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Helicity-Amplitude Relations for Vector-Meson Production from a Skyrmion Model

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Mattis has derived six relations among the partial-wave amplitudes for the reaction $\pi N \to \rho N$. From these relations we deduce that the six independent helicity amplitudes for the t-channel isospin-0 reaction may be simply expressed in terms of one unknown function. This leads to definite predictions for the ρ-meson density matrix and the absence of nucleon polarization effects, which can be tested directly, without recourse to partial-wave analysis.

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In a recent Letter, Mattis[1] has obtained a number of linear relations among the partial-wave amplitudes for the reaction $\pi N \to \rho N$, by use of a model in which the baryon is considered to be a "skyrmion." In particular, the six isospin-$\frac{3}{2}$ partial-wave amplitudes (indexed by final spin S, initial and final orbital angular momenta L and L', and total angular momentum J) are expressed as linear combinations of those with isospin $\frac{1}{2}$. I point out in this Letter that if these equations are written in terms of t-channel isospin amplitudes a remarkable simplification occurs. One finds one linear relation among the $I_t = 1$ partial-wave amplitudes, which is identical in form to that derived by Mattis for the reaction $\pi N \to \omega N$, and five relations among the $I_t = 0$ partial-wave amplitudes. If one uses the standard connection between the helicity partial-wave amplitudes and the L-S amplitudes of Mattis, these five relations enable one to express the six independent helicity amplitudes for the reaction in terms of one unknown function. The results, expressed in terms of transversity amplitudes, are even simpler, since two are identically zero, and the four remaining amplitudes are related to the unknown function by phases and numerical factors. The phenomenological consequences are immediate; one can test them directly from the experimental data without going to the trouble of a partial-wave analysis. Unfortunately, the t-channel isospin-0 reaction is $\pi^0 p \to \rho^0 p$, which cannot be measured directly. The linear combination $(\pi^+ p \to \rho^+ p) + (\pi^- p \to \rho^- p) - (\pi^- p \to \rho^0 n)$ does indeed isolate the isospin-zero contribution,[2] but it requires accurate measurements of all three reactions.

In order to demonstrate my results, I adopt the notation of Mattis and use the isospin-crossing relations of Rebbi and Slansky[3] to introduce the t-channel isospin amplitudes,

$$A_s^{3/2} = -(1/\sqrt{6})A_t^0 + \tfrac{1}{2}A_t^1,$$
$$A_s^{1/2} = -(1/\sqrt{6})A_t^0 - A_t^1. \quad (1)$$

If one inserts these amplitudes in Eqs. (14) and (15) of Mattis, one finds, after some algebra,

$$\rho_3^0(L,L-2)_{2L-1} = 0,\quad \rho_3^0(L,L+2)_{2L+1} = 0,\quad \rho_3^0(L,L)_{2L-1} = -[(2L-1)/(L+1)]^{1/2}\rho_1^0(L,L)_{2L-1}, \quad (2a,b,c)$$

$$\rho_3^0(L,L)_{2L+1} = (2+3/L)^{1/2}\rho_1^0(L,L)_{2L+1},\quad \rho_1^0(L,L)_{2L-1} = -(1+1/L)\rho_3^0(L,L)_{2L+1}, \quad (2d,e)$$

$$\rho_1^1(L,L)_{2L-1} - \rho_1^1(L,L)_{2L+1} = [(2L-1)/(L+1)]^{1/2}\rho_3^1(L,L)_{2L-1} + (2+3/L)^{1/2}\rho_3^1(L,L)_{2L+1}, \quad (2f)$$

where superscripts refer to I_t. One observes that the relation among the t-channel isospin-1 partial-wave amplitudes, (2f), is identical to Eq. (16) of Mattis, which was derived for the reaction $\pi N \to \omega N$. The latter is a pure t-channel isospin-1 reaction; hence there appears to be a universal constraint for these reactions.

The constraints in the case of the t-channel isospin-zero reaction are much stronger. Those amplitudes with $|\Delta L| = 2$ vanish, and three of the $\Delta L = 0$ amplitudes may be written as multiples of the fourth one, which I have chosen to have $S = \frac{1}{2}$ and $J = L + \frac{1}{2}$. Note that two of the relations, (2c) and (2d), are at fixed J, while (2e) relates J to $J+1$. At this point I introduce the standard relation among helicity partial-wave amplitudes and the L-S amplitudes, as given by Jacob and Wick[4]:

$$H^J_{\lambda_c\lambda_d\lambda_a\lambda_b} = \sum_{\substack{LL'\\SS'}} \Bigg[[(2L+1)(2L'+1)(2S+1)(2S'+1)]^{1/2} \times \begin{pmatrix} L & S & J \\ 0 & \lambda & -\lambda \end{pmatrix} \begin{pmatrix} S_a & S_b & S \\ \lambda_a & -\lambda_b & -\lambda \end{pmatrix} \begin{pmatrix} L' & S' & J \\ 0 & \mu & -\mu \end{pmatrix} \begin{pmatrix} S_c & S_d & S' \\ \lambda_c & -\lambda_d & -\mu \end{pmatrix} \eta \rho_{2S\,2S'}(L,L')_{2J} \Bigg], \quad (3)$$

