Signature of g Boson in the Interacting-Boson Model from g-Factor Variations

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Using projection techniques, we show that stretching occurs in the *sdg* interacting-boson model (IBM) but not in the *sd* model. As a result the *sdg* IBM allows g-factor variations in the ground-state band in accordance with recent experiments, and as such may provide a signature for the g boson.

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There has been a long controversy on the role of the g boson in the interacting-boson model (IBM). Microscopically it was found that in the region of deformation there are nonnegligible admixtures of $J=4^+$ (G) pairs in the low-lying collective states. Thus the mapping of the fermion pairs to the boson states requires a certain amount of g boson which is the image of the G pair.[1] On the other hand, phenomenological analyses of low-lying collective states indicate that the *sd* IBM is quite sufficient for the description of vast amounts of data.[2] The need for the g boson arises only for high-spin states[3] and for relatively high-lying side bands.[4] Given that for low-lying states the predictions of the *sd* and *sdg* IBM's are so similar, it becomes a matter of importance to find a nuclear property that might distinguish the two models. The recent observation of g-factor variations in the ground-state band may just provide such a property.

In the simplest case of the *sd* IBM with one-body $M1$ operator, g factors for all states are constant because the $M1$ operator is proportional to the angular momentum operator. Including higher-order terms in the $M1$ operator leads to some spin dependence in the g factors, which, however, is too small to have any practical significance.[5] Likewise, the extension to the proton-neutron IBM (IBM-2) does not lead to a large variation, because the ground-state band is almost pure in F-spin symmetry, and one gets essentially identical results for the two versions.[6] Thus the IBM with only s and d bosons is unable to explain the g-factor variations in the ground-state band. In the following we will show that including the g boson allows the boson system to stretch (i.e., amplitudes of boson operators in the intrinsic state change with spin), which introduces a spin dependence in the expectation value of the $M1$ operator.

In order to demonstrate the technique in a simple case, we will first carry out the calculation for the *sd* IBM. For the boson Hamiltonian, we take the dominant quadrupole interaction

$$H=-\kappa Q\cdot Q, \tag{1}$$

where the quadrupole operator Q contains another parameter χ:

$$Q_\mu=(s^\dagger\tilde{d}+d^\dagger s)^{(2)}_\mu+\chi(d^\dagger\tilde{d})^{(2)}_\mu.$$

Here, parentheses denote tensor coupling of operators and $\tilde{d}_\mu=(-)^\mu d_{-\mu}$. Introducing the intrinsic state for the ground-state band[7]

$$|\phi\rangle=(b^\dagger)^N|-\rangle,\quad b^\dagger=x_0 s^\dagger+x_2 d_0^\dagger,$$

and projecting to good angular momentum,[8]

$$|LM\rangle=P^L_{M0}|\phi\rangle,$$

$$P^L_{MK}=[(2L+1)/8\pi^2]\int D^{L*}_{MK}(\Omega)R(\Omega)d\Omega,$$

we evaluate the expectation value of H, Eq. (1), for a given spin:

$$\langle H\rangle_L=\frac{\langle -|b^N(-\kappa Q\cdot Q)P^L_{00}(b^\dagger)^N|-\rangle}{\langle -|b^N P^L_{00}(b^\dagger)^N|-\rangle}=-\kappa\frac{\int d\beta\sin\beta\, d^L_{00}(\beta)\langle -|b^N Q\cdot Q e^{-i\beta J_y}(b^\dagger)^N|-\rangle}{\int d\beta\sin\beta\, d^L_{00}(\beta)\langle -|b^N e^{i\beta J_y}(b^\dagger)^N|-\rangle}, \tag{2}$$

where β denotes the Euler angle and $d^L_{mm'}$ are the reduced rotation matrices. If we define the rotated intrinsic operator as

$$b'^\dagger=e^{-\beta J_y}b^\dagger e^{i\beta J_y}=x_0 s^\dagger+x_2\textstyle\sum_\mu d^2_{\mu 0}(\beta)d^\dagger_\mu,$$

the matrix element in the denominator can be easily calculated as

$$\langle -|b^N e^{i\beta J_y}(b^\dagger)^N|-\rangle=N!(\partial b'^\dagger/\partial b^\dagger)^N=N![x_0^2+x_2^2 d^2_{00}(\beta)]^N\equiv N![Z(\beta)]^N.$$

