Red Shifts and Blue Shifts of Spectral Lines Emitted by Two Correlated Sources

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It has recently been shown theoretically that correlations between fluctuations of the source distribution at different source points can produce red shifts or blue shifts of emitted spectral lines. To facilitate experimental demonstration of this effect a simple example is analyzed. It involves only two small appropriately correlated sources and brings out the essential physical features of this new phenomenon.

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I showed not long ago that the spectrum of light produced by a fluctuating source depends not only on the source spectrum but also on the correlation that may exist between the source fluctuations at different points within the domain occupied by the source.[1] This result was recently confirmed experimentally.[2] I also showed that under certain circumstances source correlations may produce red shifts or blue shifts of spectral lines in the emitted radiation.[3,4] This prediction has obviously important implications, particularly for astronomy, and it is therefore desirable to verify it also by experiment.

In this Letter I analyze theoretically one of the simplest systems that will generate spectral shifts by this mechanism; namely, two small correlated sources, with identical spectra consisting of a single line of Gaussian profile. I show that with an appropriate choice of the correlation, the spectrum of the emitted radiation will also consist of a single line with a Gaussian profile; however, this emitted line will be red shifted or blue shifted with respect to the spectral line that would be produced if the sources were uncorrelated, the nature of the shift depending on the choice of one of the parameters that specifies the exact form of the correlation coefficient. The main features of this theoretical prediction have been confirmed by Bocko, Douglass, and Knox, using acoustical rather than optical sources. An account of their experiments is given in the accompanying Letter.[5]

Let us consider two small fluctuating sources located at points P_1 and P_2. I assume that the fluctuations are statistically stationary. Let $\{Q(P_1,\omega)\}$ and $\{Q(P_2,\omega)\}$ be the ensembles that represent the source fluctuations[6] at frequency ω. Furthermore, let $\{U(P,\omega)\}$ be the ensemble that represents the field at point P generated by the two sources (Fig. 1). Each realization $U(P,\omega)$ may then be expressed in the form[7]

$$U(P,\omega)=Q(P_1,\omega)\frac{e^{ikR_1}}{R_1}+Q(P_2,\omega)\frac{e^{ikR_2}}{R_2}, \tag{1}$$

where R_1 and R_2 are the distances from P_1 to P and from P_2 to P, respectively, and $k=\omega/c$, c being the speed of light in free space. The spectrum of the field at the point P is given by

$$S_U(P,\omega)=\langle U^*(P,\omega)U(P,\omega)\rangle, \tag{2}$$

where the angular brackets denote ensemble average. On substitution from Eq. (1) into Eq. (2), we find that

$$S_U(P,\omega)=(1/R_1^2+1/R_2^2)S_Q(\omega)+[W_Q(P_1,P_2,\omega)e^{ik(R_2-R_1)}/R_1R_2+\text{c.c.}]. \tag{3}$$

Here

$$S_Q(\omega)=\langle Q^*(P_1,\omega)Q(P_1,\omega)\rangle=\langle Q^*(P_2,\omega)Q(P_2,\omega)\rangle \tag{4}$$

is the spectrum (assumed to be the same) of each of the two source distributions,

$$W_Q(P_1,P_2,\omega)=\langle Q^*(P_1,\omega)Q(P_2,\omega)\rangle \tag{5}$$

is the cross-spectral density of the source fluctuations [first paper of Ref. 6, Eqs. (3.3) and (5.9)], and c.c. denotes the complex conjugate.

The degree of spectral coherence at frequency ω, which is a measure of correlation that may exist between the two fluctuating sources, is given by the formula[8]

$$\mu_Q(P_1,P_2,\omega)=W_Q(P_1,P_2,\omega)/S_Q(\omega). \tag{6}$$

The normalization in Eq. (6) ensures that $0\le|\mu_Q(P_1,P_2,\omega)|\le 1$. The extreme value $|\mu_Q|=1$ characterizes complete correlation (complete spatial coherence) at frequency ω. The other extreme value, $\mu=0$, characterizes complete absence of correlations (complete spatial incoherence).

On substituting for W_Q from Eq. (6) into Eq. (3), we find that

$$S_U(P,\omega)=S_Q(\omega)\{1/R_1^2+1/R_2^2+[\mu_Q(\omega)e^{ik(R_2-R_1)}/R_1R_2+\text{c.c.}]\}, \tag{7}$$

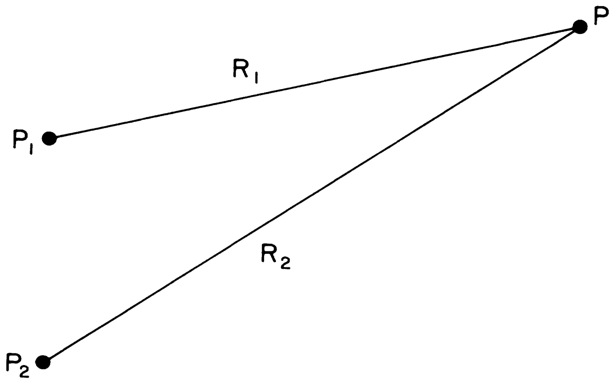

FIG. 1. Geometry and notation relating to the determination of the spectrum $S_U(P,\omega)$ of the field at P produced by two small sources with identical spectra $S_Q(\omega)$ located at P_1 and P_2.

where I have omitted the arguments P_1 and P_2 in μ. For the sake of simplicity, let us choose the field point P to lie on the perpendicular bisector of the line joining P_1 and P_2. Then $R_1=R_2$ $(=R$, say) and formula (7) reduces to

$$S_U(P,\omega)=(2/R^2)S_Q(\omega)[1+\mathrm{Re}\mu_Q(\omega)], \tag{8}$$

where Re denotes the real part.

