Neutron Diffraction by Surface Acoustic Waves

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We report an experiment demonstrating diffraction of cold neutrons by surface acoustic waves. We show that, in contrast to analogous experiments with light, the motion of the surface–acoustic-wave deformation of the surface requires significant modification of the diffraction equation applicable to stationary gratings. Diffracted beam intensities are in reasonable agreement with a simple theoretical treatment.

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The diffraction of light by the moving "grating" formed by a surface acoustic wave (SAW) is a well-known phenomenon,[1] which has been routinely used to investigate SAW propagation and attenuation characteristics.[2] We report here an experiment which demonstrates the neutron-optical analogy of this effect.

There are two main differences between the scattering of neutrons and the scattering of light by surface acoustic waves. Firstly, the refractive index[3] of matter for neutrons is very close to unity, i.e., for neutrons $1-\mu \sim 10^{-5}$, whereas for light $\mu - 1 \sim 1$. This fact necessitates the use of grazing incidence for neutron investigations of surface phenomena. Secondly, for light, the SAW may be treated as a stationary grating to a very good approximation, while for neutrons this is not the case since the neutron speeds are an order of magnitude smaller than the SAW speed. We deal with this problem by transforming from the frame in which the SAW is stationary. An important consequence of this rapid motion of the grating is that the diffraction angles are much larger than those for neutrons diffracted by a stationary grating of the same spatial periodicity as the surface acoustic wavelength.[4-6]

Kinematics.— Consider a SAW propagating along the X axis on the surface of the material which occupies the space $y \le 0$. In the primed frame in which the SAW is stationary the diffraction-grating equation for reflected neutrons incident in the X-Y plane may be written

$$k'\cos\theta_n' = k'\cos\theta' + nsK, \tag{1}$$

where k' is the neutron wave number, θ' the glancing angle of incidence, and θ_n' the angle of the nth diffracted beam ($n = 0, \pm 1, \pm 2, \ldots$). K is the SAW wave number, and the integer s is ± 1 depending on whether the propagation of the SAW and the incident neutrons have the same or opposite senses.

A Galilean transformation from the stationary to the (unprimed) laboratory frame yields the result

$$k_n \cos\theta_n = k\cos\theta + nsK, \tag{2a}$$

$$k_n^2 = k^2 + 2n\kappa K, \tag{2b}$$

where $\kappa = mu/\hbar$, m is the neutron mass, and u is the SAW speed. The second of these equations corresponds to a neutron energy change due to the absorption of n phonons, and the diffracted-beam neutron wave number is consequently subscripted.

For our experiment $K \ll (k,\kappa)$ and $(\theta, \theta_n) \ll 1$, which yields the approximation

$$\theta_n^2 = \theta^2 + 2n(K/k)[\kappa/k - s]. \tag{3}$$

The factor $\kappa/k - s$ is the ratio of the SAW velocity in the neutron rest frame to the neutron velocity in the laboratory. For cold neutrons this factor is an order of magnitude greater than unity, its value for a stationary grating of the same spatial frequency, and results in a significant enhancement of the diffraction angles.

Dynamics.— In the stationary SAW frame, the relevant Schrödinger equation is

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2 - V(x,y)\right]\Psi(x,y) = 0, \tag{4}$$

where $\hbar^2 V/2m$ is the neutron potential of the surface material. As a result of the surfce deformation, $V(x,y)$ is periodic in x with spatial frequency K, and may be expanded as a Fourier series in x. In this situation Floquet's theorem leads to a Bloch-type wave function, the product of a plane wave $\exp(ikx\cos\theta)$ and a function $u(x,y)$ periodic in x with spatial frequency K, which may also be expanded as a Fourier series in x.

