# Nonlinear Coupling of Stimulated Raman and Brillouin Scattering in Laser-Plasma Interactions

Harvey A. Rose, D. F. DuBois, and B. Bezzerides
*Los Alamos National Laboratory, University of California, Los Alamos, New Mexico 87545*
(Received 29 December 1986)

Detailed agreement with the experiments of Walsh, Villeneuve, and Baldis is obtained from a theory of the coupling of stimulated Raman scattering and stimulated Brillouin scattering which incorporates the nonlinear physics of the Zakharov model.

PACS numbers: 52.40.Nk, 42.65.−k, 52.35.Mw, 52.35.Ra

Two years ago Walsh, Villeneuve, and Baldis[1] (WVB) reported on experiments in which they observed the interaction of electron density fluctuations associated with stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS). Fluctuations associated with SRS were observed to occur only in a narrow band of densities $0.01 < n_0/n_c < 0.05$, where $n_c$ is the critical density for the $CO_2$ pump laser and $n_0$ is the plasma density at the top of the Gaussian profile; in the experiments $n_0/n_c$ was always less than 0.25.

The equations for our model[2] were given by Aldrich *et al.*[3] and can be derived from standard plasma fluid equations. We assume spatial variation only in the $x$ direction and represent the total electromagnetic vector potential $A^T$ as

$$A^T=[\tfrac{1}{2}A\exp(-i\omega_0 t)+\tfrac{1}{2}A_R\exp(-i\omega_R t)]+\text{c.c.},$$

where $\omega_R=\omega_0-\omega_p$ with $\omega_0$ and $\omega_p$ the incident light frequency and electron plasma frequency of the background density, respectively. The slowly varying envelope amplitudes satisfy

$$[-2i\omega_0\partial_t+(\omega_p^2-\omega_0^2)-c^2\partial_x^2]A=-\omega_p^2(n/n_0)A+(\omega_p^2/8\pi e n_0)(\partial_x E A_{\mathrm{R}}), \tag{1}$$

$$[-2i\omega_R\partial_t+(\omega_p^2-\omega_{\mathrm{R}}^2)-c^2\partial_x^2]A_{\mathrm{R}}=-\omega_p^2(n/n_0)A_{\mathrm{R}}+(\omega_p^2/8\pi e n_0)A\partial_x E^*. \tag{2}$$

The nonlinear source terms on the right-hand sides of these equations contain the couplings responsible for SBS and SRS.

The slowly varying envelope $E$ of the electrostatic field satisfies

$$(-2i\omega_p\partial_t+\omega_p^2 n/n_0-3v_e^2\partial_x^2)E=-\tfrac{1}{2}(\omega_p^2/c^2)(e/m)\partial_x(AA_{\mathrm{R}}^*)+\delta S_E(x,t). \tag{3}$$

The conditions for validity of these equations are discussed in more detail in Ref. 3 and include the condition $n/n_0\ll 1$. The low-frequency density *fluctuation*, $n$, obeys

$$(\partial_t^2-c_S^2\partial_x^2)n=(16\pi m_i)^{-1}\partial_x^2[|E|^2+(\omega_p^2/c^2)(|A|^2+|A_{\mathrm{R}}|^2)]+\delta S_n(x,t). \tag{4}$$

In the present study the left-hand sides of Eqs. (3) and (4) are augmented by collision operators $-2i\omega_p\times v_e(-i\partial_x)$ and $2v_i(-i\partial_x)\partial_t$, respectively. The function $v_e(k)$ is chosen to be a simple polynomial fit to the sum of collisional[4] plus Landau damping for Langmuir waves, and likewise $v_i(k)$ and $c_S^2$ [in Eq. (4)] are fitted by the ion acoustic roots of the collisionless-plasma dispersion relation for parameters of the experiment $(m_i/Zm_e=5500,\ T_e/T_i\cong 1, Z\cong 4)$.

