Four-Mode Squeezing

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The four-wave-mixing interactions produced by pumping at two well-separated frequencies can couple four different radiation modes to generate a new kind of squeezed state of light. A dual-frequency homodyne detection scheme is described and is used to observe this type of nonclassical correlation. Initial experimental results in an optical fiber show a noise level 20% below the vacuum noise level for the output of the dual-frequency detector, even though the noise at each individual detector remains above the vacuum level.

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Parametric mixing via either four-wave or three-wave nonlinear interactions has been demonstrated to generate squeezed states of light, states whose quantum noise level varies with the phase of the local oscillator used in detection and which can fall below that associated with coherent states (the vacuum noise level).[1–3] In these interactions, a pump field at a single frequency causes the signal and idler modes shifted above and below the pump frequency (or half the pump frequency, in three-wave mixing) to become coupled, in pairs—hence the name "two-mode squeezing."[4] It has been shown theoretically that pumping at two or more frequencies can induce correlations among four or more modes and thereby permit further quantum noise suppression.[5,6] We have produced and detected such "four-mode squeezing" using four-wave mixing in an optical fiber. This Letter reviews some of the properties of the four-mode squeezing produced when pump beams of two different frequencies couple four sideband modes via third-order nonlinear optical processes, discusses its detection, and describes our experimental results.

Recall that ordinary two-mode squeezing is detected by mixing of a local oscillator at the pump frequency with signal and idler modes at radio-frequency offsets $\pm\delta$ from the local-oscillator (LO) frequency (e.g., in a balanced homodyne scheme[1,3,7,8]). When a nonlinear medium is pumped at two frequencies, the sideband modes at one pump frequency become coupled to those at the other, giving a total of four coupled modes for each radio frequency δ. The most general way to detect the resultant four-mode squeezing is to homodyne with two LO frequencies, one at each pump frequency. Alternatively, one can homodyne first with an optical-frequency local oscillator (at the average pump frequency) and then with an rf local oscillator (at the difference pump frequency).[5,6] The latter detection scheme was not used in our experiment, since it does not allow the flexibility we required in order to circumvent fiber-induced phase noise.

Consider then a system of two detectors D1 and D2, as shown in Fig. 1. The detectors are separately illuminated by strong LO waves $\mathcal{E}_1$ and $\mathcal{E}_2$ at frequencies ω_1 and ω_2, respectively, which mix with signal and idler modes to produce currents $\hat{\mathcal{I}}_1$ and $\hat{\mathcal{I}}_2$ that oscillate at radio frequencies δ. When the sidebands at each of the detectors are squeezed in the ordinary, two-mode sense, the rf noise powers at detectors D1 and D2 vary with the LO phase shifts, Φ_1 and Φ_2. Now let the two detector currents be combined with a relative phase shift $2\phi_r$. (Each detector current undergoes an electronic phase shift before it is combined with the other; for present purposes, however, the common phase shift is inconse-

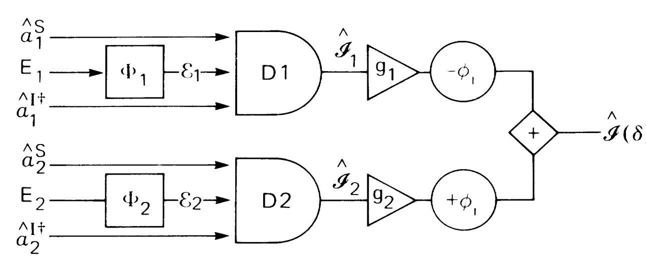

FIG. 1. A dual-frequency homodyne detector. Beams of light with multiple frequency components centered around carrier frequencies ω_1 and ω_2 are incident on detectors D1 and D2, respectively. The carrier waves at ω_1 and ω_2 have amplitudes E_1 and E_2, respectively. The sidebands above and below the carriers are described by the modal annihilation operators $\hat{a}_j^S$ and $\hat{a}_j^I$, respectively ($j=1,2$). The local-oscillator waves with amplitudes $\mathcal{E}_1$ and $\mathcal{E}_2$ are derived by phase shifting of the carriers optically by Φ_1 and Φ_2. The currents from the two detectors are amplified and electronically phase shifted as shown before being summed to yield the output current $\hat{\mathcal{I}}$.

