Instabilities and Chaos in the Polarizations of Counterpropagating Light Fields

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We show that the polarizations of counterpropagating light waves in an isotropic Kerr medium are temporally unstable when their total intensity exceeds a certain threshold value. Periodic and chaotic temporal behavior can occur in the output polarizations and under certain conditions also in the output intensities.

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Intense counterpropagating laser beams are required for the operation of useful nonlinear optical interactions such as optical bistability and phase conjugation by degenerate four-wave mixing. However, recent theoretical work has indicated that there exist regimes in which such fields are not stable. Silberberg and Bar-Joseph[1] have predicted that in the scalar approximation, counterpropagating waves interacting in a nonlinear Kerr medium characterized by noninstantaneous response can undergo oscillatory and chaotic temporal evolution. This instability results from the gain of a four-wave mixing process and the distributed feedback that results from scattering off the grating formed by the interference of the counterpropagating waves; most other examples of chaos in nonlinear optics have required external feedback.[2] Theoretical studies have also recently shown that the steady-state polarizations of counterpropagating fields in a nonlinear Kerr medium can be multivalued[3] as well as possessing a chaotic *spatial* distribution.[4] Kaplan[5] has shown that for an isotropic nonlinear Kerr medium there exist in steady state four eigenarrangements for the polarizations that remain invariant upon propagation through the material. Wabnitz and Gregory[6] have pointed out that only two of these eigenpolarizations are *spatially* stable in steady state; the stable arrangements are those with both fields linearly polarized with parallel polarizations and both fields circularly polarized and corotating. Preliminary numerical simulations of the temporal behavior of this system have been conducted,[7] but to our knowledge no one has previously investigated the temporal stability of these spatially stable eigenpolarizations.

In this Letter we examine the temporal stability of the polarizations and intensities of counterpropagating waves in an isotropic Kerr medium for the case in which the input polarizations are one of the eigenarrangements that are known to be spatially stable.[6] For definiteness, we concentrate on the case in which the two input polarizations are linear and parallel.[8] We allow the field amplitudes to be time dependent and allow the medium to possess a noninstantaneous response. We find that when the input intensities exceed a certain threshold value the output beams become temporally unstable. This instability can lead to abrupt switching of the state of polarization of the output beams or can produce oscillatory or chaotic fluctuations of the output polarizations and under some circumstances also of the intensities.

We consider the geometry shown in the inset to Fig. 1. The total complex electric field in the medium can be decomposed into its x and y Cartesian components as

$$\mathbf{E}=[E_x(z,t)\hat{\mathbf{x}}+E_y(z,t)\hat{\mathbf{y}}]e^{-i\omega t}. \quad (1)$$

Each component consists of a forward- and backward-

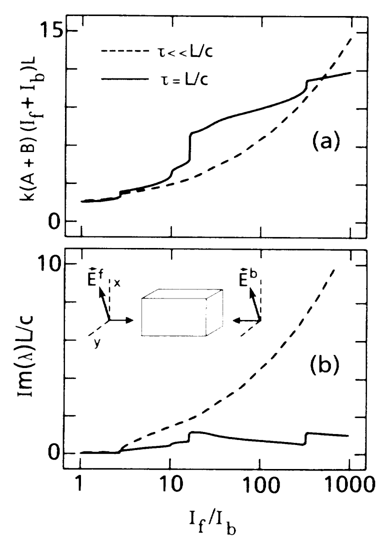

FIG. 1. (a) Total input intensity at the threshold for instability and (b) frequency of oscillation plotted as functions of the forward-to-backward input intensities for $\tau=L/c$ and for $\tau \ll L/c$. Oscillatory instabilities are predicted for input ratios greater than 2.7.

traveling part such that

$$E_i = E_i^f(z,t)e^{ikz} + E_i^b(z,t)e^{-ikz}, \tag{2}$$

where $i = x, y$, $k = \omega/c$, and c is the speed of light in the medium. The complex nonlinear polarization can similarly be represented as $\mathbf{P} = (P_x\hat{\mathbf{x}} + P_y\hat{\mathbf{y}})\exp(-i\omega t)$, where each component of the polarization amplitude can be expressed in terms of a field-dependent susceptibility tensor χ_{ij} and $P_i = \sum_{j=x,y}\chi_{ij}E_j$. For a lossless Kerr medium, χ_{ij} is assumed to satisfy the Debye relaxation equation

$$\tau\,\partial\chi_{ij}/\partial t + \chi_{ij} = (A-B)(\mathbf{E}\cdot\mathbf{E}^*)\delta_{ij} + B(E_iE_j^* E_i^* E_j), \tag{3}$$

where τ is the response time of the medium and A and B are real constants that characterize the mechanism that gives rise to the nonlinear Kerr effect. In the limit $\tau \ll L/c$, where L is the length of the medium, the susceptibility tensor attains its steady-state value[5] given by the right-hand side of Eq. (3). The electric field and the nonlinear polarization are substituted into the driven wave equation, and the slowly varying-envelope approximation is made to yield the coupled amplitude equations

$$\left(\frac{\partial}{\partial z} \pm \frac{1}{c}\frac{\partial}{\partial t}\right)E_i^{f,b} = \pm k \sum_{j=x,y} [\chi_{ij}^{(0)}E_j^{f,b} + \chi_{ij}^{(\pm 2ik)}E_j^{b,f}], \tag{4}$$

where $\chi_{ij}^{(0)}$ and $\chi_{ij}^{(\pm 2ik)}$ are the dc and $\pm 2ik$ spatial Fourier components of χ_{ij}, respectively. What distinguishes our treatment from that of previous workers[1,3,6] is that we take into account both the vector nature of the fields and the nonzero response time of the medium.

