. 3 represents the limit of measurements in cell K in the presence of gravity. Since t is based on T_λ at the midpoint of the superfluid layer, all reduced temperatures to the left of the dashed line correspond to the presence of a superfluid–normal-fluid interface within the lower half of cell K.

We define R_K, the Kapitza resistance, as the value of R_B in the limit $Q\rightarrow 0$. For $t>1.6\times10^{-5}$ R_K was simply R_B measured at any of our values of Q. For $t<4\times10^{-6}$ the power dependence was so strong that an extrapolation to zero heat flux was not attempted. Figure 4 shows R_K of both cells plotted against t for $t>4\times10^{-6}$. Measurements of the Kapitza resistance between superfluid ^{4}He and gold at low temperatures[7,13] show that R_K is approximately proportional to T^{-3}. This effect accounts for the rise in R_K for $t>2.5\times10^{-3}$. The anomalous increase by about 10% of R_K as t changes from 10^{-3} to 10^{-5} indicates that R_K is singular at T_λ. If R_K were a regular function of temperature at T_λ, then the curve in Fig. 4 would become horizontal as $t\rightarrow 0$. The remarkable similarity between R_K in cells J and K (which have different aspect ratios, ^{3}He-impurity concentrations, and assembly procedures) strongly supports the idea that the singularity is not caused by the bulk liquid, but rather is a property of the solid-liquid interfaces.

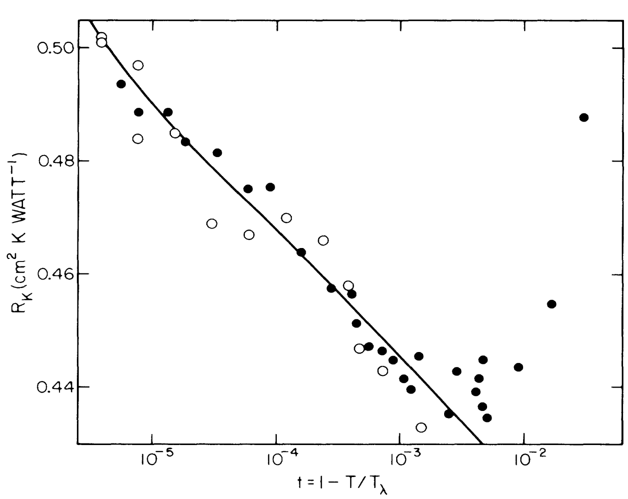

FIG. 4. Kapitza resistance R_K vs t. Solid circles are cell-K measurements, open circles are uncorrected cell-J measurements minus 0.587 cm^2 K/W. The curve is the best fit of the theory from Refs. 3 and 5 to the data.

From two-fluid hydrodynamics,[1,3] the heat current carried by superfluid counterflow is given by

$$Q_s=[(\rho_s/\rho_n)j_n-j_s]ST. \tag{2}$$

Here ρ_s and ρ_n are the superfluid and normal-fluid densities, j_s and j_n are the superfluid and normal-fluid currents, and S is the entropy. According to the theory,[3,5] the currents vanish at the solid surfaces if the conversion rate between the superfluid and normal-fluid components is finite. Hence, at the surfaces $Q_s=0$. Thus, there exist liquid layers near the surfaces, say of typical thickness ξ_K, that cannot transport the entire heat flux Q by counterflow. Within these layers, heat is in part diffusively conducted. At the top (cool) surface, normal fluid entering this layer must be converted to superfluid, and *vice versa* at the bottom (warm) surface. Thus the continuity equation for the superfluid component[1,3] must contain a sink term[5] in these regions. In one dimension it becomes

$$\partial\rho_s/\partial t+\partial j_s/\partial z=-\delta\rho_s/\tau. \tag{3}$$

Here the z axis lies along the axis of the cylindrical cell, $\delta\rho_s$ is the change of ρ_s from its value in the absence of heat flow, and τ is a characteristic time for superfluid to normal-fluid conversion. Ferrell suggests[5] that τ may be approximated by

$$\tau\approx\xi^2/2D_\psi, \tag{4}$$

where $\xi=\xi_0 t^{-\nu}$ is the correlation length with $\nu=0.672$, and where D_ψ is an order-parameter diffusivity. Sufficiently near T_λ, we expect $D_\psi\sim t^{-\nu/2}$, yielding $\tau\sim t^{-3\nu/2}$. Thus, τ becomes large as t becomes small, and $\delta\rho_s$ must grow with decreasing t in order to maintain the right-hand side of Eq. (3) at a steady-state value. The change of ρ_s by $\delta\rho_s$ is associated with a temperature difference δT which drives the heat diffusion within the layers of effective thickness ξ_K. Thus it contributes an additive term to the Kapitza resistance given by $R_K^\xi=\xi_K/\lambda$, where λ is the diffusive thermal conductivity of the layer. The prediction[5] for R_K^ξ may be written in the form

$$R_K^\xi=r(C_P/2\nu w'kS)^{1/2}t^{(1-3\nu)/2}/\lambda, \tag{5}$$

where C_P is the specific heat at constant pressure[14] and $k\simeq 2.41$ is the amplitude of the superfluid fraction[14] $\rho_s/\rho=kt^\nu$. To obtain Eq. (5), we approximated the ratio $D_\psi/(\lambda/\rho C_P)$ by the real part w' of the frequency ratio w which can be derived from thermal conductivity measurements above T_λ.[15] The coefficient r is a characteristic length, expected to be within a factor of order unity of ξ_0. We fit

$$R_K=R_K^0+R_K^\xi \tag{6}$$

to our measurements, using only R_K^0 and r as adjustable parameters. The best fit of Eq. (6) to our data is shown in Fig. 4. It gave $R_K^0 = 0.408$ cm^2 K/W and $r = 11.8 \times 10^{-8}$ cm.

The agreement between the theory and experiment in Fig. 4 is remarkable. Nonetheless, certain issues remain unresolved. The theory in its present form[5] assumes that the equilibrium superfluid fraction is constant over the entire length of the cell, while near the surface ρ_s presumably is reduced below its bulk value. For sufficiently small t, this effect will alter the prediction, Eq. (5).[16] Because of it, the singular mechanism discussed above will not be symmetric about the midpoint of the cell. At the warm end the hydrodynamic effect opposes the equilibrium superfluid-density depression, while at the cool end it enhances it.[17] Further, the remaining thermodynamic parameters, most notably λ and C_P, are also modified from their bulk values within a few correlation lengths of the surfaces. In addition, any density variation near the surface due to van der Waals interactions has so far been neglected in the theory. Finally, the strong power dependence of R_B in Figs. 2 and 3 has not yet been explained quantitatively.[17]

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