Nuclear Rotational Damping: Finite-System Analog to Motional Narrowing in Nuclear Magnetic Resonances

R. A. Broglia
The Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen Ø, Denmark, and Istituto di Fisica, Universitá di Milano, Milano, Italy, and Istituto Nazionale di Fisica Nucleare, Sezione di Milano, 20133 Milano, Italy

T. Døssing and B. Lauritzen
The Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen Ø, Denmark

and

B. R. Mottelson
NORDITA, DK-2100 Copenhagen Ø, Denmark
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From the analogy between the collective quadrupole decay among compound nuclear states in strongly rotating nuclei and the phenomenon of nuclear magnetic resonance, it is possible to formulate a model of nuclear rotational damping which exhibits interesting correspondences between nuclear and atomic statistical processes.

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It is well established that nucleons can organize their motion in nuclei, leading to quadrupole deformed shapes of the average field and to rotations of the nucleus as a whole.[1] At small rotational frequencies, the moment of inertia is only about half of the value for rigid rotation. This is because nuclei in their ground state are superfluid.

In heavy-ion reactions leading to fusion, large amounts of angular momentum and temperature can be imparted to the compound nucleus. In analogy with the cooling process of a liquid drop, nucleon evaporation can reduce the temperature of the nucleus, without much changing its angular-momentum content. (Energy loss per evaporated nucleon varies as separation energy, while angular-momentum loss per evaporated nucleon varies as I/A, where I is the total angular momentum and A the number of nucleons in the rotating system.) Thus, the excitation energy can be reduced to a point where further evaporation is no longer possible but the resulting "evaporation residue" still retains a very large amount of angular momentum. There will still be an appreciable excitation energy in the residue, but the system is "cold" in the sense that almost all of this energy is tied up in the single degree of freedom associated with the angular momentum. Such a cold drop of nuclear Fermi liquid can accomodate angular momenta up to about $100\hbar$. For higher values of the spin, the rotating nucleus will fission because of the intense centrifugal force. The loci of the lowest-energy states of a given spin is called the "yrast" line. Experimental studies of discrete lines in the quadrupole gamma-decay spectrum have identified transitions within rotational bands which extend up to around sixty units of angular momentum.

At an excitation energy of a few megaelectronvolts above the yrast line, rotational bands become very closely spaced in energy. Any single band can be viewed as a collective sequence of related states imbedded in a dense background of other (more or less complicated) states, to which it will couple by residual interactions.

This coupling leads to the stationary states of the system (the compound nucleus) which are complicated mixtures of the available unperturbed configurations. The rotational degree of freedom is "damped" in these compound states in the sense that the electric quadrupole decay of a single quantum state with angular momentum I will not go to a unique final state with $I-2$ (as for the unperturbed bands) but will exhibit a spectrum[2] of final states[3-6] (strength function) all having $I-2$.

The nuclear rotational damping of the compound nucleus has an analog[7] in the linewidth observed in the phenomenon of nuclear magnetic resonance (NMR) in condensed matter[8-10] (cf. Table I). Each nuclear spin in its lattice precessing with a Larmor frequency determined by the external magnetic field B_0 is the analog of a rotational band. The whole crystal at a given temperature, and with the magnetic dipole interaction between nearest neighbors taken into account, is the analog of the mixed-band eigenstates of the compound nucleus. The spin system is probed by an external time-dependent magnetic field perpendicular to the constant field B_0, while the analogous information on the dynamics of the nuclear rotational motion is contained in the emitted electric-quadrupole radiation. The linewidth observed in the absorption process is due to the spread in Larmor frequencies and reflects the spatial inhomogeneity of the local magnetic field. Its analog in the nuclear case is the

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TABLE I. Some elements used in the analogy between rotational damping and the linewidth of NMR. To emphasize this analogy $\hbar = 1$ has been used in this table.

