# Dynamics of Three-Dimensional Ionospheric Plasma Clouds

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(Received 10 September 1986)

The first self-consistent, three-dimensional analysis of plasma cloud evolution in the ionosphere is presented. It is demonstrated that there is a preferred perpendicular scale size associated with 3D plasma clouds given by $r_c \sim c(T_e+T_i)/eB_z V_n \Gamma_c$, where $r_c$ is the cloud radius, $T_\alpha$ is the temperature of the $\alpha$ species, $B_z$ is the ambient magnetic field, $V_n$ is the neutral wind speed, and $\Gamma_c < 1/\sqrt{2}$.

PACS numbers: 94.20.Vv

For more than two decades, the evolution of artificial plasma clouds in the near-earth space environment has been of interest to space physicists. Research in this area continues to be exciting and vigorous, especially in light of the recent Active Magnetospheric Particle Tracer Explorer mission[1] and the upcoming Combined Release and Radiation Effects Satellite mission.[2] Originally it was thought that artificial clouds would simply be a diagnostic of ambient plasma conditions; it was soon discovered that they do not simply convect because of ambient fields or winds, but, for example, can become unstable and rapidly structure. The mechanism which causes the structuring of ionospheric clouds is the $\mathbf{E}\times\mathbf{B}$ gradient drift instability.[3] This instability has been extensively studied both theoretically[4-9] and computationally.[10-14] Although a considerable amount of research has been carried out on this instability, there are several deficiencies with regard to its application to ionospheric plasma clouds. First, the bulk of theoretical analysis has focused on the short-wavelength limit (i.e., $kL \gg 1$ where $k$ is the wave number and $L$ is the density-gradient scale length associated with the cloud boundary). However, observationally the gross structuring of barium clouds is clearly in the long-wavelength regime[11] (i.e., $kL \ll 1$). And second, until quite recently, the finite parallel extent of the cloud along the ambient magnetic field has been neglected, as well as any parallel dynamical effects. Progress is being made in this area[6-9]; however, a self-consistent, long-wavelength, three-dimensional analysis of plasma cloud evolution has been lacking.

In this Letter we present the first fully three-dimensional study of plasma cloud dynamics in the long-wavelength limit. We first develop an equilibrium based upon a waterbag model. We then perform a perturbation analysis on the equilibrium. The important result of this analysis is the derivation of a stability criterion for the large-scale structuring of ionospheric plasma clouds; i.e., clouds are stable for $r < r_c$, where $r$ is the radius of the cloud and a $r_c$ is the critical radius and function of the local plasma parameters.

The general three-dimensional equations for a warm plasma cloud in a uniform magnetic field $\mathbf{B} = B\hat{\mathbf{e}}_z$ and a uniform background neutral wind $\mathbf{V}_n = V_n \hat{\mathbf{e}}_x$ (see Fig. 1) are given by[8]

$$\frac{\partial n}{\partial t} - \frac{c}{B}\nabla\phi\times\hat{\mathbf{z}}\cdot\nabla n + \frac{\partial}{\partial z}\frac{1}{e\eta_e}\left(\frac{\partial \phi}{\partial z} - \frac{T_e}{ne}\frac{\partial n}{\partial z}\right) = 0, \tag{1}$$

$$\frac{c}{B}\frac{\nu_{in}}{\Omega_i}\nabla_\perp\cdot n\nabla_\perp\phi + D_{\perp i}\nabla_\perp^2 n + \frac{\nu_{in}}{\Omega_i}\hat{\mathbf{z}}\times\mathbf{V}_n\cdot\nabla n + \frac{\partial}{\partial z}\frac{1}{e\eta_e}\left(\frac{\partial \phi}{\partial z} - \frac{T_e}{ne}\frac{\partial n}{\partial z}\right) = 0, \tag{2}$$

where $\eta_e = m_e \nu_e / ne^2$ is the parallel resistivity, $\nu_e = \nu_{ei} + \nu_{en}$, $D_{\perp i} = (\nu_{in}/\Omega_i)cT_i/eB$ is the perpendicular ion diffusion coefficient, $\nu_{ei}$ is the electron-ion collision frequency and $\Omega_\alpha$ and $\nu_{\alpha n}$ are the cyclotron and neutral collision frequencies of the species $\alpha$. Equation (1) is the electron continuity equation and (2) arises from charge neutrality ($\nabla\cdot\mathbf{J} = 0$). We consider the electrostatic limit, take $T_e$ and $T_i$ to be spatially uniform, assume $\nu_e/\Omega_e$, $\nu_{in}/\Omega_i \ll 1$ so that Hall terms can be discarded, and neglect ion parallel diffusion and perpendicular electron diffusion.