quential, and will henceforth be ignored.) The positive-frequency amplitude at frequency δ of the combined current $\hat{\mathcal{J}}$ is the complex operator

$$\hat{\mathcal{J}}^{(+)}(\delta)=e^{-i\phi_r}g_1\hat{\mathcal{J}}_1^{(+)}(\delta)+e^{i\phi_r}g_2\hat{\mathcal{J}}_2^{(+)}(\delta),\quad \hat{\mathcal{J}}_j^{(+)}(\delta)=2^{-1/2}\eta_j[\mathcal{E}_j^*\hat{a}_j^S+\mathcal{E}_j\hat{a}_j^{I\dagger}]+[\eta_j(1-\eta_j)]^{1/2}|\mathcal{E}_j|\hat{v}_j,\quad j=1,2. \tag{1}$$

Here $\hat{\mathcal{J}}_j^{(+)}$ are the positive-frequency noise-current operators for the individual detectors, g_j the detector electronic gains, and η_j the detector quantum efficiencies. The complex numbers $\mathcal{E}_j=T_jE_j\exp(-i\Phi_j)$ are the LO amplitudes at detectors D1 and D2, derived in our experiment from the pump amplitudes E_j; the factors Φ_j and T_j are real with $0<T_j\leq 1$. The operators $\hat{a}_j^S$ are annihilation operators for signal sideband modes at $\omega_j+\delta$, and $\hat{a}_j^{I\dagger}$ are creation operators for idler modes $\omega_j-\delta$. The operators $\hat{v}_j$ are quadrature-phase amplitudes[4] for the vacuum, coupled in by the imperfect detector quantum efficiencies. Factors of $(1\pm\delta/\omega_j)^{1/2}$ have been set equal to unity. The electronic noise power measured by an rf spectrum analyzer is proportional to the mean square fluctuations $\langle|\mathcal{J}^{(+)}|^2\rangle\equiv\langle\mathcal{J}^{(+)}\times\mathcal{J}^{(+)\dagger}\rangle_{\text{sym}}$, where "sym" denotes a symmetrized product.[4]

The signature of squeezing is a subunity ratio V of the detected electronic noise power associated with $\hat{\mathcal{J}}^{(+)}(\delta)$ to that obtained with coherent states incident on each detector:

$$V(\delta,\Phi_1,\Phi_2,\phi_r)=\langle|\mathcal{J}^{(+)}(\delta)|^2\rangle/\langle|\hat{\mathcal{J}}^{(+)}|^2\rangle_{\text{coh}}, \tag{2}$$

where

$$\langle|\hat{\mathcal{J}}^{(+)}|^2\rangle_{\text{coh}}=\tfrac{1}{2}[\eta_1(g_1|\mathcal{E}_1|)^2+\eta_2(g_2|\mathcal{E}_2|)^2]$$

is the mean square noise photocurrent when coherent states are incident on both detectors. With four-mode squeezing the mean square current fluctuations (noise power) $\langle|\hat{\mathcal{J}}^{(+)}(\delta)|^2\rangle$ can be smaller than the vacuum or coherent-state level ($V<1$), even when the noise powers at both detectors $\langle|\hat{\mathcal{J}}_j^{(+)}(\delta)|^2\rangle$ are above that level (i.e., even when there is no two-mode squeezing).

We have produced and observed four-mode squeezing using a single-mode optical fiber as the nonlinear medium. Wave-vector mismatch prevents the two pump waves from generating a third strong wave by four-wave mixing, but nondegenerate four-wave mixing couples the signal and idler sidebands of the two pump waves to each other. For a fiber length $l\sim 100$ m, and for visible light, the required phase-matching conditions occur when $\delta<[(\omega_1-\omega_2)\delta]^{1/2}<100$ GHz $<(\omega_1-\omega_2)$. The propagation equations for the two pump beams and the four sideband modes are[9]

