Observation of the Aharonov-Bohm Effect for $\omega_c \tau > 1$

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We have observed the Aharonov-Bohm effect in the magnetoresistance of doubly connected geometries fabricated in high-mobility GaAs/AlGaAs heterostructures. Periodic oscillations in the resistance associated with the flux's penetrating the annulus are suppressed above $\omega_c \tau \approx 5$, where ω_c is the cyclotron frequency and τ is the scattering time, while fluctuations in the resistance as a function of magnetic field are observed for $0 < \omega_c \tau < 300$, superimposed upon Shubnikov–de Haas oscillations.

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The Aharonov-Bohm (AB) effect has been observed in magnetoresistance measurements of doubly connected, disordered, one-dimensional normal metal wires smaller than the inelastic diffusion length L_ϕ. For such quantum mechanically coherent devices, the phase of the electronic wave function in one branch of the annulus is changed relative to the other by a change in the magnetic flux penetrating the annulus, resulting in constructive and destructive interference periodic in the flux. The transport properties reflect the interference.[1,2] In particular, Webb *et al.* have demonstrated that oscillations in the magnetoresistance periodic in field associated with magnetic flux penetrating an annulus ($\Phi_0 = hc/e$) can persist for fields as high as 8.0 T ($\Phi > 1000\Phi_0$) with $\omega_c \tau \ll 1$, where ω_c is the cyclotron frequency and τ is the scattering time. The periodic oscillations beat with aperiodic fluctuations due to flux penetrating the wire comprising the annulus (Φ_c). The rms amplitude of oscillation in the resistance Δr is small relative to the resistance of the device, $\Delta r/r < 0.001$, because the conductance per channel adds incoherently and the number of channels (or transverse subbands) at the Fermi energy is large, $N_T > 1000$. In this Letter, we describe the first unambiguous observation of the AB effect in the magnetoresistance of an annulus fabricated in a high-mobility GaAs/AlGaAs heterostructure. In contrast to earlier experiments in disordered normal-metal rings, we find that periodic oscillations in the magnetoresistance associated with the flux penetrating the annulus are suppressed beyond $\omega_c \tau \approx 5$, while fluctuations in the resistance are observed for $0 < \omega_c \tau < 300$ superimposed upon Shubnikov–de Haas oscillations. As a result of quantization in the transverse direction, we estimate that the current is carried by only a few ($N_T < 10$) channels at the Fermi energy (at 270 mK) with minimal elastic scattering, and correspondingly we find that the rms amplitude of oscillation is approximately 10% of the resistance of the device (at least a factor of 100 larger than that observed in normal-metal rings).

A preliminary experiment by Datta *et al.*[3] in parallel GaAs quantum wells connected through Au-Ge–alloyed contacts has revealed fluctuations in the magnetoresistance below 0.25 T which were attributed to quantum interference, but did not demonstrate the AB effect unequivocally. For instance, because of the uncertainty in the geometry of a structure determined by contacts and the 2-μm thickness of the device, the three fluctuations in resistance observed by Datta *et al.* might also be attributed to $hc/2e$ oscillations associated with weak localization in doubly connected geometries analogous to the experiment by Sharvin and Sharvin[4] in normal-metal cylinders. In contrast, our experimental results show oscillations in the resistance that persist up to $\Phi = 300\Phi_0$ and can only be associated with the fundamental flux hc/e.

The devices were fabricated by electron-beam lithography and low-voltage (75–150 V) reactive ion etching on modulation-doped $GaAs/Al_{0.33}Ga_{0.67}As$ heterostructures grown by molecular-beam epitaxy. Electron-beam lithography was used to pattern an etch mask which protects the underlying AlGaAs from a subsequent partial etch of the AlGaAs layer in a CCl_2F_2-He-O gas mixture. The anisotropic partial etch, which removes the doped layer, stops in the AlGaAs spacer layer and so laterally confines the electron gas to the region immediately beneath the etch mask. The remaining doped AlGaAs over the wire is partially depleted as a result of the pinning of the Fermi level in the band gap at the AlGaAs surface on the exposed side wall and to the presence of traps in the layer. A photograph of the 2.0-μm average diameter ring with a linewidth of 500 nm is shown in the inset to Fig. 1. The fabrication process is described in detail elsewhere.[5]

