Nonlocal Potential Measurements of Quantum Conductors

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Multiterminal measurements of magnetoresistance fluctuations in silicon inversion-layer nanostructures are extended to probe spacings $L \ll L_\phi$, the phase-preserving diffusion length. Unlike for $L > L_\phi$, the sizes of the voltage fluctuations are independent of L, and have novel correlations consistent with independent potential fluctuations of each probe. The corresponding "conductance" fluctuations $\delta G(L)$ are $\gg e^2/h$; however, this can be understood if each pair of probes effectively measures voltage fluctuations at scale L_ϕ, determined by the condition $\delta G(L_\phi) = e^2/h$.

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Electrical resistance measurements on small metal wires have revealed the existence of random quantum interference phenomena inside disordered conductors.[1,2] At low temperatures, electrons diffuse a distance L_ϕ as large as 1 μm before quantum phase information is destroyed by inelastic scattering. Because such length scales are experimentally accessible, this allows us to study concretely how measurement probes interact with a quantum system.

Our approach is to measure multicontact silicon inversion-layer devices with a narrow conducting channel and closely spaced side branches used as potential probes.[3] Our overall results are as follows. When a current I flows in the channel, reproducible quantum interference causes the voltage drop V_{ij} between two probes i and j to be a fluctuating function of magnetic field, as shown in Fig. 1 ($R_{ij} = V_{ij}/I$). In our new experiments at probe separations $L_{ij} < L_\phi$, the statistical correlations between the field traces for various i,j reveal that the electrochemical potential of each probe fluctuates independently with respect to its neighbors. The average voltage drop is proportional to L_{ij} but the root mean square amplitude of the fluctuations is independent of L_{ij}. This amplitude is the same as that of the voltage fluctuations between probes spaced approximately L_ϕ apart, which corresponds to the "universal conductance fluctuations" of magnitude e^2/h that have been of considerable recent interest.[4-6] The magnetic field scale for the fluctuations indicates the area over which magnetic flux perturbs the interference, and also is characteristic of L_ϕ.[2,5,6] Thus we conclude that, as far as the quantum conductance fluctuations are concerned, our four-terminal measurement that attempts to probe inside a region of quantum interference gives results characteristic of the system as a whole, including the measuring probes.

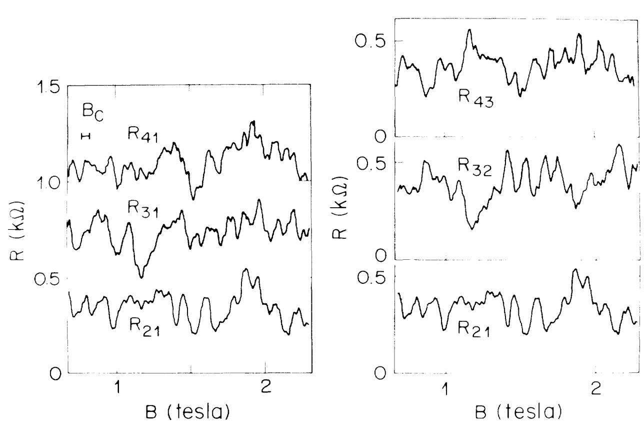

FIG. 1. Resistance measured between various pairs of probes for the short device with 0.15 μm probe spacing.

Before describing the experiment further, we consider certain fundamental issues. Quantum mechanically the two-terminal resistance of a composite resistor is not simply the sum of its component resistances measured separately.[7] But this does not describe a multiterminal measurement. Existing multiterminal quantum models[8-10] emphasize that each voltage probe connects the conducting channel to a macroscopic measurement reservoir associated with phase randomization. For slow changes of external parameters, the potential of each reservoir adjusts to cancel net current flow through the probes. For sufficiently high-impedance voltmeters, the potential differences between these macroscopic reservoirs can be logged without a significant perturbation of the system, and can therefore be added and subtracted to infer additional combinations. (The left side of Fig. 1 is formed by our adding together traces on the right side.) That simultaneous measurements with fixed probes are additive does not imply that the measurements would be unaffected by our physically connecting or disconnecting a probe, since electrons do diffuse into and out of the probes, and the change of even a single scatterer is expected to change substantially the interference patterns.[11]