An important departure from Ref. 3 and other works on Langmuir turbulence is that the calculations presented here are for *finite*, slab geometry. In this case we have found it useful to make a further decomposition of the vector potentials into slowly varying spatial envelopes $A=a_0\exp(ik_0x)+a_{\mathrm{B}}\exp(ik_{\mathrm{B}}x)$ and $A_{\mathrm{R}}=a_{\mathrm{R}}\exp(ik_{\mathrm{R}}x)$, where $k_j$'s ($j=0$,B,R) are wave vectors of the incoming laser pump and backscattered Brillouin and Raman light, respectively. The $a_j$'s are assumed to be slowly varying in space over a distance $2\pi/k_j$ and their second spatial derivatives are neglected. In the slab geometry considered here the pump wave is incident from the left at $x=0$ with amplitude specified to by *an increasing function of time:* $a_0(x=0,t)=a_{\max}(t/t_{\mathrm{rise}})^{1/2}$ for $t<t_{\mathrm{rise}}$. This corresponds to the WVB experiments where the pump power is a linearly increasing function of time with $t_{\mathrm{rise}}\cong 500$ ps. The laser field then propagates[5] to the right through the region of uniform plasma $0\le x\le L$. The boundary conditions are taken to be $a_{\mathrm{B}}(x=L,t)=a_{\mathrm{R}}(x=L,t)=0$, outgoing ion and Langmuir waves; for the latter $\partial_x E=\pm ik_E E$ at $x=L,0$, respectively, where $k_E$ is the wave number of the fastest (linear) growing Langmuir mode with group velocity $v_E$. These boundary conditions eliminate reflections in the mode $k_E$ and have little effect on the nonlinear processes which are local in space. The equations are nu-

merically solved with a general one-dimensional partial-differential-equation solver.[6]

In the experiments of WVB there are experimental and theoretical grounds to expect that the density fluctuations are excited from *thermal* equilibrium levels. Because SRS and SBS take a finite time to develop, we have found it necessary to excite the system continuously with thermal noise sources denoted by $\delta S_E$ and $\delta S_n$ in Eqs. (3) and (4).

Only the SRS and SBS active modes are noise driven. In our one-dimensional model these are in a band of wave numbers about $k_E$ $(2k_0)$ with width $\Delta k_{\mathrm{R\,(B)}} = l_{\mathrm{R\,(B)}}^{-1}$, where $l_{\mathrm{R\,(B)}}$ is the transient SRS (SBS) convective gain length to be defined in Eq. (6), evaluated at $t \simeq 100$ ps. The noise amplitudes $\delta S_E(k,t)$ $[\delta S_n(k,t)]$ of these modes are taken to be consistent with thermal fluctuations; i.e., so that in the *linearized* versions of Eqs. (3) and (4) the averaged responses $\langle |E(k)|^2 \rangle$ and $\langle |n(k)|^2 \rangle$ would be the proper thermal equilibrium values for a *three-dimensional system.*[7] In the numerical calculations a number of independent noise modes of the order of 100 was generated by use of a pseudo-random-number generator, each mode actually representing "colored" noise with a frequency spectrum

$$\langle |\delta S_{E\,(n)}(\omega)|^2 \rangle \propto \Gamma^8(\omega^2+\Gamma^2)^{-4},$$

with

$$\Gamma > 20\gamma_{\mathrm{R\,(B)}}, \nu_e, \nu_i.$$

For the purpose of calculating linear growth rates, an additional slowly-varying-spatial-envelope approximation for $E$ and $n$ can be made with respect to the wave vectors of the linearly matched modes. For a system excited by white-noise sources uniformly distributed over the interval $0 < x < L$ the fluctuations in $E$ and $n$ near $x=0$ can be shown to grow in time as

$$\langle |E(x,t)|^2 \rangle \propto (\gamma_{\mathrm{R}} t)^{-1} \exp(2\gamma_{\mathrm{R}} t), \tag{5}$$

$$\gamma_{\mathrm{R}} = \nu_{\mathrm{R}} (2L/ct_{\mathrm{rise}})^{1/2} - \nu_e(k_E),$$