We measured the four-terminal electrical resistance of three devices with three different diameters fabricated with a geometry similar to that shown in Fig. 1, using an ac resistance bridge consisting of two PAR model 124-A lock-in amplifiers operating at 14 Hz. The typical excitation voltage across the annulus was about 1 μV. The measured average diameters of nominally 2.5-, 2.0-, and 1.0-μm-diameter annuli were 2.35 ± 0.04, 1.88 ± 0.04, and 0.94 ± 0.02 μm, respectively. The samples were refrigerated in a ^{3}He-evaporation cryostat inserted into a 13.5-T superconducting solenoid. We found that wires

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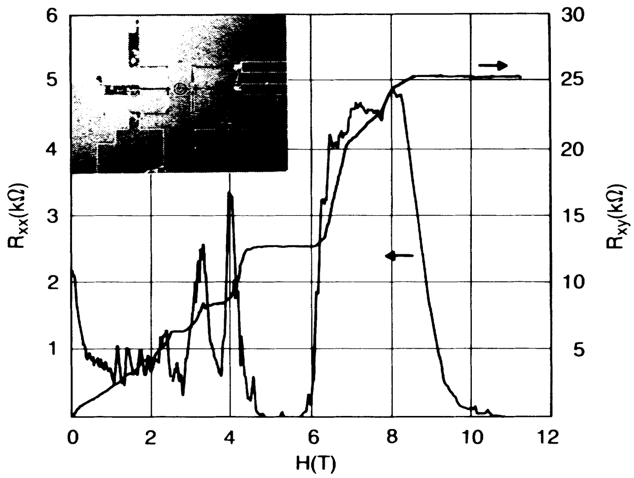
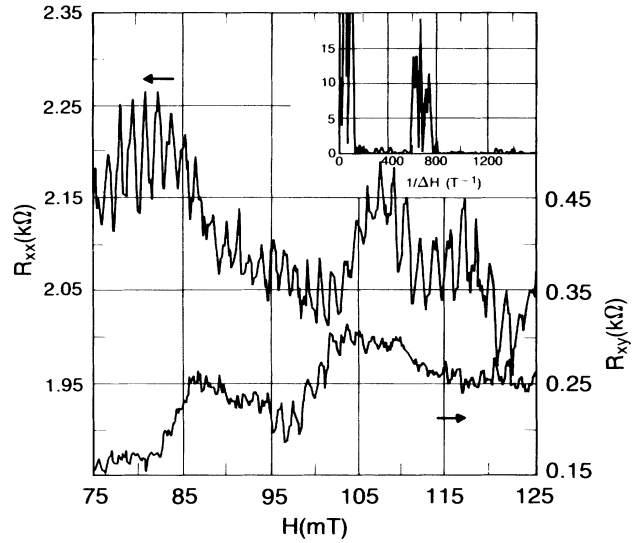


FIG. 1. The magnetoresistance of the 2.0-μm-diameter annulus measured at 270 mK. An electron micrograph of the device is shown in the inset of the figure.

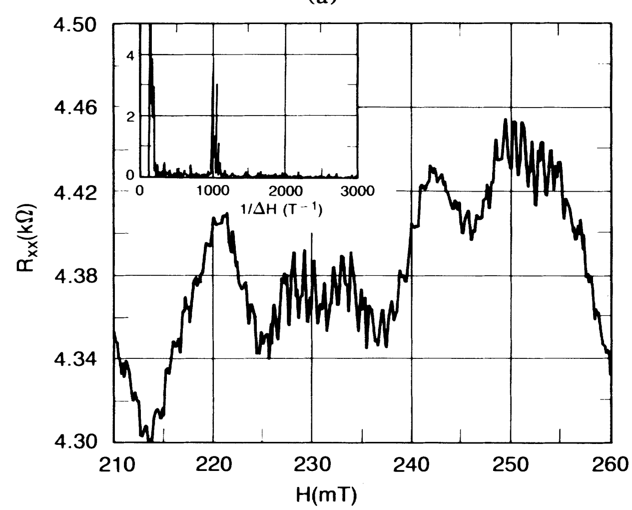
which were lithographically $< 400-450$ nm wide did not conduct at low temperature ($T < 4$ K), and so we anticipate that wires which are lithographically 500 nm wide have conducting widths less than 100 nm.[6,7]

