Resonance Fluorescence from an Atom in a Squeezed Vacuum

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The fluorescent spectrum for a two-level atom which is damped by a squeezed vacuum shows striking differences from the spectrum for ordinary resonance fluorescence. For strong coherent driving fields the Mollow triplet depends on the relative phase of the driving field and the squeezed vacuum field. The central peak may have either subnatural linewidth or supernatural linewidth depending on this phase. The mean atomic polarization also shows a phase sensitivity.

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The generation of broad band squeezed light has recently been reported using four-wave mixing in atomic vapors[1] and optical fibers,[2] optical parametric oscillation,[3] and optical bistability.[4] In the parametric oscillator a reduction of fluctuations by 60% from the normal vacuum level has been achieved. In this Letter we investigate the spectroscopic properties of an atom interacting with a broad-band squeezed vacuum field.[5,6] Gardiner has considered the radiative decay of a two-level atom interacting with such a squeezed vacuum.[7] He showed that the two polarization quadratures are damped at different rates—one at an enhanced rate and the other at a reduced rate compared to normal radiative decay. We analyze resonance fluorescence from a driven atom which is damped by a squeezed vacuum. Certain atomic properties such as the steady-state atomic polarization, saturation intensity, and fluorescent spectrum, are now phase dependent.

The Hamiltonian describing the interaction of a two-level atom with the quantized multimode radiation field and a classical driving field is given in the electric-dipole and rotating-wave approximations by

$$H=\tfrac{1}{2}\hbar\omega_A\sigma_z-(\mu E e^{-i\omega_L t}\sigma_+ +\mu^* E^* e^{i\omega_L t}\sigma_-)+H_{\mathrm{rad}}+\hbar(\sigma_+\Gamma+\sigma_-\Gamma^\dagger), \tag{1}$$

where ω_A is the atomic resonance frequency, σ_+, σ_-, and σ_z are pseudospin operators for the atom, and μ is the atomic dipole moment; H_{rad} is the free Hamiltonian for the quantized radiation field, Γ and $\Gamma^\dagger$ are operators defined in terms of the positive- and negative-frequency components of this field, respectively, and E is the amplitude of the coherent driving field with frequency ω_L. The normal treatment of resonance fluorescence takes the quantized radiation field in the usual vacuum state. We assume that it is in a broad-band squeezed vacuum state centered about the frequency ω_L. We assume that *all* of the modes coupling to the atom are squeezed so there will be not spontaneous emission into unsqueezed vacuum modes, and that the bandwidth of the squeezing is sufficiently broad that the squeezed vacuum appears as δ-correlated squeezed white noise to the atom. Then correlation functions for Γ and $\Gamma^\dagger$ can be written in the form[8]

$$\begin{aligned}&\langle\Gamma^\dagger(t)\Gamma(t')\rangle=\gamma N\delta(t-t'),\quad \langle\Gamma(t)\Gamma^\dagger(t')\rangle=\gamma(N+1)\delta(t-t'),\\ &\langle\Gamma(t)\Gamma(t')\rangle=\gamma M e^{-2i\omega_L t}\delta(t-t'),\quad \langle\Gamma^\dagger(t)\Gamma^\dagger(t')\rangle=\gamma M^* e^{2i\omega_L t}\delta(t-t').\end{aligned} \tag{2}$$

Here γ is the atomic decay rate for spontaneous emission into the *unsqueezed* vacuum, and N and M are parameters which charactize the squeezing, with $|M|^2\le N(N+1)$, where the equality holds for a minimum uncertainty squeezed state. The variances in the quadrature phases of the squeezed field at the site of the atom are $V(X_\theta)=\frac{1}{2}[N+|M|\cos(\theta-\phi)+\frac{1}{2}]$, where $M=|M|\times e^{i\phi}$; the phase ϕ will depend on details of the scheme used to generate the squeezed vacuum. For a highly squeezed $(N\gg 1)$ minimum-uncertainty state the variances in the maximally squeezed quadrature, $\theta=\phi$, and the out-of-phase quadrature, $\theta=\phi+\pi$, are $V(X_\phi)\approx N$, and $V(X_{\phi+\pi})\approx 1/16N$.