In this experiment we compare the behavior of two devices, a "short" one with 150-nm probe spacing and a "long" one with 5-μm probe spacing. Each has a 250-nm-wide channel and 50-nm-wide probes that quickly broaden to 100 nm. Because the electron density in the inversion layer is controlled by the gate, each device represents many distinct specimens. We present data for $T \sim 0.4$ K and gate voltages 6–8 V above threshold (~ 600 $\Omega/\square$, mean free path $l = 40$ nm), where the inelastic diffusion length of these devices is expected to be about 1 μm.[12] [$L_\phi = (Dt_{\text{in}})^{1/2}$, where D is the diffusion constant and t_{in} is the inelastic-scattering time.] Thus the short device contains many probes inside a single quantum unit of size L_ϕ, while the long device contains many quantum units between each pair of probes.

Our data consist of fluctuating traces of resistance R_{ij} versus magnetic field at different gate voltages. We measured the R_{ij} with 1-nA ac currents at 15 Hz, simultaneously monitoring the voltage drops between three different pairs of probes with lock-in amplifiers, subsequently constructing other combinations by additivity. Figure 2 shows the root mean square amplitude of such traces, as a function of probe spacing. Each point combines data from six different gate voltages in the range 7–8 V, and also averages together the results for pairs of probes with the same spacing. At large probe spacings, the fluctuation amplitude increases as the square root of the length. This represents the uncorrelated addition of the resistance fluctuations across many channel segments of size L_ϕ. The partial cancellation that results from such addition is the essence of the self-averaging behavior that causes the "universal" fluctuations (at scale L_ϕ) to become relatively less important in large systems.[5,6,13,14] At small probe spacings, in contrast, the amplitude remains *constant*. This change of behavior occurs at scales of order L_ϕ, and is equivalent to a change in the statistical correlations between the different additively related measurements.

To understand how the fluctuation amplitude can be independent of probe spacing, consider Fig. 1. Each trace is labeled by the closely spaced (0.15–0.45 μm)

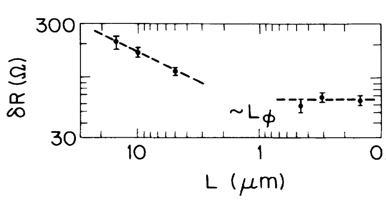

FIG. 2. Amplitude of resistance fluctuations as a function of probe spacing for the long and short devices, showing distinctly different dependence.

probes (1–4) involved in its definition. All amplitudes are comparable. The *average* resistance increases with distance between probes, but the cumulative fluctuation amplitude remains constant, because of significant cancellations. For example, the prominent feature below the label "31" is associated primarily with probe 3 (i.e., does not appear in "41" and "21"). Because probe 3 subtracts from "43" and adds to "32," this feature contributes to each with opposite sign. The correlations between adjacent traces in Fig. 2 (approximately +50% on the left and −50% on the right) are typical of the correlation coefficients of the entire data set (for closely spaced probes).

This pattern differs from the expected correlations for widely spaced probes that primarily measure the accumulated voltage drops across many independent quantum units in the channel between the probes. The correlation coefficient (normalized covariance)

$$C_{ijkl} = \sum(\Delta R_{ij}\Delta R_{kl})/[\sum(\Delta R_{ij})^2\sum(\Delta R_{kl})^2]^{1/2}$$

in such an independent-segments model should simply be the normalized overlap between channel segments: $L_{ij} \cap L_{kl}/[L_{ij}L_{kl}]^{1/2}$, which ranges from 0 for segments that do not overlap to 82% for two segments out of three.

Instead, the correlations are consistent with a simple independent-probes model: The potential of each measuring probe fluctuates *independently* with respect to its neighbors. Each trace is defined by the independent fluctuations of two probes, and the correlation coefficient between simultaneous traces for closely spaced probes averages to 0 or ±50% dependent upon whether and how a probe is shared in the relevant measurements.