We now assume that the input fields are linearly polarized in the y direction, in which case the steady-state solution for the amplitudes of the forward- and backward-traveling waves is

$$E_{x,\text{ss}}^{f,b}(z) = 0, \tag{5a}$$

$$E_{y,\text{ss}}^{f,b}(z) = I_{f,b}^{1/2}\exp[\pm ik(A+B)(I_{f,b} + 2I_{b,f})z], \tag{5b}$$

where $I_f = |E_{y,\text{ss}}^f(0)|^2$ and $I_b = |E_{y,\text{ss}}^b(L)|^2$. In previous work that treated the temporal stability of optical systems,[1,9] the amplitude of the optical field was perturbed. However, we wish to investigate the temporal stability of the field polarization, and thus we perturb the steady-state solution [Eq. (5)] by assuming the presence of a small time-dependent field polarized in the x direction so that the total complex field can be represented as

$$\mathbf{E}(z,t) = \{[f_1(z)e^{\lambda t} + f_2(z)e^{\lambda^* t}]\hat{\mathbf{x}} + E_{y,\text{ss}}^f\hat{\mathbf{y}}\}e^{i(kz-\omega t)} + \{[b_1(z)e^{\lambda t} + b_2(z)e^{\lambda^* t}]\hat{\mathbf{x}} + E_{y,\text{ss}}^b\hat{\mathbf{y}}\}e^{i(-kz-\omega t)}, \tag{6}$$

with the boundary conditions $f_{1,2}(0) = b_{1,2}(L) = 0$. This perturbed field is substituted into the coupled nonlinear equations (3) and (4) which yield a set of linearized equations for the perturbation amplitudes. These linearized equations are entirely uncoupled from the equations for the perturbation amplitudes polarized in the y direction derived in the scalar-wave theory.[1] The equations for $f_{1,2}$ and $b_{1,2}$ along with the boundary conditions allow us to solve analytically for the boundary of instability. In order to do this, we set $\text{Re}(\lambda) = 0$ and solve for the threshold intensity and the frequency offset $\text{Im}(\lambda)$ at which the perturbation occurs. We find that the threshold for instability increases as the ratio B/A decreases and that only in the limit $B/A \to 0$ is there no solution with $\text{Re}(\lambda) > 0$ corresponding to a solution that grows in time. In this Letter, we assume in all of our numerical examples the case of a medium for which $B/A = 3$. The total input threshold intensity for instability for such a medium is plotted as a function of the ratio I_f/I_b in Fig. 1(a) for the cases $\tau \ll L/c$ and $\tau = L/c$. The region above each of the curves is that for which the perturbation is temporally unstable. The oscillation frequency $[\text{Im}(\lambda)]$ of the perturbation at threshold is plotted in Fig. 1(b). We see that for input pump intensity ratios less than 2.7, this polarization instability is dc in nature $[\text{Im}(\lambda) = 0]$ for both $\tau \ll L/c$ and $\tau = L/c$. For larger values of the pump imbalance ratio, the steady-state solution [Eq. (5)] undergoes a Hopf bifurcation $[\text{Im}(\lambda) \neq 0]$. Note that even in the limit of relatively small backward intensities $(I_f/I_b > 10.0)$, the threshold for instability is considerably less than $k(A+B)I_fL = 15$ normally required for single-beam stimulated scattering involving only the forward wave.[10] A similar effect for stimulated Brillouin has been discussed by Zel'dovich.[11]

In order to determine the full dynamical behavior in the region where instability is predicted, we have numerically integrated the coupled nonlinear equations (3) and (4) in both space and time using the method of characteristics. We ramp the input fields on adiabatically to the desired steady-state input intensity and hold them at this value. Figure 2(a) shows the time evolution for the polarization state of the forward-traveling wave for the case where $I_f = I_b$. We find that when the input intensities exceed the threshold value shown in Fig. 1(a), the steady-state solution [Eq. (5)] is no longer temporally stable and the output polarizations switch to a new steady-state value. The forward and backward output