The master equation for the reduced density operator ρ of the atom following from Eqs. (1) and (2) reads

$$\dot{\rho} = -i\tfrac{1}{2}\omega_A[\sigma_z,\rho] + i[\Omega e^{i\phi_L}e^{-i\omega_L t}\sigma_+ + \Omega e^{-i\phi_L}e^{i\omega_L t}\sigma_-,\rho] + \tfrac{1}{2}\gamma(N+1)(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-)$$
$$+\tfrac{1}{2}\gamma N(2\sigma_+\rho\sigma_- - \sigma_-\sigma_+\rho - \rho\sigma_-\sigma_+) - \gamma M e^{-2i\omega_L t}\sigma_+\rho\sigma_+ - \gamma M^* e^{2i\omega_L t}\sigma_-\rho\sigma_-, \quad (3)$$

where $(\mu/\hbar)E = \Omega e^{i\phi_L}$, and 2Ω is the Rabi frequency. Optical Bloch equations may be derived from this equation. Our first interest is with the steady-state solutions to these equations. We choose polarization quadratures that are in phase and out of phase with the driving field:

$$\langle\sigma_x\rangle = \tfrac{1}{2}(\langle\sigma_-\rangle e^{i\omega_L t}e^{-i\phi_L} + \langle\sigma_+\rangle e^{i\omega_L t}e^{i\phi_L}), \quad \langle\sigma_y\rangle = (1/2i)(\langle\sigma_-\rangle e^{i\omega_L t}e^{-i\phi_L} - \langle\sigma_+\rangle e^{-i\omega_L t}e^{i\phi_L}). \quad (4)$$

Then

$$\langle\sigma_x\rangle_{ss} = \frac{1}{2N+1}\frac{1}{2}Y(\Phi)\left(\frac{\gamma_{us}}{\Gamma(\Phi)}\right)^{1/2}\frac{\delta+\Delta(\Phi)}{1+\delta^2+[Y(\Phi)]^2}, \quad \langle\sigma_y\rangle_{ss} = \frac{1}{2N+1}\frac{1}{2}Y(\Phi)\left(\gamma_{us}\frac{\Gamma(\Phi)}{\gamma_u\gamma_s}\right)^{1/2}\frac{1}{1+\delta^2+[Y(\Phi)]^2},$$
$$\langle\sigma_z\rangle_{ss} = -\frac{1}{2N+1}\frac{1+\delta^2}{1+\delta^2+[Y(\Phi)]^2}, \quad (5)$$

with

$$\delta = \frac{\Delta\omega}{(\gamma_u\gamma_s)^{1/2}}, \quad \Gamma(\Phi) = \gamma(N+|M|\cos\Phi+\tfrac{1}{2}), \quad \Delta(\Phi) = \frac{\gamma|M|\sin\Phi}{(\gamma_u\gamma_s)^{1/2}}, \quad Y(\Phi) = 2\Omega\left(\frac{1}{\gamma_{us}}\frac{\Gamma(\Phi)}{\gamma_u\gamma_s}\right)^{1/2}, \quad (6)$$

where $\Delta\omega = \omega_A - \omega_L$, and $\Phi = 2\phi_L - \phi$; $\gamma_u = \gamma(N+|M|+\tfrac{1}{2})$ and $\gamma_s = \gamma(N-|M|+\tfrac{1}{2})$ are the fast and slow polarization decay rates obtained by Gardiner,[7] and $\gamma_{us} = \gamma_u + \gamma_s$ is the corresponding decay rate for the inversion. The solutions for a normal radiatively damped atom are recovered from Eqs. (5) and (6) by setting $N = |M| = 0$.