$$\begin{aligned}
\partial E_n/\partial z&=i\chi\{|E_n|^2+2\gamma|E_m|^2\}E_n,\\
\partial\hat{a}_n^S/\partial z&=i\chi\{2(|E_n|^2+\gamma|E_m|^2)\hat{a}_n^S+(E_n)^2\hat{a}_n^{I\dagger}+2\gamma E_nE_m^*\hat{a}_m^S+2\gamma E_nE_m\hat{a}_m^{I\dagger}\}+i\hat{\Gamma}E_n,\\
\partial\hat{a}_n^{I\dagger}/\partial z&=-i\chi\{2(|E_n|^2+\gamma|E_m|^2)\hat{a}_n^{I\dagger}+(E_n^*)^2\hat{a}_n^S+2\gamma E_n^*E_m\hat{a}_m^{I\dagger}+2\gamma E_n^*E_m^*\hat{a}_m^S\}-i\hat{\Gamma}^\dagger E_n^*,
\end{aligned}\tag{3}$$

where $(n,m)=(1,2)$ or $(2,1)$. Phase noise due to thermal fluctuations of the fiber index of refraction[10] are represented by the operators $\hat{\Gamma}$ and $\hat{\Gamma}^\dagger$, and γ is a polarization correlation factor.[11] The coefficient $\chi=(12\pi\omega/nc)\chi^{(3)}f$, where $\chi^{(3)}=5\times10^{-15}$ esu is the third-order nonlinear susceptibility of fused silica, and f is a mode-overlap factor of order unity.[2,11]

At the output of the fiber, the pump and sideband amplitudes are

$$\begin{aligned}
E_n(l)&=E_n(0)e^{i(r_n+2\gamma r_m)},\\
\hat{a}_n^S(l)&=e^{i(r_n+2\gamma r_m)}\{(1+ir_n)\hat{a}_n^S(0)+ir_n\hat{a}_n^{I\dagger}(0)+2i\gamma(r_1r_2)^{1/2}[\hat{a}_m^S(0)+\hat{a}_m^{I\dagger}(0)]+i\hat{\Gamma}(r_n/\chi)^{1/2}\},\\
\hat{a}_n^{I\dagger}(l)&=e^{-i(r_n+2\gamma r_m)}\{-ir_n\hat{a}_n^S(0)+(1-ir_n)\hat{a}_n^{I\dagger}(0)-2i\gamma(r_1r_2)^{1/2}[\hat{a}_m^S(0)+\hat{a}_m^{I\dagger}(0)]-i\hat{\Gamma}^\dagger(r_n/\chi)^{1/2}\},
\end{aligned}\tag{4}$$

where the argument zero denotes the fiber input, and $r_j=\chi|E_j|^{2l}$ are the usual squeeze parameters. The ratio V of Eq. (2) is

$$\begin{aligned}
V(\delta,\Phi_1,\Phi_2,\phi_r)=1&+\frac{2\eta_1}{(1+Z^2)}[r_1\sin2\Phi_1+(2r_1^2+8\gamma^2r_1r_2+\rho r_1)\sin^2\Phi_1]\\
&+\frac{2\eta_2Z^2}{(1+Z^2)}[r_2\sin2\Phi_2+(2r_2^2+8\gamma^2r_1r_2+\rho r_2)\sin^2\Phi_2]\\
&+\frac{4(\eta_1\eta_2)^{1/2}Z}{(1+Z^2)}[\gamma(r_1r_2)^{1/2}\cos2\phi_r]\{2\sin(\Phi_1+\Phi_2)+[4(r_1+r_2)+(\rho/\gamma)]\sin\Phi_1\sin\Phi_2\},
\end{aligned}\tag{5}$$

where ρ is a normalized thermal phase-noise amplitude derived from $\hat{\Gamma}$ and $\hat{\Gamma}^\dagger$, $Z=(g_2\sqrt{\eta_2}|\mathcal{E}_2|)/(g_1\sqrt{\eta_1}|\mathcal{E}_1|)$, and the LO amplitudes $\mathcal{E}_j$ at the detectors are derived from the transmitted pump amplitudes, $E_j(l)$. The term dependent on ϕ_r in Eq. (5) reflects the fact that the correlations between the two photocurrents are an essential feature of four-mode squeezing.

