QCD and Rising Cross Sections

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We show that parton-parton scattering processes in QCD drive a rapid increase in the proton-proton scattering cross section at high energies, and use a diffraction-scattering formalism to obtain quantitative predictions for $\sigma_{\rm tot}$, $\sigma_{\rm inel}$, and $\sigma_{\rm el}$ at Superconducting Super Collider and cosmic-ray energies. We also predict a very rapid increase in the number of minijet events with $\sqrt{s}$.

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A number of authors[1-3] have observed that the growth in the pp and $\bar{p}p$ cross sections at high energies seems to be associated with the increase in the number of QCD jets. This observation has been incorporated in a simple model by approximating $\sigma_{\rm tot}$ as[1,2]

$$\sigma_{\rm tot} \approx \sigma_{\rm soft} + \sigma_{\rm QCD}, \tag{1}$$

where $\sigma_{\rm soft}$ is an underlying "soft" cross section, taken as energy-independent at high energies, and $\sigma_{\rm QCD}$ is calculated in perturbative QCD with use of a cutoff at low transverse momenta $p_\perp$. This model predicts a rapid increase in $\sigma_{\rm tot}$ with increasing center-of-mass energy $\sqrt{s}$, and can be adjusted to fit the CERN ISR, CERN collider, and cosmic-ray cross-section data[4] by increasing the low-$p_\perp$ cutoff with energy.[2] However, as shown some time ago,[5] the simple additive model for $\sigma_{\rm tot}$ violates partial-wave unitarity by large factors at Fermilab Tevatron collider and Superconducting Super Collider (SSC) energies, and must be modified.

Our objective here is the prediction of the energy dependence of $\sigma_{\rm tot}$ from the QCD parton model using a diffraction-scattering formulation of the problem which is consistent with the unitarity constraints. A similar model has been used by l'Heureux, Margolis, and Valin[3] to obtain phenomenological fits to the pp and $\bar{p}p$ total and elastic scattering data in the energy ranges of the ISR and the CERN collider. Our emphasis is on the QCD predictions rather than detailed phenomenology. We use fairly conservative parton-model input—quark and gluon distribution functions evolved according to the Altarelli-Parisi equations,[6] standard QCD cross sections for elementary $2 \rightarrow 2$ processes, and cutoffs which restrict our calculations to semihard processes—to obtain our predictions for the increase in the total, inelastic, and elastic cross sections with energy. The results shown in Fig. 1 agree rather well with collider and cosmic-ray data for the parameters discussed below. We find that the semihard QCD processes associated with minijets lead to a rapid increase in the pp and $\bar{p}p$ total cross sections with increasing $\sqrt{s}$. (The two cross sections are essentially equal for $\sqrt{s} \geq 100$ GeV.) However, the increase is much less rapid than predicted by the additive model in Eq. (1) and also significantly less rapid than is allowed by the Froissart bound, $\sigma_{\rm tot} \leq C \ln^2 s$.

Our general approach is based on a diffractive treatment of pp (or $\bar{p}p$) scattering. We regard elastic scattering as the diffractive shadow scattering associated with inelastic processes, and calculate the latter. We will use an impact-parameter representation for the scattering amplitude, and will ignore spin-dependent effects and the small real part of the scattering amplitude, both good approximations at high energies.[7] With these approximations,

