Electric and Magnetic Properties of Hot Gluons

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The dielectric constant ϵ and magnetic permeability μ for a gluon plasma are calculated from the one-loop gauge-invariant effective action. The real parts are gauge-fixing independent and agree with earlier work. The imaginary part of μ^{-1} is zero in any covariant background-field gauge, while the imaginary part of ϵ is gauge-fixing dependent and negative definite. This result indicates that there is no consistent perturbative description of gluonic plasmons on a scale $\geq (g^2T)^{-1}$.

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There is strong evidence, mainly based on computer simulations, that pure Yang-Mills theory exhibits a low-temperature confined phase and a high-temperature deconfined phase also referred to as a gluon plasma (GP). A fair number of results concerning the thermodynamics of the GP is by now well established. There is a high-temperature perturbative expansion for the free energy, which has no infrared (IR) divergences to order g^5, provided one performs a proper resummation of graphs corresponding to the insertion of the electric mass.[1] It is also understood that the perturbative expansion of the free energy breaks down at order g^6 due to uncontrollable IR divergences. It is commonly speculated that these divergences are related to the nonperturbative generation of a magnetic mass $m_{\text{mag}} \sim g^2T$, which is the (dimensionful) coupling constant of the corresponding three-dimensional theory obtained by dimensional reduction.[2,3] Moreover, in a recent work Nadkarni has shown how the high-T perturbation expansion for the electric screening mass breaks down at next-to-leading order.[4] Finally, it has been conjectured by DeTar[5] that at the scale g^2T the plasma exhibits dynamical confinements in the sense that no colored modes are propagating.

In order to understand the physics of the GP, knowledge of the static properties, i.e., the thermodynamics, is necessary but not sufficient. Of equal importance are the dynamical properties related to transport coefficients and response functions, and much work has been devoted to this subject.[6-8] In particular, the spectral properties (i.e., dispersion relations) of plasma oscillations have been studied in detail. Since these properties are related to Green's functions rather than thermodynamical derivatives of the partition function, one is faced with serious problems related to gauge dependence. In the approach advocated by Kapusta and Kajantie,[7] and by Heinz, Kajantie, and Toimela,[8] the thermal expectation value of the retarded commutator of the colored electric fields is related to a linear response function much like in QED. In this paper we shall approach the problem differently. Using the background-field method and a covariant background-field gauge,[9] we construct a gauge-invariant one-loop finite-temperature effective action Γ. This object contains all information about the plasma to lowest nontrivial order in $\hbar$, and is of the form

$$\Gamma = \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\left[\epsilon \mathbf{E}^2 - \frac{1}{\mu}\mathbf{B}^2 + \kappa \mathbf{E}\cdot(\mathbf{E}\times\mathbf{B}) + \cdots\right], \tag{1}$$

where $\mathbf{E}$ and $\mathbf{B}$ are the color field strengths and the coefficients ϵ, μ, κ, etc., depend on the rotational- and gauge-invariant operators $T\nabla^0$, $(\nabla^0)^2$, and ∇^2. The dots stand for higher-order terms in the (covariant) derivative expansion which is possible at high temperature since T provides a mass scale. The coefficients in this expansion are generalized response functions, and of particular interest are the dielectric constant ϵ and the magnetic permeability μ which determine the linear response of the plasma. One can consider Γ as a classical action and study the corresponding mean-field equations. As will be seen below, there is a serious problem in that these equations are not gauge-fixing independent. We stress that the two issues of gauge invariance and gauge-fixing independence are separate, and that usually the second does not follow from the first. For instance, although $\Gamma[A;\alpha]$ is manifestly gauge invariant, it does depend parametrically on the gauge-fixing parameter α. At zero temperature, Abbott has shown that the S-matrix elements obtained from the on-shell amplitudes are α independent and coincide with those obtained from the conventional generating function.[10] We shall return to the issue of gauge-fixing dependence below. The mean-field equations derived from (1) are nothing but the Maxwell's equations in a medium characterized by ϵ and μ,

$$\epsilon \mathbf{k}\cdot\mathbf{E}_L = 0,$$

$$(\epsilon\omega^2 - \mu^{-1}\mathbf{k}^2)\mathbf{E}_T(k) = 0, \tag{2}$$

$$(\epsilon\omega^2 - \mu^{-1}\mathbf{k}^2)\mathbf{B}_T(k) = 0, \tag{3}$$

where L and T refer to longitudinal and transverse modes, respectively. To evaluate ϵ and μ we express the effective action in terms of the gluon polarization tensor

$$\Gamma=\Gamma_0-\frac{1}{2}\int\frac{d^4k}{(2\pi)^4}A_\mu(k)\Pi^{\mu\nu}(k,\alpha)A_\nu(-k)+\cdots, \tag{4}$$

where Γ_0 is the classical action. A comparison of (1) and (4) yields

$$\epsilon(\omega,\mathbf{k},\alpha)=1-\Pi_{00}/\mathbf{k}^2,$$
$$\mu^{-1}(\omega,\mathbf{k},\alpha)=1+\frac{1}{2\mathbf{k}^2}\left[\Pi_S-3\frac{\omega^2}{\mathbf{k}^2}\Pi_{00}\right]. \tag{5}$$