The magnetoresistances, R_{xx} and R_{xy}, at 270 mK observed in the 2-μm-diameter annulus are shown in Fig. 1. Zero-resistance states are observed in R_{xx} and Hall resistance plateaus in R_{xy} corresponding to the Landau indices $N=0$, 1, 2, and 3. The positions in magnetic field of these features can be attributed to a single two-dimensional electron density of $n=(2.5\pm0.2)\times10^{11}$ cm^{-2} in contrast with the density of $(3.9\pm0.1)\times10^{11}$ cm^{-2} measured in a 2D (300×900-μm^2) Hall bridge at 4 K. At low field, however, there is a deviation in R_{xy} from the slope consistent with a density of 2.5×10^{11} cm^{-2}. The change in the carrier density in the wire from the 2D value may be due to the partial depletion of the doped layer from the AlGaAs sidewalls.[5,6] The deviation observed at low field slope may be indicative of transverse confinement.[8,9]

Figures 2(a) and 2(b) clearly show examples of the periodic oscillations in magnetic field observed in r_{xx} of the 2.5- and 2.0-μm-diameter annuli below 0.5 T, superimposed upon a slowly varying background, and the corresponding Fourier power spectrum. The periods of the high-frequency oscillations, 0.95 mT for the nominal 2.5-μm diameter, 1.4 mT for the nominal 2.0-μm diameter, and 5.8 mT for the 1-μm annulus, are in direct correspondence with the penetration of the flux $\Phi_0=hc/e$ through the average area of the annulus (within the experimental error associated with the measurement of the diameter of the annulus), and are not consistent with $hc/2e$, even if we consider the inner radius of a conducting wire 500 nm wide. In addition, oscillations with the same period were observed in R_{xy} as shown in Fig. 2(a) even though the probes used to measure R_{xy} were on the



(a)



(b)

FIG. 2. (a),(b) The periodic magnetoresistance observed in a 2.5- and 2.0-μm-diameter annulus, respectively. Fourier power spectra of the magnetoresistance of the respective annuli are shown in the insets. In (a) both R_{xx} and R_{xy} are shown.

same side of the annulus. The observation of oscillations in r_{xy} is a direct consequence of the coherence of the electronic wave function associated with the annulus and the lead frame. The rms amplitude of the periodic oscillations in the conductance at 10 mT are $0.016e^2/h$, $0.32e^2/h$, and $0.51e^2/h$ for the 2.5-, 2.0-, and 1.0-μm-diameter annuli, respectively. If we interpret the width of the hc/e fundamental to be indicative of the inside and outside trajectories around the annulus,[1] then the conducting wire widths are 80, 130, and 75 nm for the 2.5-, 2.0-, and 1.0-μm-diameter annuli, respectively.

The width of the wire may not be so simply related to the width of the Fourier transform with only a few conducting channels, however.

The fluctuations associated with flux penetrating the area of the wire comprising the annulus beat with the periodic oscillations and consistently have a much larger amplitude (by about a factor of 2). Fluctuations in the magnetoresistance due to quantum interference are observed in both R_{xx} and R_{xy} over the entire field range examined as shown by Fig. 1; however, the typical frequency of fluctuation is lower as the field increases.[7] The correlation field B_c (which is related to L_ϕ by $B_c = 2.4\Phi_0/L_\phi W$ for diffusive transport), deduced from the cutoff of the low-frequency components in the Fourier spectra, changes from about $B_c = 10$ mT near zero field to 25 mT at 1T.