The new feature brought by the squeezed vacuum is the dependence on the phase Φ. Control over Φ is available either via the phase of the squeezed vacuum field or via the phase of the coherent driving field. The steady-state averages show interesting phase dependences. The prefactor $1/(2N+1)$ which multiplies each of Eqs. (5) results from saturation of the atom by the squeezed vacuum field. Saturation of the remaining, reduced, inversion— $-1/(2N+1)$—is controlled by the coherent driving-field intensity. This intensity is normalized against a *phase-dependent* saturation intensity. Let us assume minimum-uncertainty squeezing and the strong-squeezing limit for simplicity. Then

$$[Y(0)]^2 \approx 2Y_{vac}^2, \quad [Y(\pi)]^2 \approx (1/8N^2)Y_{vac}^2, \quad (7)$$

where $Y_{vac}^2 = 8\Omega^2/\gamma^2$ is the intensity of the coherent driving field normalized by the usual saturation intensity. These limiting cases correspond to a saturation intensity $8N^2$ times the usual saturation intensity for $\Phi=\pi$, and one-half of the usual saturation intensity for $\Phi=0$. This phase dependence gives us a new control over nonlinear effects in the atom. If $\frac{1}{2}(1+\delta^2) \ll Y_{vac}^2 \ll 8N^2(1+\delta^2)$, the atomic inversion can be switched from a highly saturated state to an unsaturated state by changing Φ from 0 to π.

The effect such a phase change has on the polarization is more subtle, since $\langle\sigma_x\rangle_{ss}$ and $\langle\sigma_y\rangle_{ss}$ show phase dependences beyond those due to their saturation denominator. A careful analysis uncovers the following interesting behavior: When $\delta=0$ and $\frac{1}{2} \ll Y_{vac}^2 \ll 8N^2$, $\langle\sigma_x\rangle_{ss}$ shows a very sensitive dependence on phase, rapidly changing between $\langle\sigma_x\rangle_{ss} = +\frac{1}{2}$ and $\langle\sigma_x\rangle_{ss} = -\frac{1}{2}$ around $\Phi=\pi$, as illustrated in Fig. 1. A similar phase sensitivity is shown by $\langle\sigma_y\rangle_{ss}$ for driving-field intensities in the range $\frac{1}{2}(1+\delta^2) \ll Y_{vac}^2 \ll 8N^2(1+\delta^2)$. These observations suggest the possibility for a number of interesting nonlinear propagation effects if an extended medium with this phase-dependent polarization can be realized in the laboratory, including novel schemes for optical bistability.

The fluorescent spectrum for a two-level atom interacting with the usual vacuum has been well studied.[9-11] For strong driving fields this spectrum shows the

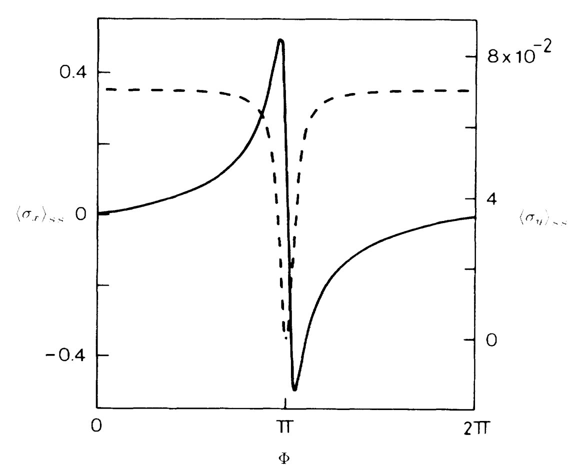

FIG. 1. Steady-state polarization as a function of the phase Φ for $N=50$ and $Y_{vac}=10$. The solid curve plots $\langle\sigma_x\rangle_{ss}$ and the dashed curve plots $\langle\sigma_y\rangle_{ss}$.

three-peaked form first predicted by Mollow.[9] We now show how this spectrum is modified when the atom interacts with a squeezed vacuum.