For each configuration of two pairs of probes defining two simultaneously measured traces, we compute the experimental correlation coefficients between those traces and average them for different gate voltages. We further average over equivalent configurations. The statistical uncertainty is typically 5%–10%, and the correlation between simultaneous measurements of long segments with pairs of probes on opposite sides of the channel averages 98%. Figure 3 shows experimental data (circles) for both long and short devices, in comparison with the model-dependent expectations (rectangles) described above. The vertical axis presents the different possible pairings of probes. Our two models are distinguished most clearly by the cases of shared probes without overlapping segments (top two configurations), and of overlapping segments without shared probes (fourth and fifth configurations). In the long devices the channel between the probes dominates the correlations, while in the short devices the probes themselves dominate.

In the short device, the magnetic field scale of the fluctuations also demonstrates that they are not a locally determined property of the channel between the probes. For the universal conductance fluctuations, a magnetic flux (field times relevant area) of order h/e is required

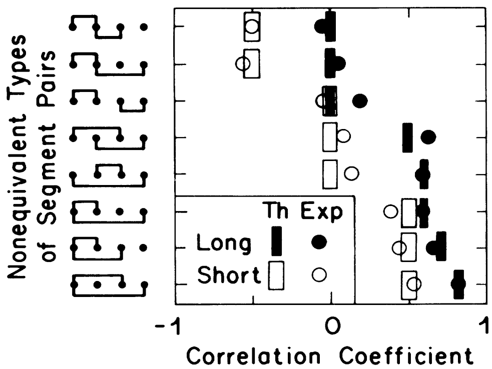

FIG. 3. "Theoretical" and experimental correlation coefficients for various nonequivalent pairs of segments. The nature of the pairs are indicated schematically on the left, in order of increasing expected correlation. The long devices ($L_{ij} > L_\phi$) agree with the theoretically justified independent-segments model, while the short devices ($L_{ij} < L_\phi$) agree with the empirically suggested independent-probes model.

to change significantly the interference effects.[4-6,15] For our closely spaced probes, $B_c = 0.05$ T, independent of segment length. This corresponds to $2.4h/e$ in an area of 0.8 μm$\times$0.25 μm, or in a 0.45 μm$\times$0.45 μm square (in view of the fact that the diffusion also spreads into the probes, which occupy considerable area). The autocorrelation functions of the long segments also have $B_c = 0.06-0.10$ T corresponding to units of channel 0.7–0.4 μm long. Thus, even though theoretical and experimental definitions of the relevant quantum areas contain possible factor-of-2 uncertainties, the crossover in behavior in Fig. 2 does correspond approximately to the scale L_ϕ, and the independently fluctuating probes at closer spacings respond to interference effects throughout a region of approximate size L_ϕ.

An independent measure of L_ϕ might be obtained from weak-localization effects, which are most clearly observed in large specimens in which the random fluctuations average away.[12] The *average* trace for the long segments, unlike for the short ones, does show a weak-localization peak at low fields. Our data unfortunately stop just short of zero field and are perturbed by significant random fluctuations, but are consistent with an L_ϕ of approximately 0.5 μm (with factor-of-2 accuracy).

Our overall results can be interpreted as showing that the spatial scale for nonlocal contributions[4,5,10] to the conductivity from random quantum interference (L_ϕ) is much greater than the spatial scale of the *average* conductivity, which is the mean free path l. Thus, when a current I is diffusing between two contacts, the randomly directed microscopic currents excited in the channel affect regions of size L_ϕ, including the potential probes, and no special physical process occurs at the attachment point. The essential physics is that the potentials of measuring reservoirs fluctuate independently (with respect to each other) for probes spaced less than L_ϕ apart. This plausible but previously unknown fact has also been observed by Benoit *et al.*[16] Figure 2 shows that the magnitude of these potential differences is approximately constant for separations up to L_ϕ. Thus, in a sense yet to be fully specified theoretically,[9,10] the measured fluctuations are characteristic of scale L_ϕ.[17] Our results are consistent with the philosophy that one cannot probe inside a quantum system without the probe affecting the measurement in an important way. Our results do not rule out the possibility, however, that less invasive probes can be devised.