Substitution of these forms reduces the Schrödinger equation (4) to the system of coupled ordinary differen-

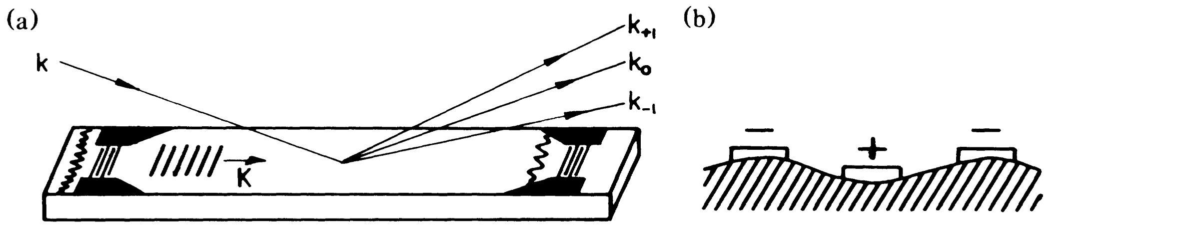

FIG. 1. (a) Schematic of neutron "grating" diffraction by surface acoustic waves. (SAW propagation sense $s=+1$.) (b) Instantaneous profile of the interdigitated electrode structure showing the applied voltage and the deformation of the piezoelectric surface.

tial equations in the surface normal coordinate y:

$$[d^2/dy^2+k_{ny}^2(y)]u_n=\sum_{m\neq 0} V_m u_{n-m}, \tag{5}$$

where $k_{ny}(y)$, a transverse momentum the same in the laboratory and the SAW rest frames, is given in terms of laboratory variables as

$$k_{ny}^2(y)=k^2\sin^2\theta+2nK(\kappa-sk\cos\theta)-(nK)^2-V_0(y). \tag{6}$$

The $\{V_i(y)\}$ and $\{u_i(y)\}$ are the Fourier coefficients of the expansions of $V(x,y)$ and $u(x,y)$. The $y\to\pm\infty$ values of $k_{ny}(y)$, k_{nyr}, and k_{nyt} give the components of the reflected and transmitted neutron-beam wave numbers normal to the surface. We note that k_c, the critical wave number for total external reflection, is related to $V_0(y)$ by $k_c^2\equiv V_0(-\infty)$.

The boundary conditions for the solution of (5) are

$$u_n(y)=\begin{cases}\delta_{n,0}\exp[-ik_{0yr}y]+r_n\exp[+ik_{nyr}y], & y\to+\infty, \quad (7a)\\ t_n\exp[-ik_{nyt}y], & y\to-\infty, \quad (7b)\end{cases}$$

where $k_{ny}>0$ if real, and $k_{ny}=i\,|k_{ny}|$ if imaginary, corresponding to an evanescent wave. The reflection and transmission coefficients, which apply in both stationary SAW and laboratory frames, are then given by

$$R_n=(k_{nyr}/k_{0yr})\,|r_n|^2, \tag{8a}$$

$$T_n=(k_{nyt}/k_{0yr})\,|t_n|^2. \tag{8b}$$

Under the conditions of our experiment the equation system (5) may be approximated by our ignoring all order other than $n=0,\pm1$ and by our giving the potential the abrupt form

$$V(x,y)=k_c^2H(A\cos Kx-y), \tag{9}$$

where H is the unit Heaviside step function, A is the amplitude of the surface wave, and k_c is the critical neutron wave number for total external reflection. (This assumption is valid provided that the effective neutron-beam wavelengths normal to the surface are greater than both the soft range of the surface potential and the amplitude of the SAW deformation.) With Eq. (9) and the smallness of $k_{ny}A$, convenient computational approximations for V_0 and $V_{\pm1}$ are

$$V_0(y)=k_c^2H(-y), \tag{10a}$$

$$V_{\pm1}(y)=(k_c^2A/2)\delta(y), \tag{10b}$$

which yield

$$R_0=|\{k_{0yr}-k_{0yt}\}/\{k_{0yr}+k_{0yt}\}|^2, \tag{11a}$$

$$R_{\pm1}=A^2k_{0yr}k_{\pm1yr}\,|\{k_{0yr}-k_{0yt}\}/\{k_{\pm1yr}+k_{\pm1yt}\}|^2. \tag{11b}$$

The k_{nyr} are real and positive when a reflected beam is kinematically allowed. The wave numbers, k_{nyr} and k_{nyt}, are given by Eq. (6) and the paragraph following it.