In the potential equation presented in (2) the parallel conductivity greatly exceeds the transverse Pederson conductivity; as a consequence the scale lengths along the magnetic field greatly exceed those in the transverse direction. It is therefore convenient to define a set of dimensionless variables in which the conductivity is isotropic and all scale lengths become comparable. For a plasma cloud of perpendicular scale length $r_c$ the appropriate parallel scale length is $L_z = r_c(\Omega_e\Omega_i/\nu_e\nu_{in})^{1/2}$, where for simplicity we take $\nu_e$ and $\nu_{in}$ to be spatially uniform. The dimensionless equations then become

$$\frac{\partial n}{\partial t} - \nabla\Phi\times\hat{\mathbf{z}}\cdot\nabla n + \frac{\nu_{in}}{\Omega_i}\frac{\partial}{\partial z}\left(n\frac{\partial \Phi}{\partial z} - \Gamma\frac{\partial n}{\partial z}\right) = 0, \tag{3}$$

$$\nabla\cdot n\nabla\Phi + \partial n/\partial y - \Gamma\partial^2 n/\partial z^2 = 0, \tag{4}$$

 

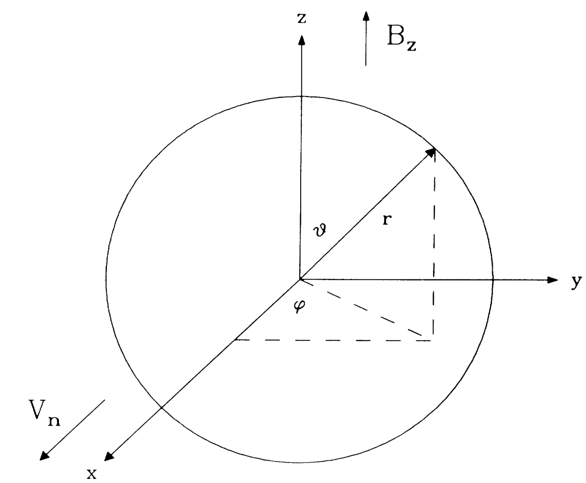


FIG. 1. Plasma configuration and spherical geometry used in the analysis.

where $\Phi=\phi+T_i \ln n/e$, $\Gamma=c(T_e+T_i)/eB_zV_nr_c$, and the remaining dimensionless variables are defined by $tV_n/r_c \to t$, $r_c\nabla_\perp \to \nabla_\perp$, $L_z\,\partial/\partial z \to \partial/\partial z$, and $n/n_b \to n$, where $n_b$ is the density of the uniform background. The parallel compression terms in (3) are small since $v_{in}/\Omega_i \ll 1$ and can be neglected. The continuity equation then simplifies to

$$\partial n/\partial t - \nabla\Phi\times\hat{\mathbf{z}}\cdot\nabla n=0. \tag{5}$$

In the limit $\partial/\partial z=0$, (4) and (5) contain no free parameters so that the transverse scale length $r_c$ is undetermined. However, when parallel dynamics are included, the parameter $\Gamma$ enters the equations. Since the remaining terms are all of order unity, $\Gamma$ can, at most, be of order unity so that the minimum scale size is simply $r_c \sim c(T_e+T_i)/eB_zV_n$. Thus, for a 3D plasma cloud, a simple but rather general dimensional analysis yields a minimum preferred scale size. Moreover, (4) and (5) provide a new set of equations which include parallel dynamics, and may be amenable to numerical simulation techniques such as contour dynamics.[13]

We now examine the stability of a waterbag plasma cloud which is a sphere of radius 1 in our dimensionless units,

$$n_0(r)=MH(1-r)+1, \tag{6}$$

where $H$ is the Heaviside step function. In the physical coordinate system the cloud is greatly extended along $z$. The equilibrium potential $\Phi_0$ of such a spherical cloud consists of two components: the first from the polarization of the cloud by the neutral winds; and the second, the ambipolar potential required to balance the electron pressure parallel to the magnetic field $B_z$. The neutral wind causes the cloud to drift with a velocity $V_0=(M/3)/(1+M/3)$ (normalized to the neutral wind speed $V_n$). In the drifting frame the potential is given by

