Stability of Modulated Couette Flow

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The stability of modulated Couette flow is studied by modulation of the angular velocity, Ω, of the inner cylinder according to $\Omega = \bar{\Omega}[1+\epsilon\cos(\omega t)]$, where $\bar{\Omega}$ is the mean angular velocity of rotation. ω is varied for several radius ratios, and ϵ is varied for one radius ratio. The results are compared to earlier experimental and theoretical results.

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The stability of modulated hydrodynamic systems has attracted increasing attention during the past several years. Much theoretical and experimental work has been done on modulated Bénard convection[1-5] and modulated Couette flow,[6-14] as well as more general unsteady flows.[15] These problems are important since the unmodulated Bénard and Couette systems are widely studied, and because of the occurrence of modulated hydrodynamics in nature, e.g., blood flow in the circulatory system and the diurnal heating and cooling of planetary atmospheres. The particular Couette-flow problem of a stationary outer cylinder and sinusoidally modulated rotating inner cylinder described by $\Omega = \bar{\Omega}[1+\epsilon\cos(\omega t)]$ was first investigated by one of us.[6,7] The data were interpreted to indicate that under certain conditions modulation stabilizes the flow, a result which conflicted with subsequent theories.[9,11-14] For this experiment stabilization or destabilization is characterized by the value of $\bar{\Omega}_c$, the mean angular velocity of the inner cylinder at which Taylor vortex flow begins. Donnelly used an electrochemical technique to measure the radial component of Taylor vortices, u_r, and took as his critical angular velocity that which corresponded to a change in the rate of growth of u_r from linear dependence on $\bar{\Omega}$ to a Landau-law dependence in which u_r is proportional to $(\bar{\Omega}^2 - \bar{\Omega}_c^2)^{1/2}$. He stated, however, that transient vortices, which appeared and disappeared during a modulation cycle, were present for a range of values of $\bar{\Omega}$ less than $\bar{\Omega}_c$. A linear dependence of u_r on $\bar{\Omega}$ followed by a square-root dependence has been explained by Hall.[10] Theories predict the onset of Taylor vortex flow which may occur at any point in a modulation cycle, and which may be transient. We have performed the current experiment to examine the details of the onset of transient vortex flow. Since there has not been complete agreement among different theories or between theory and experiment, we have considered it a matter of high priority to conduct a decisive experiment.

In this Letter we describe a new modulation experiment utilizing knowledge of the importance of ramping rate and employing better control and electronic detection methods than have previously been used. A diagram of the apparatus is given in Fig. 1. A Hewlett-Packard model 3325A synthesizer/function generator is set by computer to ramp its dc and ac output voltages with a given frequency and ϵ at a predetermined rate. This signal is sent to a voltage-controlled oscillator which drives a Superior Electric model 061-FD-312 stepping motor. Both the inner cylinder (shaded) and a Dynamics Research model C152 optical encoder are connected to the motor. The encoder outputs 1000 pulses/revolution to a Hewlett-Packard model 5328A universal counter, which determines the mean rotation rate $\bar{\Omega}$ by integrating over an integral number of modulation cycles and outputs this value to the computer. The inner and outer cylinders are kept in a water bath whose temperature is controlled by a Neslab Endocal model RTE-8 refrigerated circulating bath (not in diagram). The cylinder bath is stirred continuously and the temperature maintained to within 0.01 °C over its entire volume. The temperature, measured by a Hewlett-Packard model 2804A quartz thermometer, is important in determining the viscosity.

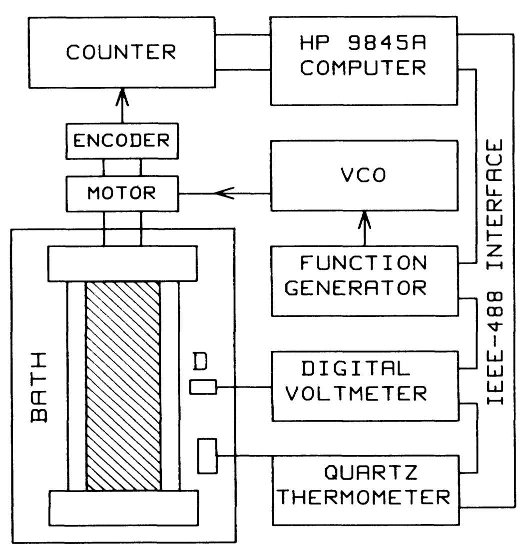

FIG. 1. Diagram of the apparatus for the modulated Couette experiment.