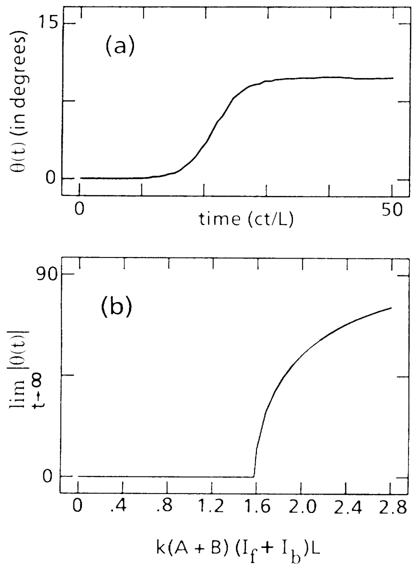

FIG. 2. (a) Temporal evolution of the polarization of the transmitted forward-traveling wave for the case of input waves with parallel linear polarizations and equal input intensities whose values $[k(A+B)(I_f+I_b)L=1.7]$ are just above the threshold for instability shown in Fig. 1(a). The output polarization remains linear, but is rotated through an angle θ with respect to the input polarization. (b) Absolute value of the rotation angle plotted as a function of the total input intensity.

intensities remain constant and the output polarizations remain linearly polarized but rotated through an angle θ with respect to the y axis. This angle can be positive or negative depending on which direction the perturbative noise drives the system. We have plotted in Fig. 2(b) the absolute value of the output angle θ as a function of the normalized input intensity. A bifurcation occurs in the steady-state solution for $|\theta|$ at $k(A+B)(I_f+I_b)L \approx 1.58$ as predicted by our stability analysis [Fig. 1(a)]. For the case of equal pump intensities and for total input intensities below 2.8, this stable steady-state behavior is

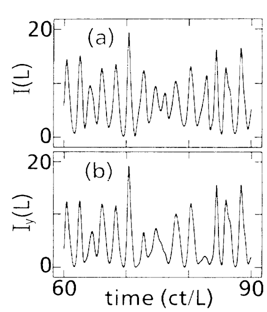

FIG. 3. Chaotic temporal evolution of (a) the total transmitted intensity and (b) the intensity of the y-polarization component for the forward-traveling wave for $\tau=L/c$, equal pump intensities, and $k(A+B)I_fL=6$.

the same regardless of the medium response time. In the limit $\tau \ll L/c$, we find that at higher input intensities this branch eventually becomes unstable and the polarization state switches to another stable branch.

When the response time is of the order of the transit time, there exists for the case of balanced pumping a second instability threshold above which both the polarization and the intensity can exhibit oscillatory and chaotic temporal behavior. Our analysis is unable to predict the onset of this new instability because of the complicated steady-state distribution of the pump fields above the first instability threshold. However, from our numerical simulations we are able to determine an approximate value of $k(A+B)(I_f+I_b)L \approx 2.8$ for this second threshold when $\tau=L/c$, which is why the curve shown in Fig. 2(b) terminates at this value of the abscissa. For a total normalized intensity just above this second threshold value, both the polarization and the total intensity oscillate periodically in time. This threshold intensity of 2.8 is smaller than that of 4.0 predicted by the scalar-wave theory and suggests that the scalar instability may be unobservable unless $B/A \to 0$. At higher intensity (Fig. 3), the output polarization and intensity fluctuate in time in an apparently chaotic fashion. We have analyzed the time series using standard methods[12] and found that it yields a positive order-two Renyi entropy, which implies that the temporal evolution is chaotic in the strict sense.

The gain-feedback mechanism that leads to the oscillatory and chaotic instability discussed above (i.e., for $\tau=L/c$) is analogous to that which leads to the instability predicted by Silberberg and Bar-Joseph, except that in our case a tensor nonlinear response grating rather than a scalar grating produces the distributed feedback. However, our stability analysis (see Fig. 1) implies that

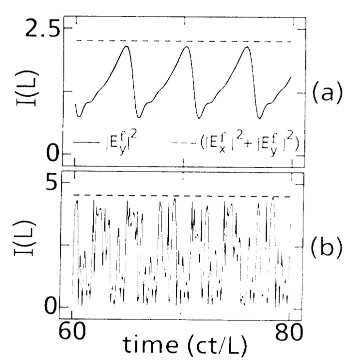

FIG. 4. Temporal evolution of the total transmitted intensity and the intensity of the y-polarization component for the forward-traveling wave for the case of unequal input intensities $(I_f/I_b=3)$ and $\tau \ll L/c$. (a) For $k(A+B)(I_f+I_b)L=3$, the system shows an oscillatory instability in its polarization and (b) for $k(A+B)(I_f+I_b)L=6$ it shows a chaotic instability. In each case, the total transmitted intensity is constant in time.

when the input waves are unequal in intensity, fluctuating temporal behavior can result even for $\tau \ll L/c$, in which limit the gain-feedback mechanism for instability cannot occur. For a pump imbalance of 3 and for a total pump intensity above the threshold value, the output polarization oscillates in time [Fig. 4(a)] and at even higher input intensities becomes chaotic [Fig. 4(b)]. The characteristic frequency of oscillation is of the order of c/L, which suggests that the mechanism which drives this instability is related to transit-time effects and not to gain at the frequency $1/\tau$ coupled with distributed feedback.

In summary, we have shown that in isotropic nonlinear Kerr media with $B \neq 0$, temporal instabilities and chaotic behavior can occur in the polarizations of counterpropagating waves even when the input fields constitute one of the spatially stable polarization configurations.

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