The spectrum of the fluorescent light in the stationary state has been calculated as the Fourier transform of the atomic autocorrelation function $\langle\sigma_+(0)\sigma_-(\tau)\rangle_{ss}$. This scheme for calculating the spectrum assumes that there exists a "window" of unsqueezed vacuum modes through which we can view the fluorescence. To be consistent we should strictly add a phase-independent damping term to the master equation to describe loss into these unsqueezed modes. However, if we assume that the solid angle occupied by the unsqueezed modes is very small we may omit this extra term. The atomic correlation function may then be calculated from Eq. (3) by using the quantum regression theorem. We give expressions for the spectrum obtained for resonant excitation, $\Delta\omega=0$, and the limiting choices of phase, $\Phi=0$ and $\Phi=\pi$.

The spectrum changes from a single peak to a triplet just as in normal resonance fluorescence; but now the threshold field amplitude $Y_{s,u}^{\text{thr}}=\gamma_{u,s}/2(\gamma_{us}\gamma_{s,u})^{1/2}$ at which this transition takes place depends on phase (the subscripts s,u and u,s are ordered so that the first applies for $\Phi=0$ and the second for $\Phi=\pi$). For $Y_{s,u}\leq Y_{s,u}^{\text{thr}}$ the spectrum is given by

$$S(\omega)=C_0\delta(\omega-\omega_A)-\frac{C_1}{\pi}\frac{\lambda_1}{\lambda_1^2+(\omega-\omega_A)^2}-\frac{C_2}{\pi}\frac{\lambda_2}{\lambda_2^2+(\omega-\omega_A)^2}-\frac{C_3}{\pi}\frac{\lambda_3}{\lambda_3^2+(\omega-\omega_A)^2},\tag{8}$$

while for $Y_{s,u}\geq Y_{s,u}^{\text{thr}}$

$$S(\omega)=C_0\delta(\omega-\omega_A)-\frac{C_1}{\pi}\frac{\lambda_1}{\lambda_1^2+(\omega-\omega_A)^2}-\frac{1}{\pi}\frac{C_R\lambda_R+C_I(\omega-\omega_A-\lambda_I)}{\lambda_R^2+(\omega-\omega_A-\lambda_I)^2}-\frac{1}{\pi}\frac{C_R\lambda_R-C_I(\omega-\omega_A-\lambda_I)}{\lambda_R^2+(\omega-\omega_A-\lambda_I)^2}.\tag{9}$$

Here

$$\begin{aligned}
C_0&=\langle\sigma_y\rangle_{ss}^2,\quad C_1=\frac{1}{4}\left[1-\frac{1}{2N+1}\frac{1}{1+Y_{s,u}^2}\right],\\
C_2&=C_R+iC_I=\frac{1}{4}\frac{\lambda_2+\gamma_{us}}{\lambda_2-\lambda_3}\left[1-\frac{1}{2N+1}\frac{1}{1+Y_{s,u}^2}\right]+\frac{1}{4}\frac{\gamma}{\lambda_2-\lambda_3}\left[1+\frac{\gamma}{\lambda_2}\right]\frac{Y_{s,u}^2}{1+Y_{s,u}^2},\\
C_3&=C_R-iC_I=\frac{1}{4}\frac{\lambda_3+\gamma_{us}}{\lambda_3-\lambda_2}\left[1-\frac{1}{2N+1}\frac{1}{1+Y_{s,u}^2}\right]+\frac{1}{4}\frac{\gamma}{\lambda_3-\lambda_2}\left[1+\frac{\gamma}{\lambda_3}\right]\frac{Y_{s,u}^2}{1+Y_{s,u}^2},
\end{aligned}\tag{10}$$

and

$$\begin{aligned}
\lambda_1&=-\gamma_{u,s},\quad \lambda_2=\lambda_R+i\lambda_I=-\tfrac{1}{2}(\gamma_{us}+\gamma_{s,u})+[\tfrac{1}{4}(\gamma_{us}-\gamma_{s,u})^2-\gamma_{us}\gamma_{s,u}Y_{s,u}^2]^{1/2},\\
\lambda_3&=\lambda_R-i\lambda_I=-\tfrac{1}{2}(\gamma_{us}+\gamma_{s,u})-[\tfrac{1}{4}(\gamma_{us}-\gamma_{s,u})^2-\gamma_{us}\gamma_{s,u}Y_{s,u}^2]^{1/2}.
\end{aligned}\tag{11}$$

