## Wilson Ratio for Kondo Lattices

Zou and Anderson's (ZA) remark[1] that "in the presence of spin-orbit coupling, only one selected linear combination of six local $f$ states can hybridize with a conduction state of a given $\mathbf{k}$ and spin" is unexceptionable. However, their calculation of the magnetic susceptibility using such properly hybridized states runs into both experimental and theoretical problems.

ZA would like to explain why the Wilson ratio—the dimensionless ratio of the $T \rightarrow 0$ magnetic susceptibility and the coefficient of the linear term in the specific heat—of heavy-fermion systems is smaller than that for isolated Kondo impurities. They construct appropriate spin-orbit (s.o.) coupled bands and then calculate the *Pauli susceptibility* with the magnetic moment $\mu_{\text{eff}}$ at the *Fermi surface.* $\mu_{\text{eff}}$ is less than $\mu$, the total $f$ moment because s.o. effects in hybridization lift the orbital degeneracy. For Ce ions ($J=\frac{5}{2}$) they find $\mu_{\text{eff}}^2 \approx 0.18\mu^2$. The Pauli susceptibility is $O(\mu_{\text{eff}}^2/T_{\text{K}})$ where $T_{\text{K}} \approx [N^*(\epsilon_{\text{F}})]^{-1}$ is also the characteristic bandwidth of the heavy-fermion band. But all the hybridization matrix elements in their band-structure calculation are of order $T_{\text{K}}$ (modified by the appropriate Clebsch-Gordan coefficients). Therefore, all the six $f$ bands for $J=\frac{5}{2}$ are within an energy range of $O(T_{\text{K}})$. The interband contribution to $\text{Im}\chi(q,\omega)$, also called the Van Vleck or orbital susceptibility, occurs in an energy range $O(T_{\text{K}})$ itself. This means that the total magnetic moment $\mu$ is recovered in virtual excitations covering an energy range of $O(T_{\text{K}})$ and the interband contribution to the susceptibility is $\approx(\mu^2-\mu_{\text{eff}}^2)/T_{\text{K}}$. ZA have ignored this contribution. The total susceptibility remains essentially unaltered from its value in the absence of s.o. effects in hybridization.

ZA's reduction of susceptibility would arise if there was a parameter besides $T_{\text{K}}$ which pushed some bands to energy $\gg T_{\text{K}}$. In that case the total-moment sum rule would not be satisfied over energies of $O(T_{\text{K}})$. However, experiments[2] on $UPt_3$ and $UBe_{13}$ demonstrate that at low temperatures magnetic fluctuations with energies less than $4T_{\text{K}}$ nearly exhaust the sum rule.

At a more general level, we question the view, taken by ZA and many others, that the essential many-body physics of the Kondo lattice is that of the single-impurity problem. For isolated impurities the Wilson ratio is a universal quantity. Even for the two-Kondo-impurity problem[3] it is nonuniversal, i.e., dependent on the values of the coupling constants including Ruderman-Kittel-Kasuya-Yosida interactions. The magnetic correlations between the local moments affect the susceptibility. Indeed, heavy-fermion systems are characterized by antiferromagnetic correlations[4] between local moments. The sum rules on $\text{Im}\chi(\mathbf{q},\omega)$ imply that if the energy scale for magnetic fluctuations does not vary rapidly with the wave vector, as experiment shows, the bulk susceptibility $\chi_0$ must then be suppressed relative to the corresponding single-impurity value. The explanation of the observed long-wavelength susceptibility, we believe, lies in the interaction effects beyond those in the single-impurity problem.

*Note added.*—In their reply, ZA imply that although calculations using their original model do not support their conclusions, more subtle interpretations would do so. They argue that if one-electron band theory were done first, relatively large interband gaps and therefore a small Van Vleck (as well as Pauli) susceptibility would ensue. Many-body effects are then claimed to lead to a renormalization of the Pauli susceptibility alone. However, since the many-body effects are due to a $U$ parameter which is much larger than even the one-electron band splittings, the Van Vleck term is also renormalized. More generally, our arguments are based on sum rules and survive the order in which different terms in the Hamiltonian are considered.

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