$$\Phi_{0w}=V_0 r\sin\theta\sin\phi(1-r^{-3})H(r-1). \tag{7}$$

The ambipolar potential $\Phi_{0a}(r,\theta)$ is produced by the term proportional to $\Gamma$ in (4). Using standard techniques, we find

$$\Phi_{0a}=a_2^+P_2(\cos\theta)/r^3,\quad r>1, \tag{8a}$$

$$\Phi_{0a}=a_0^-+a_2^-P_2(\cos\theta)r^2,\quad r<1, \tag{8b}$$

where

$$a_2^+=-\Gamma[M+\tfrac{2}{3}(M+1)\ln(M+1)]/(M+\tfrac{5}{2}),$$

$$a_2^-=-\Gamma[M-\ln(M+1)]/(M+\tfrac{5}{2}),$$

$$a_0^-=(\Gamma/3)\ln(M+1),$$

and $P_2$ is the second-order Legendre polynomial. This ambipolar potential causes the cloud and surrounding plasma to rotate around the axis of the sphere which is aligned with $\mathbf{B}$.

We now investigate the linear stability of the equilibrium defined in (6)–(8). The evolution of the plasma cloud can be described by the evolution of the local radius of the plasma boundary $R=1+\tilde{R}(\theta,\phi,t)$,

$$n=MH(\tilde{R}+1-r)+1, \tag{9}$$

where $\tilde{R}$ is the perturbed radius. The density perturbation, $\tilde{n}=-\tilde{R}\,\partial n_0/\partial r$, is zero away from the equilibrium cloud boundary at $r=1$. Thus (4) implies that the perturbed potential $\tilde{\Phi}$ satisfies the equation

$$\nabla^2\tilde{\Phi}=0 \tag{10}$$

everywhere except $r=1$. The linearized equations for $\tilde{R}$ and $\tilde{\Phi}$ are obtained by use of the continuity equation to solve for $\tilde{R}$ and then by derivation of a set of jump conditions for $\tilde{\Phi}$ and $\tilde{\Phi}'$ at the boundary. Substituting the expression for $n$ in (9) into the continuity equation, we obtain an equation for $\tilde{R}$,

$$\frac{\partial\tilde{R}}{\partial t}+\frac{\partial\tilde{\Phi}}{\partial\phi}+\frac{\partial}{\partial\phi}(\tilde{R}\Phi_0')+\cot\theta\frac{\partial\Phi_0}{\partial\theta}\frac{\partial\tilde{R}}{\partial\phi}=0, \tag{11}$$

where the prime denotes a derivative with respect to $r$. Since $\tilde{R}$ is independent of $r$, (11) must be continuous across the boundary at $r=1$ or

$$\left(\tilde{\Phi}+\tilde{R}\Phi_0'+\cot\theta\tilde{R}\frac{\partial\Phi_0}{\partial\theta}\right)\Bigg|_-^+=0, \tag{12}$$

where $\pm$ denote $r=1\pm\varepsilon$, respectively, with $\varepsilon\to 0$. Equation (12) requires that $\tilde{\Phi}$ undergo a jump at the boundary of the cloud. Similarly, we integrate the linearized version of (4) across the boundary to obtain the discontinuity in the slope of $\tilde{\Phi}'$,

$$n_0\tilde{\Phi}'\Big|_-^+ + M\Gamma\cot^2\theta\frac{\partial^2\tilde{R}}{\partial\phi^2}+\frac{3M}{M+3}\frac{\cos\phi}{\sin\theta}\frac{\partial\tilde{R}}{\partial\phi}-\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\tilde{R}\left(n_0\frac{\partial\Phi_0}{\partial\theta}\right)\Big|_-^+ + M\Gamma(2\cos^2\theta-1)\frac{\partial^2\tilde{R}}{\partial\theta^2}$$
$$+M\left(\Gamma\cot\theta-7\Gamma\cos\theta\sin\theta+\frac{3}{M+3}\cos\theta\sin\phi\right)\frac{\partial\tilde{R}}{\partial\theta}-2M\tilde{R}\left(\Gamma P_2+\frac{3}{M+3}\sin\theta\sin\phi\right)=0. \quad (13)$$