Nuclear rotation	NMR
Rotational band of "pure" configuration	Larmor precession of spin in *homogenous* and *constant* external field
$\Delta\omega_0$ = fluctuation in rotational frequency (for given I) for different "pure" bands	$\Delta\omega_0$ = variation in Larmor precession at different sites
Origin of $\Delta\omega_0$: (1) special single-particle orbits contributing significant part of total angular momentum; fluctuations in $I(\omega)$ associated with occupancy/nonoccupancy of these orbits; (2) vibrational fluctuations in nuclear shape	Origins of $\Delta\omega_0$: (1) local magnetic impurities; (2) magnetic (dipole) interactions with neighboring molecules
Stochastic averaging mechanism: Mixing of pure configurations to form compound nucleus; mixing measured by spreading width Γ_μ of pure configuration	Stochastic averaging mechanism: diffusion, characterized by hopping time τ.
Single quantum state emits quadrupole radiation with line width (strength function) $\Gamma_{\rm rot} \sim (\Delta\omega_0)(\Delta\omega_0/\Gamma_\mu)$ (in limit $\Gamma_\mu \gg \Delta\omega_0$), or $\Gamma_{\rm rot} \sim \Delta\omega_0$ (in limit $\Gamma_\mu < \Delta\omega_0$)	Ensemble of spins has NMR absorption line with width $\Gamma_{\rm NMR} \sim (\Delta\omega_0)(\Delta\omega_0\tau)$ (in limit $\tau \ll (\Delta\omega_0)^{-1}$), or $\Gamma_{\rm NMR} \sim \Delta\omega_0$ [in limit $\tau > (\Delta\omega_0)^{-1}$]

rotational damping width $\Gamma_{\rm rot}$ of the electromagnetic quadrupole decay, which has its origin in the spread in the rotational frequencies of the manifold of bands.

While the damping width of collective motion of a many-body system is, as a rule, an increasing function of the temperature, the nuclear magnetic resonance line becomes narrower with increasing T. This phenomenon, known as motional narrowing,[11] arises because higher temperatures imply shorter diffusion times between different lattice sites. Time inhomogeneity is able to average out spatial inhomogeneity.

It is noted that motional narrowing occurs quite generally in systems where the frequency of a resonant process may be influenced by time-dependent perturbations. Another example, in the case of infinite systems, is provided by the effect lattice vibrations have on the line shape of excitons in solids.[12] Motional narrowing is also predicted to be present in the damping of the rotational motion in nuclei, at least within some temperature interval.

From the above discussion, it emerges that the rotational damping width $\Gamma_{\rm rot}$ is controlled by two quantities: the spread in rotational frequencies $\Delta\omega_0$ of the unperturbed states and the time (hopping time) a compound nucleus state stays in any single rotational band (diffusion time for atoms to move between lattice sites). This last quantity is connected to the range of energies over which the rotational bands are mixed by the residual interaction ($\tau = \hbar/\Gamma_\mu$).

In keeping with the analogy to NMR, the rotational damping width $\Gamma_{\rm rot}$ can be written as $\Gamma_{\rm rot} = 2\hbar/T_2$, where T_2 is the time for the different bands (the nuclear spins in NMR) to lose phase coherence (dephasing time). There are two different regimes for the dephasing time.[10,13] If the spread in frequencies $\Delta\omega_0$ is large compared with the diffusion rate ($2\Delta\omega_0 > \tau^{-1}$), dephasing results directly from the differences in rotational frequencies:

$$T_2 = (2\Delta\omega_0)^{-1}, \quad \text{if } 2\Delta\omega_0 > \tau^{-1}. \tag{1a}$$

The factor of 2 results from the quadrupole nature of the radiation.

In the opposite extreme of very short diffusion time, the dephasing results from a random walk extending over T_2/τ different sites and, thus,

$$T_2 = (2\Delta\omega_0)^{-2}\tau^{-1}, \quad \text{if } 2\Delta\omega_0 \ll \tau^{-1}. \tag{1b}$$

The above relations applied to the nuclear rotational damping imply

$$\Gamma_{\rm rot} = \begin{cases} 2(2\hbar\Delta\omega_0), & \Gamma_\mu < 2\hbar\Delta\omega_0, \\ 2(2\hbar\Delta\omega_0)^2/\Gamma_\mu, & \Gamma_\mu \gg 2\hbar\Delta\omega_0. \end{cases} \tag{2}$$