Because of the nonlocality, care must be exercised in the interpretation of the experimental quantities $R_{ij} = V_{ij}/I$ and $G_{ij} = I/V_{ij}$ as resistances and conductances. They are a property of the entire multiprobe structure, *not just of the channel between the attachment points of the probes*. In some cases, such as when the voltage fluctuations between two closely spaced probes become larger than the average voltage drop between them, even the experimental definition is problematic. Nevertheless, it is important to note the experimental implications of the short-length-scale behavior, expressed in terms of these quantities, in connection with the universal conductance fluctuation theory.[4,5]

Previous calculations of conductance fluctuations are based on an ideal two-terminal model of the measurement, in which a disordered conductor of length L is connected via perfectly ordered leads to two phase-randomizing reservoirs. Such a configuration basically measures current fluctuations at constant voltage, whereas a multiterminal configuration measures voltage fluctuations at constant current. In the two-terminal theory, no phase-preserving excursions into the leads and back are allowed. Thus, at $T=0$, L_ϕ is by definition equal to L, the distance between the "leads." In this case, the conductance fluctuation is predicted to have a universal amplitude of approximately e^2/h, in reasonable agreement with the experimental G_{ij} for probe spacings $L_{ij} = L_\phi$ that we found in Ref. 6. There we also found good agreement with the theory for $L > L_\phi$, which gives conductance fluctuations smaller than e^2/h, by interpreting L as our probe spacing L_{ij}. In Ref. 6 we noted that some of our data corresponded to probe spacings $L_{ij} < L_\phi$ with $\delta G > e^2/h$. But by comparing these limited data with an unwarranted extrapolation of the long-length-scale theory, we missed the crossover to the interesting new regime encountered here, and claimed an incorrect extra half-decade agreement with the existing universal-conductance-fluctuation theory. A correct extension of the theory to scales $L < L_\phi$ will clearly have to take the multiterminal configuration into account.

As a final point, we note that the probe-spacing dependence of both the G_{ij} and R_{ij} changes at scale L_ϕ. On the assumption that the fluctuations are small compared to the average, δG_{ij} is just $\delta R_{ij}/\langle R_{ij}\rangle^2$, where the average $\langle R_{ij}\rangle$ over fields is proportional to L_{ij}.[18] Even if the R_{ij} were completely correlated so that the δR_{ij} were proportional to L_{ij}, the requirement that probes separated by L_ϕ correspond to $\delta G = e^2/h$, together with the additivity discussed above, would result in a rigorous lower bound for δG_{ij} of $(L_\phi/L_{ij})(e^2/h)$. In fact, we have seen that δR_{ij} becomes independent of probe spacing. Thus δG_{ij} varies as $(L_\phi/L_{ij})^2(e^2/h)$, in contrast to its $(L_\phi/L_{ij})^{3/2}$ dependence at long length scales.[17] In our present experiment, δG_{ij} is $20e^2/h$ for the most closely spaced (0.15 μm) probes. It follows the inverse-square dependence on spacing, and extrapolates to a conductance fluctuation of e^2/h at 0.66 μm, again roughly consistent with our estimates of L_ϕ.

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[15]Numerical simulations (Ref. 5) show that the half-width B_c of the autocorrelation function is about $2.4h/e$ divided by the device area (width times L_ϕ) for the smaller of two devices. That value of 2.4 gave a reasonable account of our data in Ref. 6; this coefficient is an obvious source of uncertainty in our knowledge of L_ϕ.

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[17]Factor-of-2 uncertainties in various coefficients are not resolved by these measurements.

[18]For our data set, the three nominally identical short segments have average resistances of 336, 386, and 369 Ω, corresponding to 2.44 kΩ/μm, compared to 2.46 kΩ/μm for the long segments.