$$\sigma_{\rm tot} = 4\pi \int_0^\infty db\, b[1 - e^{-\chi(b,s)}], \tag{2a}$$

$$\sigma_{\rm el} = 2\pi \int_0^\infty db\, b[1 - e^{-\chi(b,s)}]^2, \tag{2b}$$

$$\sigma_{\rm inel} = 2\pi \int_0^\infty db\, b[1 - e^{-2\chi(b,s)}], \tag{2c}$$

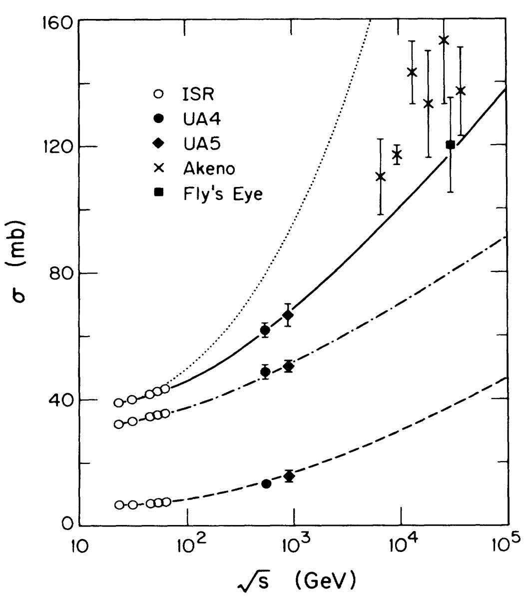

FIG. 1. Energy dependence of the proton-proton total cross section (solid curve), inelastic cross section (dot-dashed curve), and elastic cross section (dashed curve) in the diffractive QCD model in the text. The prediction of the additive model, Eq. (1), is given for the same parameters by the dotted curve. The data are from Refs. 4.

where the eikonal function $\chi(b,s)$ is real.

The key observation for our purposes is that the factor $[1-e^{-2\chi}]$ in Eq. (2c) is the probability that at least one of the two protons is broken up in a collision at an impact parameter b. We will calculate the contribution to this breakup probability of semihard collisions between the constituents of the protons using the probability-based QCD parton model. For simplicity, we will consider only gluons in the following discussion, but have used a complete description of the constituent collisions in our numerical calculations.

The number of gluon-gluon collisions in a pp collision at impact parameter b is given in the QCD parton model by

$$n(b,x)=\int d^2b'\int dx_1\int dx_2\int d|\hat{t}|\,(d\sigma_{gg}/d|\hat{t}|)(\hat{s},\hat{t})G(x_1,|\hat{t}|,|\mathbf{b}-\mathbf{b}'|)G(x_2,|\hat{t}|,\mathbf{b}'). \tag{3}$$

Here x_i is the fraction of the momentum of the parent proton carried by gluon i, $d\sigma_{gg}/d|\hat{t}|$ is the gluon-gluon scattering cross section, $\hat{s}$ and $\hat{t}$ are the Mandelstam variables for the two-gluon collision, and $G(x,|\hat{t}|,b)dx\times d^2b$ is the number of gluons in the interval dx and the transverse area element d^2b a distance b from the center of the proton.

If $n(b,s)\ll 1$ (it isn't), $n(b,s)$ can be interpreted as the probability of a gluon-gluon scattering in a pp collision at impact parameter b. Since such a collision will almost certainly break up at least one of the protons if the momentum transfer $|\hat{t}|$ is sufficiently large, $n(b,s)$ can also be interpreted as the probability of an inelastic pp collision. The latter interpretation leads to the additive model for $\sigma_{\rm tot}$ as we will show later.

For $n(b,s)\geq 1$, a straightforward mean-free-path-type argument which involves the longitudinal spatial parton distributions gives the probability $\bar{P}$ that the two protons have *not* undergone a semihard inelastic scattering at the parton level as

$$\bar{P}_{\rm QCD}(b,s)=e^{-n(b,s)}. \tag{4}$$

Thus, allowing also for soft inelastic processes not describable in terms of parton-parton scattering, the probability P of an inelastic collision is

$$P=1-\bar{P}_{\rm soft}\bar{P}_{\rm QCD}. \tag{5}$$

Comparing this expression with Eq. (3c) where P appears as $[1-e^{-2\chi}]$, we find that

$$\chi(b,s)=\chi_{\rm soft}(b,s)+\chi_{\rm QCD}, \tag{6}$$

where

$$\chi_{\rm QCD}(b,x)=\tfrac{1}{2}n(b,s). \tag{7}$$

We will assume in the following calculations that the function $G(x,|\hat{t}|,b)$ factors, $G(x,|\hat{t}|,b)\approx G(x,|\hat{t}|)\times\rho(b)$, with $G(x,|\hat{t}|)$ the usual gluon distribution in the proton, and $\rho(b)$ the probability density for finding a gluon in the area d^2b at impact parameter b. This factorization is consistent with the usual parton picture and the QCD evolution of well-localized partons with small x.