Surface waves were generated on a SAW delay line of conventional geometry consisting of photolithographed interdigitated electrode transducers[7] configured upon an ST-X-cut quartz crystal substrate[8] (see Fig. 1). In order to define the neutron reflection area and to damp any spurious SAW reflections, Gd_2O_3 paint covered the region behind the SAW-generating transducer, as well as the entire region of the receiving transducer. The wave-carrying area available for neutron illumination was 40 mm (in the X propagation direction) by about 5 mm wide. The device was driven at 34.5 MHz, which, for an X-propagating SAW on an ST quartz surface ($u=3158$ m s^{-1}), corresponds to a wavelength of 91.5 μm. At maximum power ($\simeq 100$ mW) the device generated an appropriately averaged SAW amplitude of 13.5 Å over this area as deduced from measurements of diffracted laser light.[2] During the experiment the SAW amplitude was monitored by the measurement of the previously calibrated voltage at the delay-line receiving electrodes.

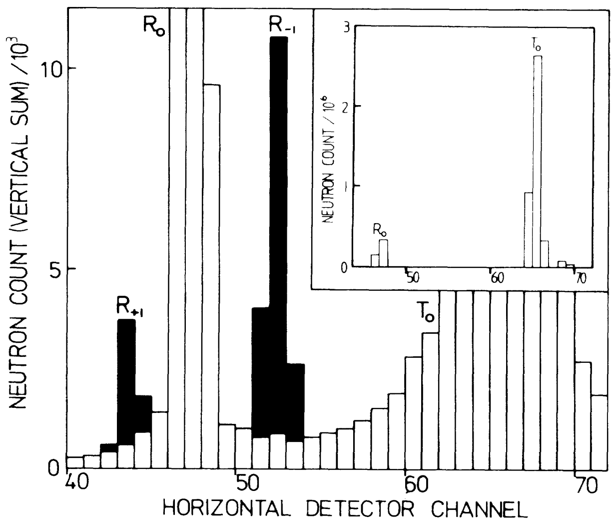

FIG. 2. Typical data set (11.0-Å neutrons, $s=+1$, $\theta=16.3$ mrad). The black areas represent the diffraction signal difference between SAW-on and SAW-off data collection. Inset: Zeroth-order beam structure of the same region.

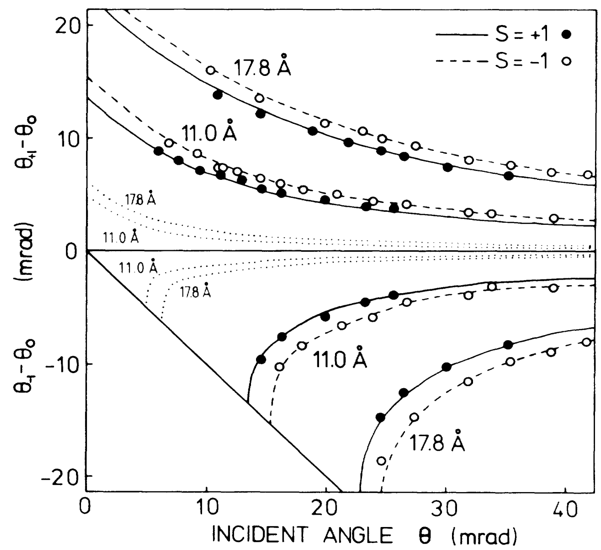

FIG. 3. First-order diffraction angles vs glancing angle of incidence. Our experimental data are compared with theory [Eq. (3)] for the two senses of SAW propagation ($s=\pm 1$). The dotted lines are the predictions of Eq. (3) for a stationary grating. (Error bars lie within data points.)