The working fluid is typically 35% glycerol, 4% Kalliroscope rheoscopic fluid, 1% stabilizer, and 60% distilled water. Viscosity measurements are made, by use of Cannon-Fenske viscometers No. 50 and No. 100, at two temperatures very close to the temperature at which the experiment will be run. A linear fit to these values is used to determine small changes in viscosity due to temperature drift recorded during the experiment, so that we can calculate the Taylor number, $N_{\rm Ta}=2\bar{\Omega}^2 R_1 d^3/\nu^2$, where $\bar{\Omega}$ is the mean angular velocity of the inner cylinder, R_1 and R_2 are the cylinder radii, $d=R_2-R_1$, and ν is the kinematic viscosity. The viscosity measurements which are made before and after the experiment may differ by as much as 1%, but the results of our experiments are repeatable over a period of weeks. This implies that changes in viscosity, which result from the experiment, occur very soon after the fluid is put into the annulus, and that the viscosity of the fluid during the experiment corresponds to the measurements taken after the fluid is removed from the apparatus. The final viscosity measurement is used in the analysis, and T_c is recalculated accordingly.

The onset of Tayor vortex flow is detected by a Texas Instruments model TIL139 source and sensor assembly, which emits an infrared beam that is reflected by Kalliroscope flakes[16] in the fluid and detected by a photocell. The flakes are $6\times30\times0.07$-μm^3 guanine platelets. In laminar flow they align broadside to the detector, and give a high reflectance of the infrared beam. When vortices appear the flakes acquire a new orientation and the reflectance, which is averaged over an integral number of modulation cycles, drops dramatically. Figure 2 shows the change in average reflectance signal, at the onset of

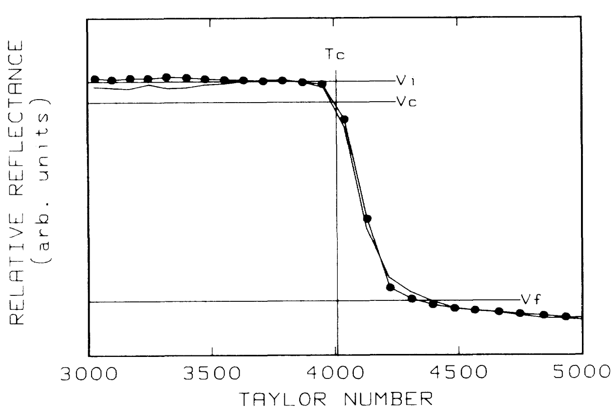

FIG. 2. Results of run for $\eta=R_1/R_2=0.88$, $\epsilon=0.5$, $\omega=3.142$ rad/sec. Data corresponding to increasing Taylor number are indicated by the line with dots. Data for decreasing Taylor number are indicated by the plane line. Note that T_c was recalculated when the viscosity was recalculated (see text). V_i, V_f, and V_c are the reflectances before transition, after transition, and at critical, respectively. The drop from V_i to V_f corresponds to about 20% of the magnitude of V_i.

Taylor vortex flow, which is measured as a function of Taylor number. We record the value of reflectance before and after the transition, and take as T_c the value corresponding to a 10% drop in reflectance from its upper to its lower value. In some runs the upper or lower value is somewhat unclear, and this uncertainty is reflected in our error bars. Other contributions to error are uncertainty in the viscosity, uncertainty due to differences in T_c when runs are repeated, and the difficulty in interpolating the $N_{\rm Ta}$ at which the critical reflectance occurs.

A number of checks were made on the functioning of the apparatus. $\bar{\Omega}$ was visually measured over an integral number of modulation cycles. Agreement between electrical and visual measurements was within 0.2%. T_c was also verified visually.

There is a small distortion in the modulation wave form. During a modulation cycle the maximum Ω is at most 0.3% greater and the minimum Ω at most 0.3% less than they would be if the wave form were a perfect sine wave. The detector was generally placed close to the middle of the annulus. Several runs were made with the detector at different locations. It was found that results changed only when the detector was more than $\frac{3}{4}$ of the way from the middle to one end of the annulus. Measurements were made for a range of aspect ratios, $\Gamma=L/d$, where L is the height of the annulus. For $\eta=R_1/R_2=0.719$, Γ was varied from 32.14 to 32.87, for $\eta=0.95$, Γ was 167.95, while for $\eta=0.88$, Γ was varied from 58.4 to 69.31. These changes in Γ caused no detectable change in the measured T_c.