where $\nu_{\mathrm{R}}$ is the homogeneous SRS growth rate[3] with a similar expression for $\langle [n(x,t)]^2 \rangle$ with $\nu_{\mathrm{R}} \to \nu_{\mathrm{B}}$, $\nu_e(k_E) \to \nu_i(2k_0)$ which is valid when $\gamma_{\mathrm{B}} < 2k_0 c_S$. These expressions are valid in a "transient convective regime," where[7] $L/c < t_{\mathrm{R\,(B)}} < t < L/v_E$ $(c_S)$. The nonlinear phenomena discussed below occur for times within this regime. This convective response is exponentially growing in time only after a transient period $t > t_{\mathrm{R\,(B)}} \simeq (\gamma_{\mathrm{R\,(B)}} + \nu_{e\,(i)})^{-1}$. Thus another novel effect treated here is that nonlinear *saturation occurs before the system reaches the linear asymptotic regime usually associated with absolute instability.* The convective gain length mentioned above is found from the analysis leading to Eq. (5) to be

$$l_{\mathrm{R\,(B)}} \cong (2ct_{\mathrm{rise}}L)^{1/2} (\nu_{\mathrm{R\,(B)}} t)^{-1}. \tag{6}$$

In Fig. 1 we present a summary of data from a large number of computer runs of the probability $P(W > W_0)$, as a function of $n_0/n_c$, that the Langmuir fluctuation energy, $W$, in the leftmost portion of the slab will exceed a value $W_0$, which is above the thermal level by a certain factor. This local Langmuir energy is averaged over about 3 wavelengths of the pump laser. For fixed values of $n_0/n_c$ and other macroscopic parameters there are large fluctuations in the Langmuir response due to fluctuations between realizations of the low-frequency thermal noise which enter into the exponential dependence of the Langmuir response on the SRS growth rate, which is modified by the SBS fluctuations as discussed below. The rms fluctuation levels are indicated by error bars in Fig. 1; these are largest in the transition region of $n_0/n_c$ between $P(W > W_0) \cong 1$ and $P(W > W_0) \cong 0$.

For the slab of length $L \simeq 15\lambda_0$ and $v_{\mathrm{osc}}/c = 0.035$, we see that Langmuir fluctuations at a level about $10^3$ above the thermal level will occur with the probability of 90% or more only in the range $0.01 \pm 0.005 < n_0/n_c < 0.040 \pm 0.005$. Increasing the pump power by a factor of 4 to $v_{\mathrm{osc}}/c = 0.07$ moves the upper transition density to $n_0/n_c = 0.050 \pm 0.005$. This transition density appears to correlate with the condition $\gamma_{\mathrm{R}}/\gamma_{\mathrm{B}} \approx 2.0$. The ratio $\gamma_{\mathrm{R}}/\gamma_{\mathrm{B}}$ decreases with increasing density because of the decrease of $\nu_{\mathrm{R}}$ which is proportional to $k_E^{1/2}$, the increase of collision damping in $\nu_e(k_E)$, and the experimentally

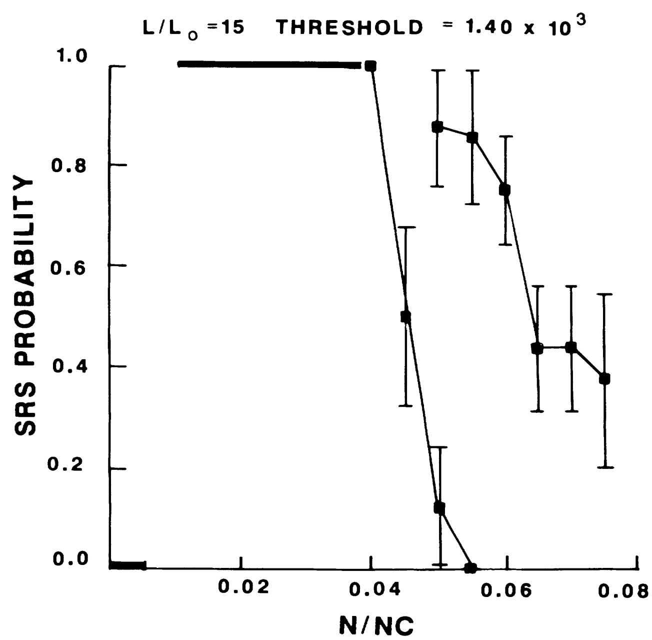


FIG. 1. Probability $P(W > W_0)$ that Langmuir energy in the leftmost portion of slab exceeds the thermal energy by a factor of 1400, vs $n_0/n_c$. Note: For $n_0/n_c \lesssim 0.01$ this probability drops to zero because of strong Langmuir damping. The curve to the left is for $V_{\mathrm{osc}}/c = 0.035$ and the curve on the right for $V_{\mathrm{osc}}/c = 0.070$.