The experiment was performed at the Institut Laue-Langevin High-Flux Reactor, at neutron wavelengths centered on 11.0 and 17.8 Å (5% FWHM, triangular distribution). The critical wave number for total external reflection of neutrons on quartz is 7.29×10^{-3} Å^{-1}, yielding critical angles (θ_c) for these wavelengths of 12.8 and 20.7 mrad. The SAW device was mounted in critical-reflection geometry[9] on the small-angle scattering spectrometer[10] D17. Incident-beam collimation of about 1 mrad was achieved between a 2-mm slit, situated at the neutron guide-tube exit 2.6 m from the device, and the projected width of the SAW surface, $\simeq 1$ mm at the operating angles. The output counts of the D17 multidetector, located 3.4 m downstream from the device, were summed in the direction perpendicular to the plane of reflection in order to average out the spatial intensity variations of the beam emerging from the guide tube. With a flux of about 10^6 n cm^{-2} s^{-1} a single data set takes several hours of beam time to acquire. The 5-mm channel width of the detector results in a resolution of 1.5 mrad.

To improve the extraction of the diffraction signal from the normal background (and from the sometimes very close and much more intense zeroth-order beams), the detector output and the rf drive to the SAW device were synchronously gated. This produced two detector outputs for comparison, SAW off (no diffraction) versus SAW on (diffraction).

A typical detector output is shown in Fig. 2.

The small-angle form of the SAW diffraction formula, Eq. (3), was tested for both experimental wavelengths, in both SAW propagation senses ($s=\pm 1$), out to several times the critical angle. These results are presented in Fig. 3; within experimental error the agreement with Eq. (3) is excellent. The expected magnification of diffraction angles over the stationary case is clearly demonstrated, as is the dependence upon SAW propagation sense.

The dependence of the diffracted-beam intensity upon SAW amplitude was checked and, within experimental errors, was found to be directly proportional to A^2 as predicted by Eq. (11b).

To test the angular intensity dependence, the diffracted-beam counts were normalized against the reflected-beam counts. The predicted values of $R_{\pm 1}/R_0$ obtained from Eq. (11) for a SAW amplitude of 13.5 Å are compared with the measurement for 11.0-Å neutrons in Fig. 4. The agreement for the 17.8-Å data is equally good. Some discrepancies are evident at higher angles of incidence, as a result no doubt of more direct probing of the actual surface-potential structure by the incident neutrons.

Previous experiments on the diffraction of neutrons by optical gratings,[4–6] while demonstrating the correct diffraction angles, had more significant discrepancies because of shadowing and the sharpness of the grating groove profile. The smoother sinusoidal form of the SAW surface deformation is responsible for the better agreement between theory and experiment in our case.

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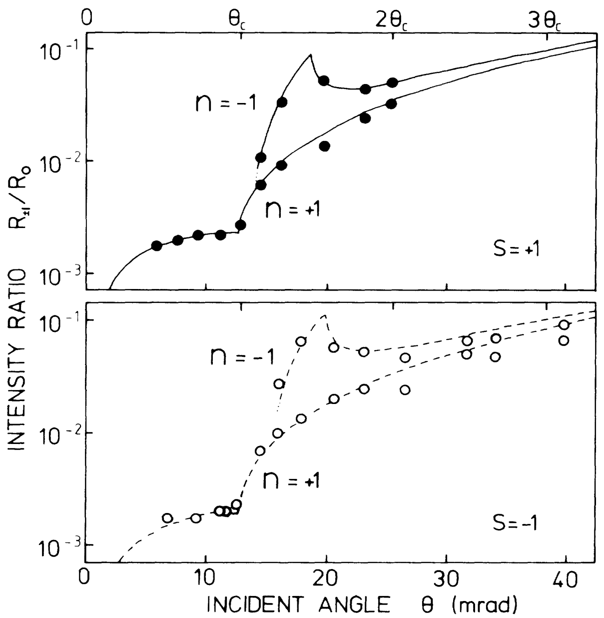

FIG. 4. Ratios of first-order to reflected (zeroth-order) beam intensities vs glancing angle of incidence, for 11.0-Å neutrons for the two senses of SAW propagation.

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