We will also assume that $\rho(b)$ is approximately the same distributon as determined from the proton electric form factor $G_E(k_\perp^2)$,

$$\rho(b)\approx\frac{1}{(2\pi)^2}\int d^2k_\perp G_E(k_\perp^2)e^{i\mathbf{k}_\perp\cdot\mathbf{b}}, \tag{8}$$

where[8]

$$G_E(k_\perp^2)\approx[1+k_\perp^2/\nu^2]^{-2},\quad \nu^2\approx 0.71\ ({\rm GeV}/c)^2. \tag{9}$$

This assumption—essentially that the partons and electric charge in the proton have similar distributions—is a modern version of the ideas on the "matter distribution" of the proton proposed twenty years ago[9] by Wu and Yang and by Byers and Yang. It can be motivated by noting that the constituent quark distribution drives the gluon distribution through QCD processes, and conversely, so that the spatial distributions of the observed quarks and gluons are tightly coupled.

Using the input above, we find that the effective area of overlap of the parton distributions in the colliding protons is

$$A(b)=\int d^2b'\rho(|\mathbf{b}-\mathbf{b}'|)\rho(b')=\tfrac{1}{8}(\nu^2/12\pi)(\nu b)^3K_3(\nu b), \tag{10}$$

where $K_3(z)$ is an exponentially decreasing hyperbolic Bessel function. The eikonal function $\chi_{\rm QCD}(b,s)=\frac{1}{2}\times n(b,s)$ factors, and is given by

$$\chi_{\rm QCD}(b,s)=\tfrac{1}{2}A(b)\sigma_{\rm QCD}(s), \tag{11}$$

where $\sigma_{\rm QCD}$ is the parton-parton elastic scattering cross section

$$\sigma_{\rm QCD}(s)=A(b)\int_{Q_{\rm min}/\sqrt{s}}^{1}dx_1\int_{x_1}^{1}dx_2\int_{Q^2_{\rm min}}^{Q^2_{\rm max}}d|\hat{t}|\,\frac{d\sigma_{gg}}{d|\hat{t}|}(\hat{s},\hat{t})G(x_1,|\hat{t}|)G(x_2,|\hat{t}|), \tag{12}$$

and $Q^2_{\rm max}=\min(x_1^2 s,x_1x_2s/2)$ for gluons. The ranges of x_1, x_2, and $|\hat{t}|$ used in the evaluation of $\sigma_{\rm QCD}(s)$ have been chosen so that interference between initial and final state radiation is small, and the G's are standard distribution functions.

We have evaluated $\chi_{\rm QCD}$ numerically using the QCD-evolved distribution functions of Eichten *et al.*[6] [set 1 with

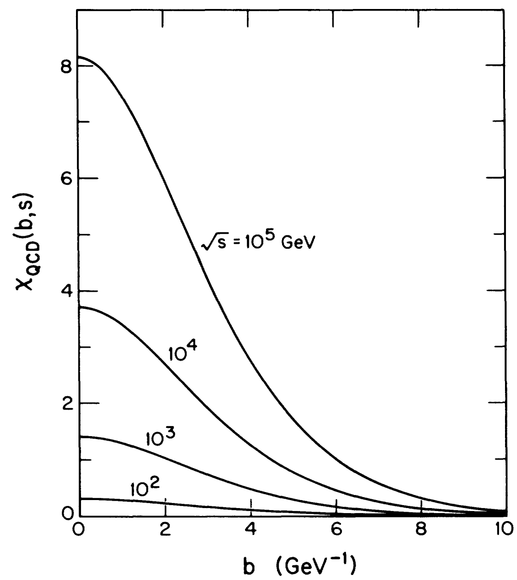

FIG. 2. $\chi_{QCD}(b,s)$ vs b for $\sqrt{s}=10^2,10^3,10^4,10^5$ GeV.

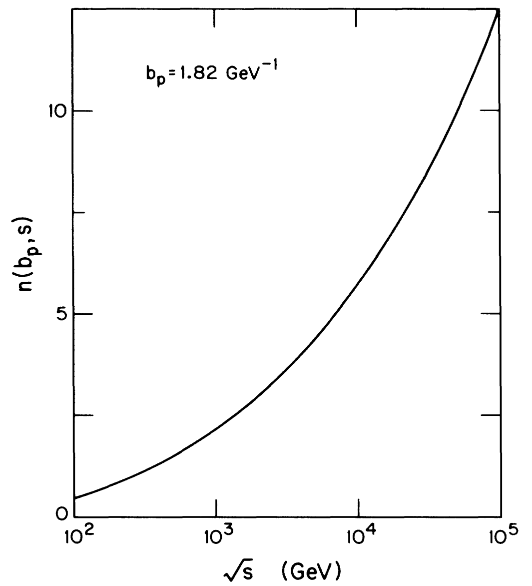

FIG. 3. The most probable number of parton-parton collisions with $p_\perp^2 > 2$ (GeV/c)^2 as a function of $\sqrt{s}$. Each collision gives two semihard jets.