observed shortening of the plasma scale length with increasing density. For $n_0/n_c \lesssim 0.01 \pm 0.005$, $\gamma_R/\gamma_B$ quickly decreases because of strong Landau damping in $\nu_e(k_E)$; we have not calculated detailed probabilities at these lower densities but there seems to be little question that the dropoff of Raman fluctuations at lower densities is due to Landau damping.

In Fig. 2 the spatial profiles of the $|E|^2$, $n$, and $|A_R|^2$ are shown at various times for $n_0/n_c = 0.045$. At early times the spatial modulation of $|E|^2$ has the same periodicity as the unperturbed SBS sound wave; this is a "Bloch wave"[3] of the Langmuir field in the periodic "potential" of the sound wave with an overall spatial modulation determined by the boundary conditions and finite damping. In Fig. 2(a) we see the ponderomotive force of $E$ field beginning to dig self-consistent density wells in the process of driven collapse.[3,8] The collapsing field is strongly dissipated at this stage because of the rapid increase of $\nu_e$ with $k$ and because of ion inertia,[8] the $E$ field "burns out" in the cavity[8] leaving density cavities in Fig. 2(b) unsupported by a ponderomotive force. Note that the strongest Langmuir fluctuations somewhat away from $x=0$ reach the collapse and burnout stage earliest, followed by the collapse and burnout of the neighboring cavitons on either side. The maximum $|E|^2$ attained in these simulations corresponds to an electron-density fluctuation of about 10% which is to be compared with WVB's estimate of 1%, which is, however, a *temporal and spatial average*.

Figure 2(c) shows the time histories of the normalized total Langmuir energy, the total energy in low-frequency fluctuations, and the Brillouin backscattered signal $|A_B(x=0,t)|^2$. The Langmuir fluctuations associated with SRS are excited in a short pulse, as seen experimentally, which is terminated by Langmuir collapse and burnout. The low-frequency fluctuations and backscattered signal associated with SBS follow the Langmuir pulse in time. *In this regime the SRS fluctuations are*