The new features of this spectrum are most easily analyzed for minimum uncertainty squeezing in the large squeezing limit. We compare the incoherent spectra for very weak and very strong fields; in both of these limits the coherent intensity is negligible in comparison to the integrated intensity under the incoherent spectrum. For weak fields ($2Y_{\text{vac}}^2\ll 1$) the incoherent spectrum is comprised of two Lorentzians, each accounting for an equal integrated intensity—a broad Lorentzian with linewidth (half width at half maximum) $\gamma 2N$, and a narrow Lorentzian with subnatural linewidth $\gamma/8N$. This weak-field spectrum is insensitive to phase. In the strong-field limit a three-peaked structure begins to be resolved for $Y_{\text{vac}}^2\sim N^2$ and the spectrum becomes phase dependent. For $\Phi=0$ it has a central Lorentzian peak with linewidth $\gamma 2N$, and sidebands with one-half of this width. All three peaks have equal height, so that the integrated intensity under the central peak is equal to the sum of the intensities under the two sidebands. For $\Phi=\pi$ the central peak has the subnatural linewidth $\gamma/8N$, and the sidebands retain a broad linewidth $\gamma 2N$. The integrated intensity under the central peak is again equal to the sum of the intensities under the sidebands. In the large squeezing limit, the narrow central peak is extremely phase sensitive; a phase shift from $\Phi=\pi$ of order $1/N$ is sufficient to significantly broaden the subnatural linewidth. This extreme sensitivity is not present, however, for more moderate squeezing, and the spectrum still shows a dramatic phase dependence. Figure 2 illustrates this phase dependence for $N=0.2$.

Our analysis has assumed *all* vacuum modes are squeezed. This is not an essential assumption, although the maximum size of the effects will be reduced if the atomic relaxation is divided between phase-sensitive decay induced by squeezed vacuum modes and phase-insensitive decay induced by unsqueezed modes. If some vacuum modes remain unsqueezed, there will be a nonzero subnatural linewidth for perfect squeezing rather than the vanishing linewidth given by the present cal-

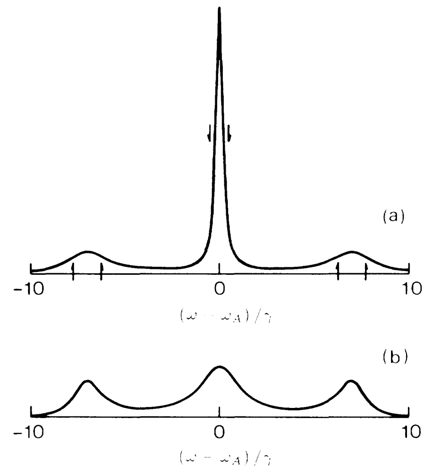

FIG. 2. Incoherent fluorescent spectrum for $N=0.2$ and $Y_{\text{vac}}=10$: (a) $\Phi=\pi$, (b) $\Phi=0$. The arrows indicate linewidths for ordinary resonance fluorescence.

culation.

We have analyzed the problem of resonance fluorescence for an atom which is damped by a squeezed vacuum and shown that the squeezing introduces a phase dependence to the steady-state polarization, inversion, and fluorescent spectrum. At high driving-field intensities a spectrum comprising a broad central peak and broad sidebands can be changed into one having a central peak with subnatural linewidth by changing the phase of the driving field by $\pi/2$.

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