To gain some insight into the nature of the $\mathbf{E}\times\mathbf{B}$ instability in this geometry, we examine the local dispersion relation by assuming $\tilde{\phi}^{\pm},\tilde{R}\sim\exp(ik_\theta\theta+ik_\phi\phi-i\omega t)$. Since $k_\theta$ introduces the parallel dynamics into the equations which we expect to be stabilizing[6,8,9] while the instability is driven by $k_\phi$, we assume that $k_\phi\gg k_\theta\gg 1$. From (11), we therefore find $\tilde{\Phi}^{\pm}\sim\exp[\mp k_\phi(r-1)]$, where we have taken the solutions which decay away from the boundary. Equations (11)–(13) then yield the local eigenvalue

$$\omega=i\gamma_0\cos\phi/\sin\theta+k_\phi V_d-[M\Gamma/(M+2)]\cot^2\theta k_\phi^2, \quad (14)$$

where

$$V_d(\theta,\phi)=[3V_0\sin\theta\sin\phi-\tfrac{3}{2}a_2^+(5\cos^2\theta-1)-(M+1)a_2^-]/(M+2)$$

and $\gamma_0=k_\phi 3M/(M+2)(M+3)$. The first term on the right-hand side of (14) produces instability on the back side of the cloud[3] where $\cos\phi>0$. The remaining terms cause the mode to propagate in the $\phi$ direction. The important feature of (14) to notice is that the local propagation velocity depends on $\theta$. The differential propagation prevents the mode from retaining its flutelike character (i.e., $\partial/\partial\theta=0$) by "shearing" the striations and thereby forcing $\partial/\partial\theta\neq 0$.

To examine this effect on the instability, we retain $\partial/\partial\theta\neq 0$ but consider it small compared with $k_\phi$. We derive an equation for $\tilde{R}$ around $\theta=\pi/2$ where the shear is weakest ($\partial\omega/\partial\theta\sim 0$). To lowest order,

$$\partial^2\tilde{R}/\partial\theta^2+(1+i\delta)k_\phi^2(\theta-\pi/2)^2\tilde{R}+[\omega-k_\phi V_d(\theta=\pi/2)-i\gamma_0\cos\phi](M+2)\tilde{R}/M\Gamma=0, \quad (15)$$

where $\delta\ll 1$. The parameter $\delta$ arises from parallel diffusion in the continuity equation and has been included to insure that the proper evanescent solution to (15) can be identified. The bounded solution to (15) is given by $\tilde{R}\sim\exp[ik_\phi(1+i\delta/2)(\theta-\pi/2)^2/2]$ which becomes increasingly oscillatory (because of the shearing of the striations) as $\theta$ deviates from $\pi/2$. The local propagation frequency and growth as functions of the angle $\phi$ are given by

$$\omega=i\gamma_0[\cos\phi-\tfrac{1}{3}\Gamma(M+3)]+k_\phi V_d(\theta=\pi/2,\phi). \quad (16)$$

The shear enters the dispersion relation in (16) through the dissipative term $\Gamma$.

In a previous investigation[9] of the stability of a cylindrical plasma cloud to modes with $k_z\neq 0$, we found that exponentially growing, localized modes formed at a finite angle $\phi_0$ where the local propagation velocity of the perturbations vanished. In the present 3D equilibrium no such localized solutions exist since (16) is first order in $k_\phi$. However, the importance of this previous calculation was not that a localized solution was found but that at a finite angle $\phi_0$ the diamagnetic propagation and fluid flow combined to produce a nonpropagating mode which could grow without convecting to the stable front side of the cloud. A similar nonpropagating mode exists for the present equilibrium at the angle $\phi_0$ defined by $V_d(\theta=\pi/2,\phi_0)=0$ or

$$V_0\sin\phi_0=(M+1)a_2^-/3-a_2^+/2. \quad (17)$$

The growth rate $\gamma$ of the instability at this angle is given by

$$\gamma=\gamma_0[\cos\phi_0-\Gamma(M+3)/3]. \quad (18)$$

Equations (17) and (18) constitute a dispersion relation for the $\mathbf{E}\times\mathbf{B}$ instability for the 3D equilibrium under consideration.

Simple expressions for the growth rate $\gamma$ can be obtained in the limits of $M\gg 1$ or $M\ll 1$. For $M\gg 1$, we find $\sin\phi_0=-M\Gamma/3$ so that $\gamma=3k_\phi[(1-M^2\Gamma^2/9)^{1/2}-M\Gamma/3]/M$. The mode is stable for $\Gamma>3/\sqrt{2}M$. In the limit $M\ll 1$, we find $\sin\phi_0=\Gamma$ or $\gamma=k_\phi M[(1-\Gamma^2)^{1/2}-\Gamma]/2$ and the mode is stable for $\Gamma>1/\sqrt{2}$.