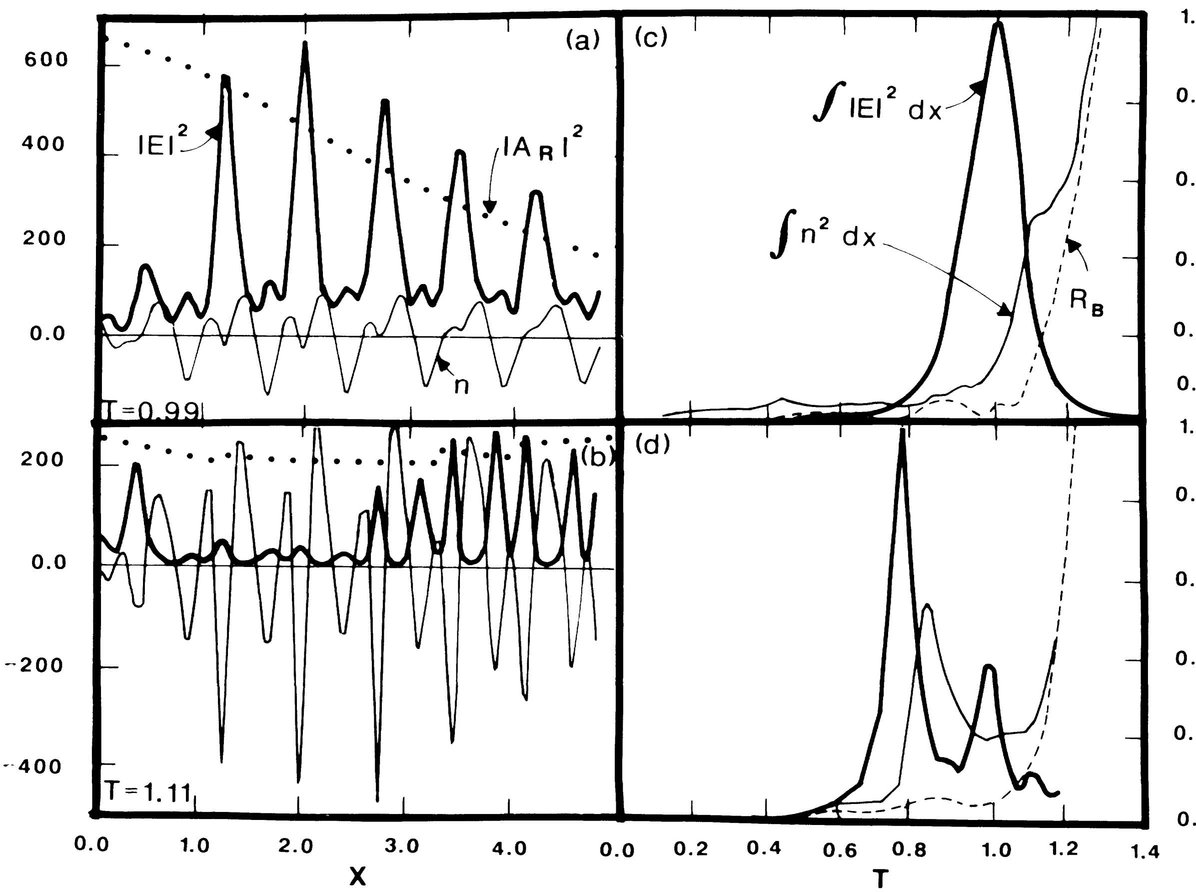


FIG. 2. (a),(b) Spatial profiles of $|E|^2$, $n$, and $|A_R|^2$ for various times for $n_0/n_c = 0.045$, $L = 15\lambda_0$, and $V_{osc}/c = 0.035$. The units are $4\pi n_0 T_e/459$ for $|E|^2$, $n_0/2754$ for $n$, and arbitrary units for $|A_R|^2$, $91\lambda_{De}$ for the spatial unit, and 140 psec for the temporal unit. The total slab length was $15\lambda_0 \simeq 150$ $\mu$m but we show only the leftmost portion $0 \le x \lesssim 3\lambda_0$. (c) Temporal history of total Langmuir energy, total ion wave energy, and SBS backscatter reflection coefficient (in arbitrary units) for the case above. The unit of time is 150 ps. (d) Same but for $V_{osc}/c = 0.07$, $n_0/n_c = 0.055$. Here the unit of time is 132 ps. The mean square thermal fluctuations were $\langle |E|^2 \rangle_{thermal} \simeq 0.1$ and $\langle n^2 \rangle_{thermal} \simeq 900$.

*terminated by collapse and the density fluctuations remaining from collapse provide an enhanced noise from which SBS then grows.* The scenario in this density regime is different than the conjecture by WVB that it is the SBS fluctuations which turn off the SRS. In the present calculations we cannot follow the subsequent behavior of SBS since nonlinearities which will ultimately saturate SBS are not included in our model. The SBS fluctuations following collapse will strongly suppress any further SRS activity because of the detuning mechanism discussed below. We note that for the parameters chosen in Figs. 2(c) and 2(d) the timing of the SRS and SBS pulses are in agreement with the experiment. In Fig. 2(d) we show the time histories for a higher value of $v_{\rm osc}$ for which the SRS pulse begins earlier and is shorter in duration in accord with the observations. The overall scenario in Figs. 2(a)–2(d) is typical of the SRS-active density regime seen in Fig. 1. For intermediate densities between the transition zones $n_0/n_c \cong 0.005$ and $n_0/n_c \simeq 0.04$ we can follow the growth of the collapsing cavitons to levels corresponding to the "detection threshold" $W_0$ using the Zakharov model and, therefore, the curves $P(W > W_0)$ are accurately predicted by this model. However, the actual fluctuation levels ultimately obtained in collapse are not accurately predicted by this model except in the transition zones where SRS is relatively weak.