Figure 3 depicts the magnetoresistance observed near 150 and 300 mT in the nominally 1- and 2-μm-diameter rings, respectively. The periodic oscillations in the resistance are appreciably damped as magnetic field increases. A similar observation was made in the nominally 2.5-μm-diameter annuli as well. We found that the amplitude of the periodic oscillations in the resistance of the 2.5-, 2-, and 1-μm annuli were suppressed beyond about 300, 160, and 300 mT, respectively. The Larmor radius $r_c = (\hbar c/eH)^{1/2}$ at 300 mT is 47 nm, while 160 mT corresponds to $r_c = 64$ nm. The AB effect is apparently suppressed when $r_c \approx W/2$, where W is obtained from the width of the hc/e fundamental in the Fourier spectrum.[10] The anomalous feature in the low-field ($H < 0.3$ T) Hall effect occurs in the same field range as the suppression of the AB effect. Previously[7] this feature was interpreted to be representative of the crossover between confinement associated with the magnetic potential and depletion.

Table I summarizes the parameters measured in the three annuli: the carrier density inferred from the slope of R_{xy} below 0.3 T, the high-field carrier density associated with the $R_{xy} = h/2e^2$ Hall plateau, the width deduced from the Fourier spectra, the zero-field, four-point resistance, and the number of area squares times the width. The resistances of the wires comprising the annuli scaled linearly to within 10% after we accounted for variations in diameter, and width (deduced from hc/e fundamental) if the carrier density is obtained from the low-field Hall effect. With use of the width deduced from the width of the hc/e fundamental in the Fourier transform, the data on the three devices examined are consistent with a mobility of 5.0×10^5 cm^2 V^{-1} s^{-1} which is about twice as large as the mobility measured in a 2D Hall bridge, $\mu = (3.0 \pm 0.2) \times 10^5$ cm^2 V^{-1} s^{-1} at 4 K. Moreover, if we assume after Berggren[8] that an infinite square well is a realistic model for the confinement potential in the absence of a magnetic field, use the high-field carrier density to determine the Fermi energy, and assume that the width of the confinement potential is given by W in Table I, we find 2, 4, and 2 transverse subbands at the Fermi energy for the 2.5-, 2.0-, and 1.0-μm diameters, respectively, and the calculated carrier densities in the absence of a field are $(1.3, 1.8, \text{and } 1.5) \times 10^{11}$ cm^{-2} in correspondence with the slope in R_{xy} below 0.3 T. Alternatively, if the carrier density is independent of field, the deviation of the Hall resistance from a slope characteristic of the high-field carrier density results from the effect of confinement when $\Delta E\tau > 1$, where ΔE represents the quantized energy due to transverse

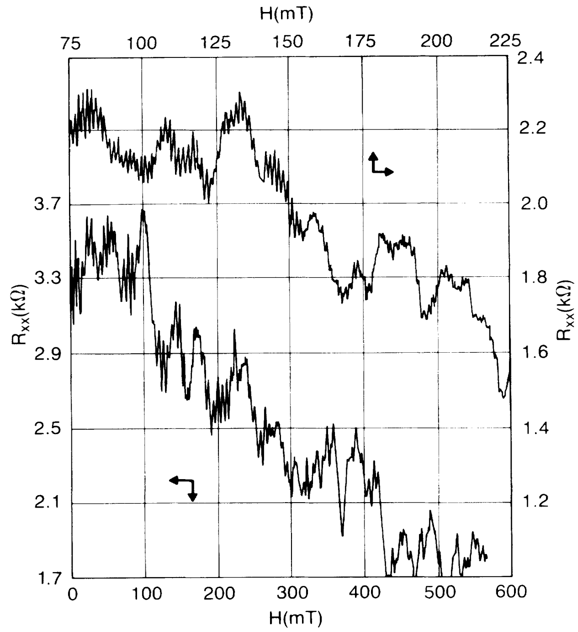


FIG. 3. The magnetoresistance observed in the nominally 2-μm (upper curve) and 1-μm (lower curve) annuli.

TABLE I. A list of the measured parameters of the three rings examined.