The abruptness of the transitions was explained in the following way. When the vortices first appear, they are present for only part of a cycle, i.e., they are transient vortices. It takes the Kalliroscope flakes some time to reorient to the laminar flow, however, and if the period of modulation is less than this time an image of the vortices, which we call a ghost, remains after secondary flow has ceased. A measurement of the time it takes for this ghost to disappear was made without modulation for $\eta=0.95$ by discontinuously lowering $N_{\rm Ta}$ from 3762 to 3191, i.e., from the Taylor vortex regime to the laminar regime. It took 9 sec for reflectance to rise to its saturated value. With the other radius ratios shear in the annulus was less and it took even longer for the flakes to realign. The detected transition is, as a result, abrupt and unmistakable.

Two phenomena may compete to move the detected T_c away from its theoretical value. The first is the pumping of Taylor vortices by the Ekman vortices at the ends. This tends to lower T_c from the value it would have with infinite cylinders. Within the limits given above, however, changes in Γ have a negligible effect on measured T_c. The second is the limit of any detection system. The instability must grow to finite size to be detectable. This moves the measured T_c to a higher value.

Park, Crawford, and Donnelly have reported on the

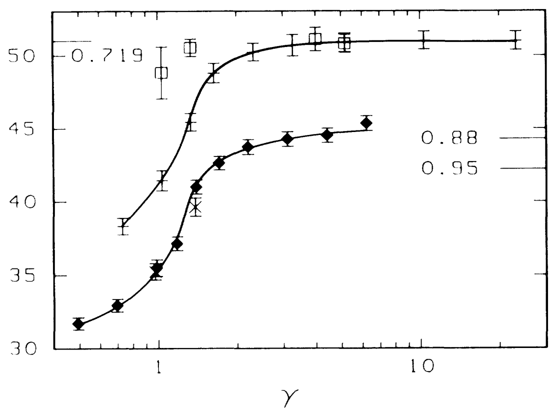

FIG. 3. Critical Reynolds number, $R_c=\bar{\Omega}_c R_1 d/\nu$, as a function of $\gamma=(d^2\omega/2\nu)^{1/2}$ for $\epsilon=0.5$, $\eta=0.719$ (plusses), $\eta=0.88$ (lozenges), and $\eta=0.95$ (crosses); and for $\epsilon=0.2$, $\eta=0.719$ (squares). Cubic splines are fitted to the data. Horizontal lines on the sides of the graph indicate critical Reynolds numbers for no modulation for each value of η (see text).

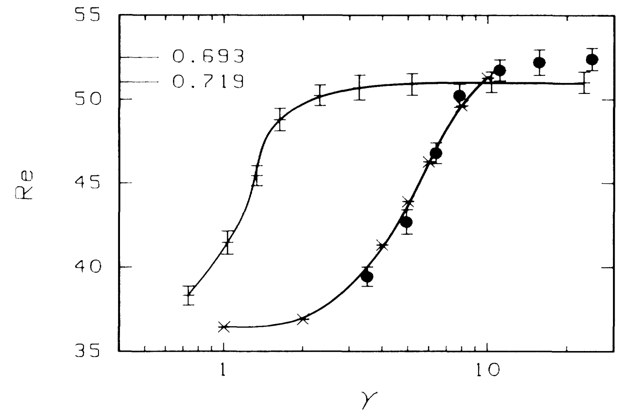

FIG. 4. R_c as a function of γ for $\eta=0.719$, $\epsilon=0.5$ before normalization (plusses), and after normalization and shift in γ (circles) (see text), and Carmi and Tustaniwskyj's theoretical values (Ref. 14) for $\eta=0.693$, $\epsilon=0.5$.

importance of ramping rates in Couette experiments.[17] In our experiments we took measurements while holding N_{Ta} constant, quasistatically ramping between measurements. After surpassing T_c we quasistatically decreased N_{Ta} and repeated our measurements. If we ramped too quickly, hysteresis resulted, its magnitude increasing with ramping rate. Following Park *et al.* we adopted a dimensionless ramping rate $a^*=(R_1 d^2 L/\nu^2)^*(d\bar{\Omega}/dt)$. Our method put an upper limit on a^*, since it set ramping rate between measurements exclusive of the time spent making measurements. A typical upper limit for a^* was 6.