In the experimental apparatus, radiation from the 647- and 676-nm laser lines in singly ionized krypton is coupled into a 114-m single-mode optical fiber held near 2 K. The low temperature is necessary to suppress thermally excited noise processes, and a resonant phase modulator is used to broaden the spectrum of each laser line to avoid the stimulated Brillouin effect.[2] The strong $(M > 10)$ phase modulation does not interfere with the squeezing, because the relative phases of all significant excitations are preserved. Typical power levels at the fiber exit were 140–160 mW for the 674-nm line and 50–60 mW for the 676-nm line.

After passing through the optical fiber, the two transmitted wavelengths are separated by a prism. One of these beams is incident directly on an FND-100 photodiode in a carefully constructed heat sink (i.e., $\Phi_1 = 0$, $T_1 = 1$). The other wavelength is reflected from a phase-shifting cavity[2,11] and detected with a second photodiode $(-\pi/2 \lesssim \Phi_2 \lesssim \pi/2,\ 0.06 \lesssim T_2 \lesssim 0.3)$. Both electrical signals are amplified, and the D2 signal is subjected to a variable phase shift before the two currents are combined. The combined signal is resolved on a spectrum analyzer set to the 56-MHz minimum of the fiber phase-noise spectrum.[2] The dc currents produced by the two detectors were measured with digital voltmeters. A voltage parametrizing the phase shift Φ_2 of the optical cavity was also recorded.

The parameter Z in Eq. (5) depends on the relative amplification factors, detector quantum efficiencies, and LO intensities, and hence varies as Φ_2 is varied. Its value was calibrated against the ratio of dc photocurrents from the two detectors by illumination of the detectors with incandescent light and laser radiation that had not been coupled through the fiber. This procedure also allowed the vacuum noise level to be determined to within $\pm 2\%$.

Two series of experimental runs were made, distinguished by whether the less powerful beam (type-I run) or the more powerful beam (type-II run) was directed toward the phase-shifting cavity and detector D2. With light coupled through the fiber, the noise level at $\Phi_2 = 0$ rose to $(2 \pm 2)\%$ (type I) or $(9 \pm 2)\%$ (type II) above the vacuum level as a result of interaction of the phase-modulated pump spectrum with residual dispersion in the cavity. No ϕ_r dependence was detectable at

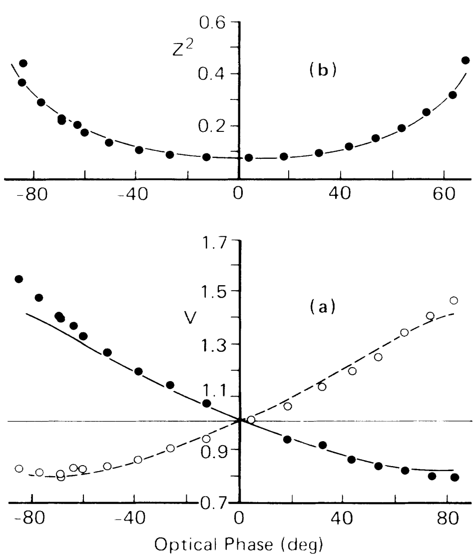

FIG. 2. Normalized total noise power as a function of phase angle Φ_2 for a four-mode squeezing experiment with $r_1 > r_2$ and $\Phi_1 = 0$. The solid circles in (a) show the noise normalized to the vacuum noise level (the solid horizontal line at $V = 1.0$) for $\phi_r = \pi/2$, while the open circles show similar results for $\phi_r = 0$. The solid and dashed lines are fits by Eq. (5) for the two cases. The value of Z^2 as a function of Φ_2 appears in (b). The radii of the open and closed circles reflect the 2% experimental uncertainty of these noise-power measurements.