We note in passing that when either $\mu_Q(\omega)\equiv 0$ (mutually completely uncorrelated sources) or when $\mu_Q(\omega)\equiv 1$ (mutually completely correlated sources), the spectrum $S_U(P,\omega)$ of the field at the point P will be proportional to the spectrum $S_Q(\omega)$ of the source fluctuations. However, in general this will not be the case. In fact, it is clear from formula (8) that the field spectrum may differ drastically from the source spectrum, the difference depending on the behavior of the correlation coefficient $\mu_Q(\omega)$ as a function of frequency.

Suppose now that the spectrum of each of the two sources consists of a single line of the same Gaussian profile,

$$S_Q(\omega)=Ae^{-(\omega-\omega_0)^2/2\delta_0^2}, \tag{9}$$

where A, ω_0, and δ_0 $(\ll\omega_0)$ are positive constants. Suppose further that the correlation between the two sources is characterized by the degree of spectral coherence

$$\mu_Q(\omega)=ae^{-(\omega-\omega_1)^2/2\delta_1^2}-1, \tag{10}$$

where a, ω, and δ $(\ll\omega_1)$ are also positive constants. In order that expression (10) is a degree of spectral coherence, I must also demand that $a\leq 2$. On substituting from Eqs. (9) and (10) into Eq. (8), I obtain the following expression for the spectrum of the field at the point P:

$$S_U(P,\omega)=\frac{2Aa}{R^2}e^{-(\omega-\omega_0)^2/2\delta_0^2}e^{-(\omega-\omega_1)^2/2\delta_1^2}. \tag{11}$$

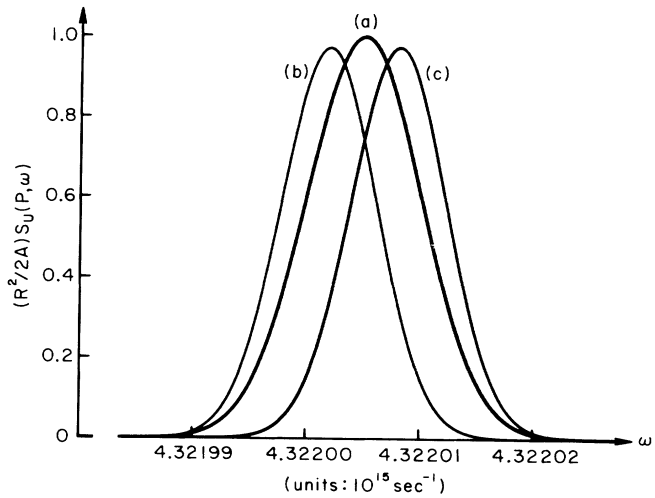

FIG. 2. Red shifts and blue shifts of spectral lines as predicted by formula (12). The spectrum $S_Q(\omega)$ of each of the two source distributions is a line with a Gaussian profile given by Eq. (9) with $A=1$, $\omega_0=4.32201\times 10^{15}$ sec^{-1} (Hg line $\lambda=4358.33$ Å), $\delta_0=5\times 10^9$ sec^{-1}. (a) The field spectrum $S_U(P,\omega)$ at P when the two sources are uncorrelated $(\mu_Q\equiv 0)$. (b),(c) The field spectra at P when the two sources are correlated in accordance with Eq. (10), with $a=1.8$, $\delta_1=7.5\times 10^9$ sec^{-1}, and (b) $\omega_1=\omega_0-2\delta_0$ (red-shifted line), (c) $\omega_1=\omega_0+2\delta_0$ (blue-shifted line).

By straightforward calculation one can show that this expression may be rewritten in the form

$$S_U(P,\omega)=A'e^{-(\omega-\omega_0')^2/2\delta_0'^2}, \tag{12}$$

where

$$A'=(2Aa/R^2)e^{-(\omega_1-\omega_0)^2/2(\delta_0^2+\delta_1^2)}, \tag{13}$$

$$\omega_0'=(\delta_1^2\omega_0+\delta_0^2\omega_1)/(\delta_0^2+\delta_1^2), \tag{14}$$

and

$$1/\delta_0'^2=1/\delta_0^2+1/\delta_1^2. \tag{15}$$

On the other hand, were the two sources uncorrelated, the correlation coefficient μ_Q would have zero value and we would then have, according to Eqs. (8) and (9),

$$[S_U(P,\omega)]_{\mathrm{uncorr}}=(2A/R^2)e^{-(\omega-\omega_0)^2/2\delta_0^2}. \tag{16}$$

Comparison of Eq. (12) with Eq. (16) shows that although both the spectral lines have Gaussian profiles, they differ from each other. Since according to Eq. (15) $\delta_0'<\delta_0$, the spectral line from the correlated sources is narrower than the spectral line from the uncorrelated sources. Further, we can readily deduce from Eq. (14) that

$$\omega_0'\lessgtr\omega_0$$

according as

$$\omega_1 \lessgtr \omega_0.$$

Hence if $\omega_1 < \omega_0$ the spectral line (12) produced by the correlated sources is centered on a lower frequency than the spectral line (16) from two uncorrelated sources, i.e., it is *red shifted* with respect to it; and if $\omega_1 > \omega_0$ the spectral line (12) is *blue shifted* with respect to the spectral line (16). Figure 2 illustrates these results by simple examples.

The preceding considerations show clearly the possibility of generating, by means of correlations between source fluctuations, either red shifts or blue shifts of lines in the spectrum of radiation emitted by sources that are stationary with respect to an observer.

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