The slab model is a crude approximation to the measured Gaussian density profile and the associated (but unmeasured) velocity profile. We have found the best fit for the data to occur for $L \simeq 15\lambda_0$. Preliminary calculations which account for the actual profiles indicate *growth rates* compatible with a slab of this length although the actual spatial region of Langmuir activity can be significantly larger.

For higher densities, $\gamma_{\rm R}/\gamma_{\rm B}$ decreases and *SRS is detuned by the growing SBS density waves.*[3] We should note that we observe that time-stationary density ripples with a spatial envelope appropriate to SBS in slab geometry have relatively little effect on the SRS *growth rates* while significantly altering the spatial Langmuir wave forms and the associated "Bloch wave" frequency dispersion $\lambda_{k_E}(t)$. However, the exponential growing SBS density waves do have a significant effect on SRS growth rates[3] when the time-dependent SRS frequency mismatch $\Delta\omega = \omega_0 - \omega_{\rm R} - \lambda_{k_E}(t)$ increases by an amount comparable to the SRS growth rate $\gamma_{\rm R}$ during an $e$-folding time $\gamma_{\rm R}^{-1}$; i.e., when $(d\Delta\omega/dt)\gamma_{\rm R}^{-1} \geq \gamma_{\rm R}$. If this condition is satisfied at a time before the Langmuir waves have reached a significant amplitude then the SRS instability is detuned with no further Langmuir wave growth or collapse.[7] A spatially modulated pattern for $|E|^2$ similar to that in Fig. 2(a) is observed which does not grow to a large amplitude or collapse as in Fig. 2(b).

If the amplitude of $n(t)$ can be externally enhanced by our applying a signal at $x = L$ in the backscattered mode then SRS could be further suppressed. We have found that with the boundary condition $a_{\rm B}(x=L,t) = fa_0(x=0,t)$ for $f^2 > 0.001$ the SRS response can be suppressed altogether for the conditions of Fig. 1 for $n_0/n_c = 0.04$.

In conclusion we assert that this model of the nonlinear coupling of SRS to SBS is in remarkable agreement with the WVB experiments. We have shown here and in Ref. 3 that the competition between SRS and SBS is controlled by the ratio of the *actual* growth rates $\gamma_{\rm R}/\gamma_{\rm B}$, whatever they may be.

We gratefully acknowledge useful conversation with H. Baldis, J. Bernard, D. Forslund, C. M. McKinstrie, W. Rozmus, and D. Villeneuve. This research is supported by the U.S. Department of Energy and partially supported by U.S. Office of Naval Research Contract No. DP-F85-28.

[1] C. J. Walsh, D. M. Villeneuve, and H. A. Baldis, Phys. Rev. Lett. **53**, 1445 (1984).

[2] V. E. Zakharov, Zh. Eksp. Teor. Fiz. **62**, 1745 (1972) [Sov. Phys. JETP **35**, 908 (1972)].

[3] C. M. Aldrich, B. Bezzerides, D. F. DuBois, and Harvey A. Rose, Comments Plasma Phys. Controlled Fusion **10**, 1 (1986). (Note a slight change of notation between these papers: $A_{\rm R}$ in the present work equals $A_-$ in the earlier work.)

[4] We have used the expression for collisional damping $\nu_e = \frac{1}{2}\nu_{ei}$ using the form for $\nu_{ei}$ given by R. E. Turner *et al.*, Phys. Rev. Lett. **54**, 189 (1985).

[5] We generally find that pump depletion is negligible in the regimes considered in this work.

[6] James M. Hyman, Los Alamos National Laboratory Report No. LA-7595-M, 1979 (unpublished).

[7] Harvey A. Rose, D. F. DuBois, and B. Bezzerides, to be published.

[8] G. D. Doolen, D. F. DuBois, and Harvey A. Rose, Phys. Rev. Lett. **54**, 804 (1985); David Russell, D. F. DuBois, and Harvey A. Rose, Phys. Rev. Lett. **56**, 838 (1986).