A similar calculation for the numerator, utilizing the commutation relations

$$[b,Q_\mu]=\delta_{\mu 0}x_2 s+(x_0+\langle 2\mu 20|2\mu\rangle\chi x_2)\tilde{d}_\mu,$$

$$[Q_\mu,b^\dagger]=(-)^\mu d^2_{-\mu 0}(\beta)x_2 s^\dagger+x_0 d^\dagger_\mu+\chi x_2\textstyle\sum_{\mu'}\langle 2\mu+\mu' 2-\mu'|2\mu\rangle d^2_{\mu' 0}(\beta)d^\dagger_{\mu+\mu'},$$

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gives

$$\langle - | b^N Q\cdot Q(b^\dagger)^N | -\rangle = NN![Z(\beta)]^{N-2}\sum_\mu\{Z(\beta)[5x_0^2+(1+\chi^2)d_{00}^2(\beta)x_2^2]\delta_{\mu 0} + (N-1)[\delta_{\mu 0}x_0x_2+(x_0x_2+\langle 2\mu 20|2\mu\rangle\chi x_2^2)d_{\mu 0}^2(\beta)]^2\}.$$

For the evaluation of the integrals, we use the Gaussian approximation, which is valid for large N,

$$[Z(\beta)]^N \simeq (\mathbf{x}\cdot\mathbf{x})^N \exp(-\beta^2/\Gamma),$$

where $\mathbf{x}=(x_0,x_2)$ and $\Gamma = 2\mathbf{x}\cdot\mathbf{x}/3Nx_2^2$.

Extending the β integration to ∞ and using the integral formula[9]

$$\int_0^\infty d\beta \sin\beta P_L(\cos\beta)\exp(-\beta^2/\Gamma) = \Gamma/2 - \Gamma^2[1+3L(L+1)/2]/12 + \Gamma^3[1+15L(L+1)/4+15L^2(L+1)^2/8]/120+\cdots,$$

we obtain the following expression for $\langle H\rangle_L$ to order $1/N$:

$$-\langle H\rangle_L/\kappa = N^2[(2x_0x_2+\bar\chi x_2^2)/\mathbf{x}\cdot\mathbf{x}]^2 + N[3x_0^4-4\bar\chi x_0^3x_2+(2\bar\chi^2+8)x_0^2x_2^2+(3\bar\chi^2+1)x_2^4]/(\mathbf{x}\cdot\mathbf{x})^2 - L(L+1)[-4x_0^4-8\bar\chi x_0^3x_2-(3\bar\chi^2-12)x_0^2x_2^2+8\bar\chi x_0x_2^3+\bar\chi^2x_2^4]/(12x_2^2\mathbf{x}\cdot\mathbf{x}), \quad (3)$$

where $\bar\chi = -(2/7)^{1/2}\chi$. In the SU(3) limit, $\bar\chi = 1/\sqrt{2}$, $\mathbf{x}=(1/\sqrt{3},\sqrt{2}/\sqrt{3})$, and Eq. (3) correctly reproduces the well-known result

$$-\langle H\rangle_L/\kappa = 2N^2+3N-3L(L+1)/8,$$

obtainable from the quadratic Casimir operator of the SU(3) group. Minimizing $\langle H\rangle_L$ with respect to (x_0,x_2) is most simply done by setting $x_2=1$ (since $\mathbf{x}\cdot\mathbf{x}=1$) and differentiating the resulting expression with respect to x_0. We obtain for x_0 the equation