Our results resemble the predictions of Carmi and Tustaniwskyj,[14] and so for consistency we convert T_c to critical Reynolds number, $R_c=\bar{\Omega}_c R_1 d/\nu=(T_c/2)^{1/2}$. Runs were made with three different inner cylinders: $\eta=0.95$, 0.88, 0.719, $R_2=2.54$ cm. For each inner cylinder a run was made without modulation, and several runs were made with different values of ω for a given ϵ. Critical values are plotted as a function of $\gamma=(d^2\omega/2\nu)^{1/2}=d/\delta$, where δ is the Stokes layer thickness. Figure 3 shows our results for three radius ratios with $\epsilon=0.5$ and one with $\epsilon=0.2$. The solid lines on the sides of the graph give the critical Reynolds numbers for no modulation, $R_c^0(\eta=0.719)=(51.02\pm1.2)\%$, $R_c^0(\eta=0.88)=(44.28\pm1.13)\%$, and $R_c^0(\eta=0.95)=(42.2\pm1.45)\%$. Results are qualitatively independent of η, i.e., the curves shift with R_c^0. Figure 4 compares our results for $\eta=0.719$, $\epsilon=0.5$, to the results of Carmi and Tustaniwskyj for $\eta=0.693$, $\epsilon=0.5$. In this figure we have also normalized our values for $\eta=0.719$, $\epsilon=0.5$, to values for $\eta=0.693$, $\epsilon=0.5$, by multiplying all R_c by $R_{c\mathrm{th}}^0/R_c^0$, where $R_{c\mathrm{th}}^0$ is Carmi and Tustaniwskyj's critical value for no modulation, $R_{c\mathrm{th}}^0=52.53$. We then multiplied the experimental γ by 4.8. The normalization gives values that we would expect to get if we used Carmi and Tustaniwskyj's η, 0.693. Multiplying γ by 4.8 (or equivalently, multiplying ω by 23) places these values on the curve connecting Carmi and Tustaniwskyj's values. We see that the qualitative shape of our curve is the same as that of Carmi and Tustaniwskyj, but that our results for a given R_c are shifted to lower γ (equivalently, for a given γ we find a transition at a higher R_c). This shift may be due to the finite amount of time which is needed for a disturbance to grow to a detectable size, i.e., a longer period than that given by the theoretical γ is required to detect a given R_c. It is clear that, according to both the theory and experiment, modulation destabilizes the flow, the amount of destabilization depending on γ. In both cases it is found that, at high γ, modulation has little effect on the flow. The disturbance due to modulation is confined to a small region close to the inner cylinder, and the bulk of the fluid is unaffected. For low γ the instability is able to grow if the instantaneous Reynolds number, N_{Re}, exceeds R_c^0 for long enough during the modulation cycle. This can be understood in two ways. First, a disturbance which can grow in steady flow at a given N_{Re} will be able to grow in the quasisteady flow. Second, the modulation-induced viscous wave, which propagates across the gap, causes a distortion in the centrifugal gradient which would be present in the absence of modulation. At slow enough modulations, however, the wave has very little phase difference across the gap and the centrifugal gradient which allows growth of an instability is effectively the same as it would be at that instantaneous N_{Re} in the absence of modulation. We therefore see an asymptotic limit of R_c at low γ of $R_c^0/(1+\epsilon)$ as predicted by Carmi and Tustaniwskyj,[14] and found by Thompson[8] in his study of the asymptotic frequency limits.

A short visual study of the stability against transition to wavy vortex flow as a function of modulation was performed. Some interesting problems were encountered in this study. There is no simple signature on a spectrum analyzer by which one can recognize the appearance of the wavy mode, since the angular velocity of the waves varies as the inner cylinder speed varies. In addition, after the Taylor vortices appear, they shift slightly to achieve a stable configuration. If the time required for this stable configuration to be reached is large compared to the time during which Taylor vortex flow and wavy vortex flow are present, it is difficult to distinguish the vortex shift and waves on the vortices. Preliminary results at low enough frequency indicate that modulation is destabilizing with respect to wavy vortex flow in the same way in which it is destabilizing with respect to Taylor vortex flow. The wavy mode is transient and appears at a lower mean angular velocity of the inner cylinder with modulation than without.

The authors feel that it would be fruitful to pursue this course of study for a greater parameter space, particularly for rotation of the outer cylinder with and without modulation.

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