where $\eta = 1$ or -1 when $L + L' - S - S' - S_a + S_b - S_c + S_d$ is even or odd. For our reaction $S_a = 0$, $S_c = 1$, $S_b = S_d = S = \frac{1}{2}$. Evaluating the three-$j$ symbols, and using the five constraints written above, one obtains the following expressions for the six helicity partial-wave amplitudes in terms of the quantity $\rho_3^0(L,L)_{2L+1}$, which I shall henceforth abbreviate as a_L, where L is defined to be $J - \frac{1}{2}$:

$$H^J_{0,1/2,1/2} = \left[a_L - \frac{L+2}{L+1} a_{L+1} \right] / \sqrt{3}, \quad H^J_{0,-1/2,1/2} = -\left[a_L + \frac{L+2}{L+1} a_{L+1} \right] / \sqrt{3}, \quad (4a,b)$$

$$H^J_{-1,-1/2,1/2} = -H^J_{0,1/2,1/2}/\sqrt{8}, \quad H^J_{1,1/2,1/2} = -H^J_{0,-1/2,1/2}/\sqrt{8}, \quad (4c,d)$$

$$H^J_{1,-1/2,1/2} = -(\sqrt{\tfrac{3}{8}}) \left\{ \left[\frac{L+2}{L} \right]^{1/2} a_L - \frac{[L(L+2)]^{1/2}}{L+1} a_{L+1} \right\}, \quad (4e)$$

$$H^J_{-1,1/2,1/2} = (\sqrt{\tfrac{3}{8}}) \left\{ \left[\frac{L+2}{L} \right]^{1/2} a_L + \frac{[L(L+2)]^{1/2}}{L+1} a_{L+1} \right\}. \quad (4f)$$

These helicity partial-wave amplitudes may then be used to calculate the helicity amplitudes via the usual formula, with θ the production angle,[4]

$$H_{\lambda_c\lambda_a\lambda_d\lambda_b}(\theta,0) = \textstyle\sum_J (2J+1) H^J_{\lambda_c\lambda_d\lambda_a\lambda_b} d^J_{\lambda\mu}(\theta), \quad (5)$$

where $\lambda = \lambda_a - \lambda_b$, and $\mu = \lambda_c - \lambda_d$. If one replaces the Wigner d functions by the sums of derivatives of Legendre polynomials, following Goldberger and Watson,[5] one finds the remarkable result

$$H_{0,1/2,1/2} = -(4/\sqrt{3})\sin(\theta/2)F(\theta), \quad (6a)$$

$$H_{-1,-1/2,1/2} = (2/\sqrt{6})\sin(\theta/2)F(\theta), \quad (6b)$$

$$H_{1,-1/2,1/2} = -(\sqrt{6})\sin(\theta/2)F(\theta), \quad (6c)$$

$$H_{0,-1/2,1/2} = -(4/\sqrt{3})\cos(\theta/2)F(\theta), \quad (6d)$$

$$H_{1,1/2,1/2} = (2/\sqrt{6})\cos(\theta/2)F(\theta), \quad (6e)$$

$$H_{-1,1/2,1/2} = -(\sqrt{6})\cos(\theta/2)F(\theta), \quad (6f)$$

where the unknown function F is defined by

$$F(\theta) = \sin(\theta) \textstyle\sum_L (1 + 1/2L) a_L P'_L(\cos\theta). \quad (7)$$

An even greater simplification takes place if one expresses these results in terms of transversity amplitudes (with the spin-quantization axis chosen normal to the plane of production). One finds

$$T_{1,1/2,-1/2} = T_{-1,-1/2,1/2} = 0, \quad (8a)$$

$$T_{0,1/2,1/2} = i(2/\sqrt{3}) e^{-i\theta/2} F, \quad (8b)$$

$$T_{0,-1/2,-1/2} = i(2/\sqrt{3}) e^{i\theta/2} F, \quad (8c)$$

$$T_{1,-1/2,1/2} = -i(8/\sqrt{6}) e^{i\theta/2} F, \quad (8d)$$

$$T_{-1,1/2,-1/2} = -i(8/\sqrt{6}) e^{-i\theta/2} F. \quad (8e)$$

From this it follows that the ρ-meson spin-density matrix, expressed in the transversity basis, has the form

$$\begin{pmatrix} \frac{4}{9} & 0 & 0 \\ 0 & \frac{1}{9} & 0 \\ 0 & 0 & \frac{4}{9} \end{pmatrix}$$

for any production angle. The corresponding decay angular distribution for the ρ meson into two pions is then of the form

$$(\tfrac{4}{3} - \cos^2\beta)/4\pi,$$

where β is the angle between the pion momentum and the normal to the production plane. There is no azimuthal dependence. Another important consequence is that there can be no observable effects associated with the target nucleon polarization, since all helicity amplitudes have the same phase (I am referring here to single-spin asymmetries, not to two-spin effects).

I thus find that the predictions of Mattis amount to a specification of the decay angular distribution of the ρ meson independent of energy and production angle, as well as imposing that there be no observable single-spin effects. If the t-channel isospin reaction were directly observable it would be easy to test such predictions. The best we can do is point out, following Beder,[2] that the linear combination $(\pi^+ p \to \rho^+ p) + (\pi^- p \to \rho^- p) - (\pi^- p \to \rho^0 n)$ is pure t-channel isospin zero, and therefore should satisfy the constraints we have found.

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