$$N^2(2x_0+\bar\chi)(x_0^2+\bar\chi x_0-1)+N[-\bar\chi x_0^4+(\bar\chi^2+1)x_0^3+3\bar\chi x_0^2+(2\bar\chi^2-3)x_0] - L(L+1)(x_0^2+1)[x_0^5+\bar\chi x_0^4+2x_0^3+4\bar\chi x_0^2+(\bar\chi^2-3)x_0-\bar\chi]/6=0,$$

which can be solved order by order. For our purposes it is sufficient to note that the polynomial of the $L(L+1)$ term contains the factor $x_0^2+\bar\chi x_0-1$ and, hence, the solution for x_0 is independent of L. This proves the intuitively obvious result that there can be no stretching in the *sd*-boson system, that is, the boson system does not respond to the rotation by changing the character of the intrinsic state.

Extension of the above analysis to the *sdg*-boson system is straightforward though tedious. The quadrupole operator is replaced by

$$Q_\mu = (s^\dagger\tilde d + d^\dagger s)_\mu^{(2)} + \beta(d^\dagger\tilde d)_\mu^{(2)} + \gamma(d^\dagger\tilde g + g^\dagger\tilde d)_\mu^{(2)} + \delta(g^\dagger\tilde g)_\mu^{(2)},$$

and the intrinsic state by $b^\dagger = x_0s^\dagger + x_2d_0^\dagger + x_4g_0^\dagger$. Calculation of $\langle H\rangle_L$ follows similar lines to Eqs. (2) and (3). The final result is given by

$$\begin{aligned}-\langle H\rangle_L/\kappa = {}& N^2f^2 + N\{f^2(x_0^2+10x_2^2+31x_4^2)/3h \\ & + [5x_0^2+(1+7\bar\beta^2/2+7\bar\gamma^2/2)x_2^2+(35\bar\gamma^2/18+77\bar\delta^2/20)x_4^2]/\mathbf{x}\cdot\mathbf{x} \\ & - [22x_0^2x_2^2+40\bar\beta x_0x_2^3+164\bar\gamma x_0x_2^2x_4+124\bar\delta x_0x_2x_4^2+29\bar\beta^2x_2^4/2 \\ & + 100\bar\beta\bar\gamma x_2^3x_4+71(2\bar\gamma^2+\bar\beta\bar\delta)x_2^2x_4^2+184\bar\gamma\bar\delta x_2x_4^3+113\bar\delta^2x_4^4/2]/3h\mathbf{x}\cdot\mathbf{x}\} \\ & - L(L+1)\{f^2(x_0^2+37x_2^2+121x_4^2)\mathbf{x}\cdot\mathbf{x}/12h^2 \\ & - [10x_0^2x_2^2+19\bar\beta x_0x_2^3+80\bar\gamma x_0x_2^2x_4+61\bar\delta x_0x_2x_4^2+7\bar\beta^2x_2^4 \\ & + 49\bar\beta\bar\gamma x_2^3x_4+35(2\bar\gamma^2-\bar\beta\bar\delta)x_2^2x_4^2+91\bar\gamma\bar\delta x_2x_4^3+28\bar\delta^2x_4^4]/3h^2\}, \end{aligned} \quad (4)$$

where $\bar\beta = -(2/7)^{1/2}\beta$, $\bar\gamma = (2/7)^{1/2}\gamma$, $\bar\delta = -(10/3\sqrt{77})\delta$, and

$$f = (2x_0x_2+\bar\beta x_2^2+2\bar\gamma x_2x_4+\bar\delta x_4^2)/\mathbf{x}\cdot\mathbf{x}, \quad h = 3x_2^2+10x_4^2.$$

In the SU(3) limit, we have $\bar{\beta}=(11/14)(5/7)^{1/2}$, $\bar{\gamma}=(9/7)(2/7)^{1/2}$, $\bar{\delta}=(5/7)^{3/2}$, and $\mathbf{x}=(1/\sqrt{5},2/\sqrt{7},1/\sqrt{35})$, and Eq. (4) reduces to

$$-\langle H\rangle_L/\kappa=\tfrac{5}{7}[4N^2+3N-3L(L+1)/16],$$

in agreement with the result obtained from the Casimir operator.