Nominal diameters (μm)	$n_{H=0.3\,T}$ (10^{-11} cm^{-2})	$n_{i=2}$ (10^{-11} cm^{-2})	W (nm)	$R_{xx}(H=0)$ (kΩ)	$\square W$ (μm)
1.0	1.10 ± 0.2	2.3 ± 0.2	75	3.23	2.74
2.0	1.6 ± 0.2	2.5 ± 0.2	130	2.14	3.46
2.5	0.9 ± 0.2	1.9 ± 0.2	80	6.42	3.84

confinement. According to this interpretation, the wire widths are 210, 250, and 150 nm for the 1-, 2-, and 2.5-μm diameters, respectively, and the mobility varies by 30% about 1.4×10^5 cm^{-2} V^{-1} s^{-1}. Although the mobility in the wire may be enhanced over the 2D value because scattering is suppressed in a strictly 1D wire,[11] (i.e., only large-wave-vector scattering, $\delta k = 2k_F$, is allowed), the uncertainty in the actual width of the wire and the low-field carrier density preclude a definitive estimate of the mobility. If $\mu = 5.0\times10^5$ cm^2 V^{-1} s^{-1} then $\omega_c\tau = 1$ for $H = 16$ mT, whereas $\omega_c\tau = 1$ for $H = 70$ mT if $\mu = 1.4\times10^5$ cm^2 V^{-1} s^{-1}. Consequently, the periodic oscillations in the magnetoresistance are suppressed beyond $\omega_c\tau \approx 5$, while the fluctuating background persists even into the extreme quantum limit where $\omega_c \approx 300$.

The suppression of the AB effect is not anticipated theoretically. A general theorem by Byers and Yang[12] which appeals to the gauge invariance of the wave function states that all the physical properties of the annulus are periodic in the flux through the annulus independent of the field, but in the actual experiment the magnetic field penetrates both the annulus and the wires comprising the annulus, and so invalidates the conditions of the proof. Stone and Imry[13] have observed that the contribution to the conductance due to two different pairs of trajectories around the annulus does not have a fixed relative phase. Consequently, if the magnetic field penetrates the wire comprising the annulus, the contribution of the various trajectories to the resistance are unaffected even though a different flux is enclosed by each of the various trajectories. According to Stone[14] the effect of the flux penetrating the wires comprising the annulus is only to modulate the periodic component associated with the flux penetrating the annulus. The model developed by Stone and others[15] which describes the AB effect in transport and the fluctuating magnetoresistance background, however, relies on a WKB form for the electronic wave function which is strictly valid only for $\omega_c\tau < 1$. The AB effect may be suppressed at $r_c \approx W/2$ because the wave function is localized within one branch of the annulus as Datta and Bandyopadhyay propose.[16] It is possible that the magnetic potential dominates the transverse confinement near 300 mT and localizes the wave function to either one of the two branches so that the quantum interference associated with the flux penetrating the annulus vanishes.

In summary, we have observed the AB effect in the magnetoresistance of three doubly connected geometries fabricated in high-mobility GaAs/AlGaAs heterostructures. Periodic oscillations in the resistance associated with the flux penetrating the annulus can persist for magnetic fields such that $\omega_c\tau \approx 5$, and the amplitude of oscillation is approximately 10% of the resistance of the device. We find that the Larmor radius corresponding to the field at which the AB effect is suppressed is simply related to the width of the hc/e fundamental in the Fourier spectrum of the magnetoresistance.

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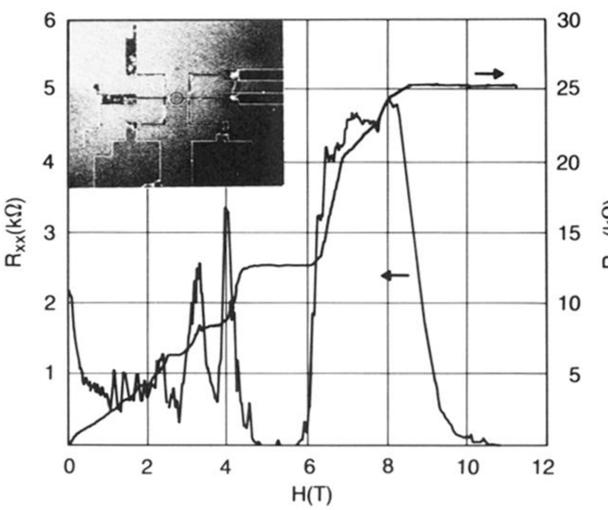

FIG. 1. The magnetoresistance of the 2.0-μm-diameter annulus measured at 270 mK. An electron micrograph of the device is shown in the inset of the figure.