The actual calculation of Π_{00} and $\Pi_S=\sum_i\Pi_{ii}$ in a general covariant background-field gauge is rather lengthy and is described elsewhere.[11] To discuss plasmons, we shall consider the kinematical region $T\gg\omega\gg|\mathbf{k}|$. In this limit the general results given in Ref. 11 reduce to

$$\mathrm{Re}\epsilon=1-\frac{\omega_p^2}{\omega^2}\left[1+\frac{3}{5}\frac{\mathbf{k}^2}{\omega^2}+\cdots\right], \tag{6}$$

$$\mathrm{Im}\epsilon=-\theta(\omega-k)\frac{\omega_D^2}{4\pi\omega T}[11+\tfrac{1}{4}(\alpha-1)^2], \tag{7}$$

$$\mathrm{Re}(\mu^{-1})=1-\tfrac{2}{5}\omega_p^2/\omega^2+\cdots, \tag{8}$$

$$\mathrm{Im}(\mu^{-1})=0, \tag{9}$$

where, for the gauge group SU(N),

$$\omega_p^2=\tfrac{1}{3}\omega_D^2=\tfrac{1}{9}Ng^2T^2, \tag{10}$$

and α is the gauge-fixing parameter ($\alpha=1$ corresponds to Feynman and $\alpha=0$ to Landau background-field gauge). Note that the real parts of ϵ and μ are gauge-fixing independent and that the imaginary part of μ vanishes for all α. Equations (7) and (9) imply that the plasmon frequency is not only gauge invariant but also gauge-fixing independent.[12] The dispersion relation is, however, not real, and with the notation

$$\omega=\omega_p-i\gamma, \tag{11}$$

the plasmon damping constant γ is given by

$$\gamma_L=\tfrac{1}{2}\omega_p\,\mathrm{Im}\epsilon,$$
$$\gamma_T=\tfrac{1}{2}\omega_p[\mathrm{Im}\epsilon-(\mathbf{k}^2/\omega_p^2)\mathrm{Im}\mu], \tag{12}$$

$$\gamma=\gamma_T=\gamma_L=-\frac{Ng^2T}{24\pi}[11+\tfrac{1}{4}(\alpha-1)^2]. \tag{13}$$

The negative sign of γ corresponds to antidamping, i.e., plasmons with exponentially growing amplitudes.[13] Before we proceed to discuss this rather surprising result, we shall comment on some objections that have been raised against the use of covariant gauges to calculate plasmon properties. The first objection maintains that covariant gauges do not allow for a Hamiltonian formulation and that this implies that a linear-response theory is not possible.[7] However, a covariant Hamiltonian approach (based on Becchi-Rouet-Stora–invariant physical states) does exist,[14] and has been used to derive thermal Green's functions in covariant gauges.[15] The second objection concerns the lack of gauge invariance. Although this criticism is clearly valid for ordinary covariant gauges, it does not apply to our calculation which is manifestly gauge invariant. Thus we can see no reason for not taking the result in (13) for γ as seriously as the results found in temporal and Coulomb gauges. Now we must face the fact that γ is gauge-fixing dependent and thus at least superficially without any physical significance. (Note that in neither the axial- nor the Coulomb-gauge calculation was the possible gauge-fixing dependence of the plasma parameters studied, since a singular δ-function–type gauge was understood throughout. It is very likely that gauge-fixing dependence is a problem in any gauge.) One way to proceed is to impose further consistency conditions on Γ in order to define the mean-field equations uniquely.[16] This was done recently by Vilkovisky in the context of the invariance of quantum effective actions under reparametrizations of the classical background field.[17] His main observation was that the definition of the classical field in terms of expectation values of the quantum field is not unique. The origin of this arbitrariness is that, unlike the action S which is a scalar on the space of gauge-field configurations, the source term $J_\mu Q^\mu$, where Q^μ is the quantum field, has no such geometrical meaning. Vilkovisky's method consists in replacing the source term with a scalar object which coincides with JQ at tree level. To one loop, one can show that the resulting "unique effective action" is identical to the background effective action defined by (1) in Landau gauge (i.e., $\alpha=0$).

However, since Vilkovisky's action simply amounts to setting $\alpha=0$ in (8)–(10), the problem of antidamping remains. Unstable solutions are physically unacceptable, and so we seem to face two possibilities. The first possibility is simply a breakdown of the linear approximation, in which case nonlinear terms like $\mathbf{E}\cdot(\mathbf{E}\times\mathbf{B})$, etc., in (1) become important and could provide damping. In this case one might still have a consistent perturbative (albeit nonlinear) description of gluonic plasmons. The second possibility is that (13) signals a real breakdown of the loop expansion, already at the one-loop level. Note that the (anti)damping occurs at the scale g^2T where one expects a nonperturbative generation of a magnetic mass and the onset of dynamical color confinement. Static color charges are deconfined and screened at high temperatures, but dynamical excitations such as plasma waves seem to have an entirely different behavior. We can speculate that the antidamping of the plasmons is yet another signal of the breakdown of perturbation theory at the magnetic scale g^2T. If so, even a crude description of the transport properties of the quark-gluon plasma might require nonperturbative techniques. Indeed, collective motion, whether analyzed by linear-response theory (plasmons) or by hydrodynamics (flow and shocks), is essentially a long-wavelength phenomenon, and thus sensitive to the IR structure of hot QCD. The consequences of this are numerous. For instance, both the equation of state and the quark-gluon chemistry for the plasma might have to be seriously revised. Transport and hydrodynamical equations should be sought for a fluid of color-singlet excitations rather than free quarks and gluons. Signatures for the plasma formation have to be reconsidered accordingly.

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