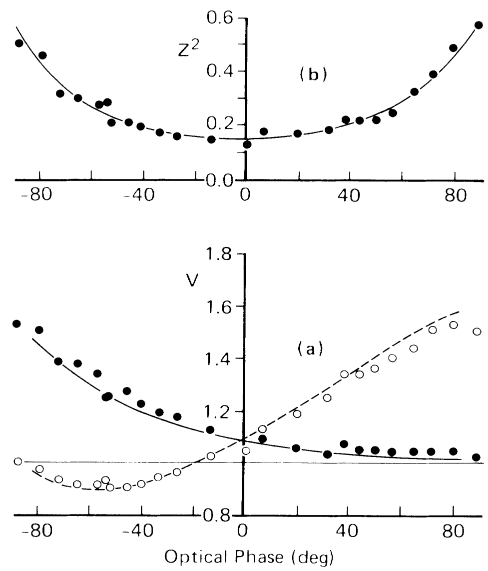

FIG. 3. Total noise power normalized to the vacuum level vs Φ_2 for the case where $r_1 < r_2$ and $\Phi_1 = 0$. (a) The solid circles and solid line are the experimental data and a fit by Eq. (5) for $\phi_r = \pi/2$, while the open circles and dashed line are the data and fit for $\phi_r = 0$. The asymmetry between these two cases results from the effect of two-mode squeezing of the ω_2 sidebands. (Two-mode squeezing of the ω_1 sidebands is unobservable at $\Phi_1 = 0$.) The deviation of the data points from the vacuum level $(V = 1.0)$ at $\Phi_2 = 0$ reflects the excess noise caused by instability of the rf oscillator driving the phase modulator necessary to suppress stimulated Brillouin oscillation. (b) The variation of the parameter Z^2 with Φ_2. The radii of the open and solid circles reflect the 2% experimental uncertainty.

$\Phi_2=0$. The D2 noise level varied with Φ_2 as predicted by Eq. (5). This noise level was not significantly below the vacuum level, as a result of fiber-produced thermal phase noise [ρ in Eq. (5)] and of the phase fluctuations added to this beam by coupling to the other pump beam.

Figure 2(a) shows the total detector system noise in a type-I run, plotted as a function of Φ_2 for $\phi_r=0$ and $\phi_r=\pi/2$ (i.e., with the D1 and D2 outputs added and subtracted). The transmitted power levels were 150 mW at D1 and 60 mW just before the phase-shifting cavity. The noise level falls 20% below the vacuum level at $\Phi_2 \approx -\pi/3$ and $\phi_r=0$ as well as at $\Phi_2 \approx \pi/2$ and $\phi_r=\pi/2$. This is strong evidence for four-mode squeezing. Figure 2(b) shows the dependence of Z^2 on the phase shift Φ_2. With these data, we were able to fit Eq. (5) to the points of Fig. 2(a). The parameters corresponding to the solid and dashed lines are $r_1=0.4$, $r_2=0.1$, $\eta_1=0.75$, $\eta_2=0.3$, $\rho=2.0$, and $\gamma^2=0.65$. These values are consistent with previous measurements on this fiber and detection system.[2]

In Fig. 3, data for a type-II run are shown. The asymmetry between the $\phi_r=0$ and $\phi_r=\pi/2$ plots is characteristic of four-mode squeezing and is emphasized when the phase of the more powerful beam is varied. When $\phi_r=0$, the parametric mixing interaction responsible for conventional two-mode squeezing reduces the net noise level near the phase of the four-mode squeezing minimum, while when $\phi_r=\pi/2$ the two effects work against one another. Both the solid and dashed lines correspond to Eq. (5) with $r_1=0.15$, $r_2=0.4$, $\eta_1=0.6$, $\eta_2=0.4$, $\rho=1.2$, and $\gamma^2=0.35$.

Phase noise due to thermal fluctuations of the fiber index of refraction[10] is the major obstacle in the observation of significant two-mode squeezing in fibers. This phase noise was eliminated at D1 by our choosing $\Phi_1=0$; values for Φ_2, the LO powers, and the relative rf gain were chosen so that the negative correlation between the two photocurrents dominated over the thermal phase-noise contribution from D2. Under these conditions appreciable quantum noise suppression due to four-mode squeezing could be observed. Four-mode squeezing thus offers two important advantages over two-mode squeezing: the additional noise suppression obtained when more than one pump frequency is used,[5,6] and the increased immunity to phase noise from light scattering of the pump wave in four-wave mixing.

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