The spreading width $\Gamma_{\rm rot}$ of the individual configurations in the compound nucleus is the analog of the diffusion time in NMR and will in general depend on excitation energy U of the states considered. Simple estimates of these dependences can be made for the Fermi-gas model[14]; in this model we have temperature

$$T \sim U^{1/2};$$

number of quasiparticles

$$v \sim T \sim U^{1/2};$$

spreading width for a single quasiparticle (phase space),

$$\Gamma_{v=1} \sim U^2;$$

spreading width for a typical configuration with v quasiparticles,

$$\Gamma_\mu \sim v\Gamma_{v=1}(U/v) \sim U^{3/2}.$$

The total angular momentum of rotational bands can be parametrized by the linear expression $I = \mathcal{J}\omega_0 + i$. Here the first term is due to the collective rotation of the nucleus as a whole with rotational frequency ω_0. The second term expresses the fluctuations in the total angular momentum at a given rotational frequency associated with exceptional contributions from individual orbits.

Fluctuations in the rotational frequency (for fixed I) are expected to increase slightly with excitation energy reflecting the fact that fluctuations in the nuclear shape as well as in the alignments associated with many quasiparticle configurations increase with increasing temperature:

$$(\Delta\omega_0)^2 \sim T \sim U^{1/2}. \tag{3}$$

Combining (3) with (1) we obtain for the rotational damping

$$\Gamma_{\rm rot} \sim \begin{cases} U^{1/4}, & \Gamma_\mu < 2\hbar\Delta\omega_0, \\ U^{-1}, & \Gamma_\mu \gg 2\hbar\Delta\omega_0. \end{cases} \tag{4}$$

These estimates lead to the variation of $\Gamma_{\rm rot}$ shown schematically in Fig. 1. The decrease of $\Gamma_{\rm rot}$ with increasing excitation energy is the nuclear analog of motional narrowing in NMR. Quantitative estimates[3] of the coefficients appearing in the above expression lead to estimates of $\Gamma_{\rm rot} \sim 100$ keV in the region of the maximum, consistent with the tentative experimental evidence[5,6] on this damping effect.

The simple expressions (2) and (4) are likely to fail at high excitation energies. Indeed, there are expected to be I-dependent terms in the two-body interaction whose effect will increase with T, as a consequence of the increase of the phase space.

From the above discussion we can conclude that the system of atoms in a crystal with nuclear spins precessing under the influence of a constant magnetic field and interacting weakly through magnetic dipole forces is a valid analogy to the compound states of strongly rotating nuclei. From it a simple yet realistic model of rotational damping can be extracted.

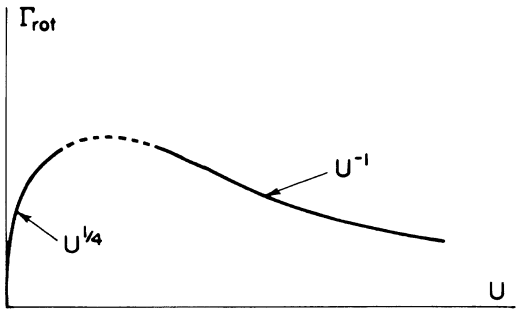

FIG. 1. Schematic variation of the rotational damping width as a function of the excitation energy U above the yrast line.

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[14]It should be emphasized that the Fermi-gas model can at best provide only a general orientation as to the statistical properties of the nuclear excitation spectra, since shell structure and the finite nuclear size play an important role in the spectra even at excitation energies of tens of megaelectronvolts. In particular the dependence $\Gamma_{v=1} \sim U^2$ is not a very satisfactory description of the observed attenuation of one-particle motion in the nucleus. The observed rather stronger damping at low excitation energy is apparently related to the importance of low-energy collective shape oscillations that are not included in the Fermi-gas description. See P. F. Bortignon, G. F. Bertsch, and R. A. Broglia, Rev. Mod. Phys. **55**, 287 (1983); P. F. Bortignon, R. A. Broglia, G. F. Bertsch, and I. Pacheco, Nucl. Phys. **A460**, 149 (1986).