$\Lambda_{\overline{MS}}=200$ MeV for $|\hat{t}|>5$ (GeV/c)^2, and the $Q^2=5$ (GeV/c)^2 distribution for small values of $|\hat{t}|$. The result for $Q^2_{\min}=2$ (GeV/c)^2 is shown in Fig. 2. The number of semihard parton collisions in a pp collision at impact parameter b is $n(b,s)=2\chi_{QCD}(b,s)$. This number is quite large at high energies, and so the interpretation of $n(b,s)$ as a breakup probability is untenable. However, just this interpretation underlies the additive model for σ_{tot} in Eq. (1). If we expand the exponential in Eq. (2c) on the assumption that $\chi(b,s)$ is small, and note that

$$2\pi\int_0^\infty db\, bA(b)=1, \tag{13}$$

we find a cross section of just the form given in Eq. (1), with σ_{QCD} given by Eq. (12) as expected. This model violates probability considerations, or equivalently, partial-wave unitarity, by large factors at high energies.

In order to calculate σ_{tot}, we need to specify χ_{soft} as well as χ_{QCD}. We will use the diffraction scattering model introduced by Durand and Lipes[10] and Chou and Yang[11] in which χ_{soft} is of the form

$$\chi_{soft}(b,s)=C(s)A(b) \tag{14}$$

with $A(b)$ related to the proton densities and form factors through Eq. (10). The ideas which underlie the model are similar to those used here, but it does not attempt to give a fundamental description of the soft interactions. The model correctly predicted the diffraction structure later observed in unpolarized and polarized pp elastic scattering,[10] and with some refinements, gives a good description of the pp and $\bar{p}p$ data through CERN collider energies.[3,11]

In our calculations, we have ignored the slow increase in $C(s)$ with $\sqrt{s}$ observed up to the ISR region—possibly the result of soft QCD interactions—and simply fixed C at a value which reproduces σ_{tot} at $\sqrt{s}\approx 43$ GeV, where the value of χ_{QCD} is very small. We then chose the cutoff momentum transfer $Q^2_{\min}=2$ (GeV/c)^2 to roughly reproduce $\sigma_{tot}(\bar{p}p)$ at $\sqrt{s}=546$ GeV. The extrapolations of our results for the pp cross sections to SSC and cosmic-ray energies are shown in Fig. 1. The total cross section predicted by the diffractive model increases much less rapidly with $\sqrt{s}$ than that predicted by the additive model.

The energy dependence of σ_{tot} is related to the rapid increase in the parton distribution functions at the small values of x which are accessible at large $\sqrt{s}$, and is not simply parametrized. The predicted ratio σ_{el}/σ_{tot} of elastic to total cross sections increases monotonically from ~ 0.175 at $\sqrt{s}=50$ GeV to ~ 0.33 at the energy of the SSC (40 TeV) because of the very rapid growth of χ_{QCD} with $\sqrt{s}$, and would approach the "black disk" ratio $\frac{1}{2}$ at much higher energies. This "blackness" of the proton at high energies is a direct consequence of QCD.

Finally, in Fig. 3, we show the most probable number of semihard parton collisions as a function of energy, corresponding to pp collisions at the impact parameter $b=1.82$ GeV^{-1} which maximizes the number distribution function $2\pi bA(b)n(b,s)$. Each collision gives two jets with transverse momenta $p_\perp > 1.4$ GeV/c. Thus, from the figure we expect a most probable number of ≈ 9 parton-parton collisions and ≈ 18 semihard jets, some with quite large $p_\perp$ and $p_\parallel$, in a pp collision at the SSC. The variation in the number of minijets for collisions at different impact parameters is quite large, and will lead to large fluctuations in particle multiplicities. A typical SSC event, however, will contain many semihard jets, concentrated in the central region.

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