More generally, for any $M$ the $\mathbf{E}\times\mathbf{B}$ instability is stable for $\Gamma$ exceeding a threshold, $\Gamma_c$. The parameters $\Gamma_c$ and $\phi_0$ can be calculated from (19) and (20) and are shown as functions of $M$ in Fig. 2. $\Gamma_c$ approaches a maximum value of $1/\sqrt{2}$ for $M\ll 1$ and is asymptotic to $3/\sqrt{2}M$ for $M\gg 1$. Clouds with $\Gamma>\Gamma_c$ are stable and those with $\Gamma<\Gamma_c$ are unstable. Moreover, we note that $\phi_0\sim 45°$ for $M\ll 1$ and that $\phi_0\sim -45°$ for $M\gg 1$. Thus, the angle of mode localization with respect to the ambient neutral wind is a function of $M$. This feature may be an observable but would probably be difficult to detect experimentally.

We apply this criterion to ionospheric barium clouds. For typical clouds released at 180 km we note that $T_e\simeq T_i\simeq 0.1$ eV, $B\sim 0.5$ G, $V_n\sim 50$ m/sec, and $M\sim 2$–10

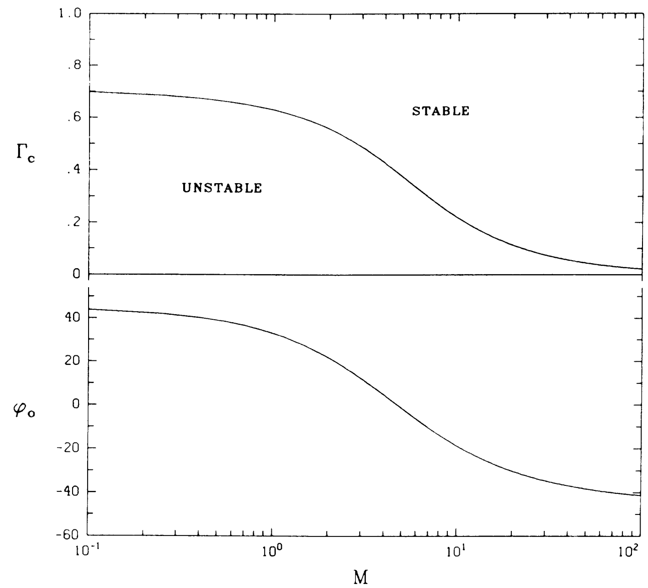


FIG. 2. Plots of $\Gamma_c$ and $\phi_0$ (degrees) vs $M$. Clouds are stable for $\Gamma > \Gamma_c$ and unstable for $\Gamma < \Gamma_c$. Also, note that $\phi_0$ is a function of $M$.

(which corresponds to $\Gamma_c \sim 0.2-0.6$). Also, we note that $\nu_{in} \sim 1\ \mathrm{sec}^{-1}$ and $\Omega_i \sim 160\ \mathrm{sec}^{-1}$ so that the condition $\nu_{in}/\Omega_i \ll 1$ is well satisfied. Thus, we find the critical radius of barium clouds to be $r_c \sim 130-400$ m, which is consistent with observations.[15]

In conclusion, we have developed the first self-consistent, three-dimensional analysis of plasma cloud evolution in the ionosphere on the basis of a waterbag model. We initially argued, on the basis of a simple dimensional argument, that there is a minimum preferred perpendicular scale size associated with 3D plasma clouds given by $r_c \sim c(T_e + T_i)/eB_z V_n$. We subsequently confirmed this scaling with a detailed analysis, and demonstrated that clouds with $r_c < c(T_e + T_i)/eB_z V_n \Gamma_c$ are stable to large-scale structuring by the $\mathbf{E}\times\mathbf{B}$ gradient drift instability. This critical scale size is consistent with observations. Finally, we add that this analysis could also be used as a prototype calculation for other plasma flute modes in a three-dimensional geometry. For example, the Rayleigh-Taylor instability which is believed responsible for the generation of equatorial spread $F$,[16] and the Kelvin-Helmholtz instability which has been suggested as a source of turbulence in the high-latitude ionosphere and magnetosphere,[17] could be affected by 3D dynamics especially in regard to coupling to regions of different conductivities (e.g., $D$ region, $E$ region).

This research was supported by the Defense Nuclear Agency.

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