Setting $x_0=1$ in Eq. (4) and varying $\langle H\rangle_L$ with respect to x_2 and x_4 leads to two coupled nonlinear equations which have to be solved numerically. Nevertheless, progress can be made by noting the general form of the solutions:

$$x_2=x_2^0[1+y_2/N+Z_2L(L+1)/N^2],$$
$$x_4=x_4^0[1+y_4/N+Z_4L(L+1)/N^2], \qquad (5)$$

where x_2^0 and x_4^0 denote the leading-order solutions and the coefficients $\{y,z\}$ are obtained from the set of nonlinear equations.

Numerical study of the coefficients $\{y,z\}$ shows that they vanish only in the SU(3) limit; that is, the structure of the yrast intrinsic state is independent of L only in that limit. Thus the *sdg*-boson system, in general, exhibits stretching. The SU(3) limit corresponds to the absolute minimum of $\langle H\rangle_L$ (in x_0,x_2, and x_4) simultaneously for all L. The boson system being at the bottom of the well has no way of stretching. Away from this limit (i.e., a different choice of β, γ, and δ), the intrinsic state has a different minimum for each L.

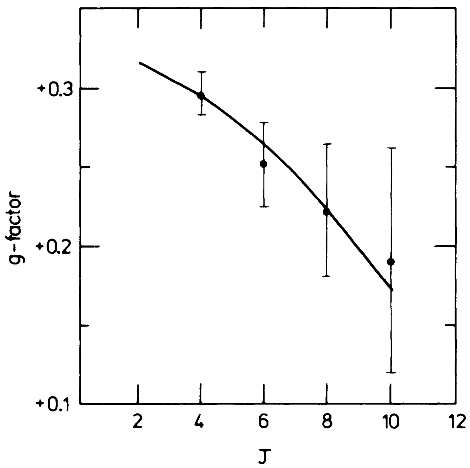

FIG. 1. Comparison of the *sdg* IBM results for g factors of the ground-state band with experiment (Ref. 11).

Next, we implement the foregoing results in the calculation of g factors. Microscopically, g factors of d and g bosons are expected to differ substantially because the latter is less spin saturated. A convenient parametrization is[10]

$$g(L)=g+g'\Delta g, \qquad (6)$$

where g represents the part proportional to the angular momentum, and g' measures the defect between the d- and g-boson g factors. Δg is the reduced matrix element of the operator $(g^\dagger\tilde{g})^{(1)}$, and is given by[10]

$$\Delta g=(\tfrac{5}{3})^{1/2}[N/(N-1)]^2x_4^2/h. \qquad (7)$$

Substituting Eq. (5) in Eq. (7), we obtain to order $1/N^3$

$$\Delta g=(\tfrac{5}{3})^{1/2}[N/(N-1)]^2(x_4^2/h)[1+6x_2^2(y_4-y_2)/hN+6x_2^2(Z_4-Z_2)L(L+1)/hN^2]. \qquad (8)$$

In order to facilitate comparison with experiment, we combine Eqs. (6) and (8) in the form

$$g(L)=g_0+g_LL(L+1). \qquad (9)$$

In Fig. 1, Eq. (9) is compared with the recent g-factor measurements of ^{166}Er.[11] The parameters used in the fit are $g_0=0.325$ and $g_L=-0.0014$. The value of g_L depends on g' in Eq. (6) and the parameters of the quadrupole operator, β, γ, and δ. Since a precise determination of these parameters requires a detailed knowledge of the side bands which is lacking at the moment, it is not possible to give an estimate of g_L. However, it is certainly within the parameter range of the Hamiltonian.

In conclusion, we offer an explanation for the g-factor variations in the ground-state band, based on stretching of the *sdg*-boson system with increasing spin. Further, since the *sd*-boson system cannot stretch, this phenomenon may provide a signature for the g boson in the relatively low-lying levels ($E_x<1400$ MeV) of the ground-state band.

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