

Length-Independent Voltage Fluctuations in Small Devices

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Conductance fluctuations in one-dimensional lines of length L shorter than the phase-coherence length L_ϕ are *not* universal but diverge as L^{-2} . Using the Onsager relations and voltage additivity, we show that the voltage fluctuations are independent of the distance between voltage probes. The antisymmetric (Hall-type) contribution to the voltage fluctuations is constant for all values of L . Measurements of the voltage fluctuations and correlation function between different regions in Au and Sb lines confirm these results.

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It has recently been shown that the low-temperature transport properties of very small normal-metal wires exhibit unexpected quantum interference behavior. In the limit where the phase-coherence length L_ϕ of the electrons is comparable to the sample size L , the conductance exhibits sample-specific¹ random fluctuations as a function of magnetic field or Fermi energy.²⁻⁷ Theoretical calculations have shown that the rms value of the fluctuations is "universal"^{3,4} and $\langle \Delta G \rangle \approx e^2/h$ when $L \approx L_\phi$. These conductance fluctuations have been observed in a wide variety of systems⁸⁻¹⁴ for $L \gtrsim L_\phi$ and found to be in quantitative agreement with the theory. The magnitude of the fluctuations decreases with energy averaging⁵⁻⁸ and ensemble averaging^{5,6,12,15} over incoherent regions. The results of the universal conductance theory were derived by calculation³ of the transmission coefficient for a long sample connected by two perfect leads to infinite reservoirs. The phase-coherence length in this model can never be larger than the distance between reservoirs. On the other hand, most experiments employ a four-probe measuring configuration and it is possible to fabricate voltage probes which are separated by distances by less than L_ϕ . The connecting leads are generally fabricated from the same material and must be included in the computation of the fluctuations. Without losing phase coherence, electrons can propagate into the leads up to a distance L_ϕ . In this Letter we show that in the limit $L_\phi > L$, where L is the distance between voltage probes of a four-wire measurement, the conductance fluctuations diverge as $\Delta G \sim (e^2/h)(L_\phi/L)^2$.

In a typical experimental arrangement, a constant current is applied to a long line and the voltage between two points separated by a distance L is measured as a function of the applied magnetic field, H . The root mean square (rms) value of the voltage fluctuation ΔV is given by $\Delta V = IR^2 \Delta G$, where R is the classical average resistance between the voltage probes, and ΔG is the rms value of the conductance fluctuations. For the case $L > L_\phi$, a classical approach^{5,12} assumes that the fluctuations in each part of the sample of length L_ϕ are uncorrelated so that the square of the fluctuations is additive.

This yields the result $\Delta V \propto \Delta V_\phi (L/L_\phi)^{1/2}$ with $\Delta V_\phi = IR_\phi^2 e^2/h$ where R_ϕ is the resistance of a wire of length L_ϕ . Using the relation $R = (L/L_\phi)R_\phi$ we obtain $\Delta G \propto (e^2/h)(L/L_\phi)^{-3/2}$. This classical addition of quantum fluctuations has been proven theoretically¹² and by experiments on arrays of rings¹⁵ and single lines.¹²

Using some simple general properties, we will demonstrate that for $L < L_\phi$ the conductance fluctuations must have a different dependence on the ratio L/L_ϕ than discussed above. The rms voltage fluctuations ΔV measured between two points x_2 and x_3 on a given line can be described by a function $\Delta V(L)$ that depends only on the distance $L = x_3 - x_2$ (translational invariance). If we consider three points 2,3,4 on a line separated by distances L_1 and L_2 , as shown in Fig. 1, voltage additivity requires that $V_{23} + V_{34} + V_{42} = 0$. Since the fluctuation of one term is always less than or equal to the sum of the fluctuations of the two others, we can write

$$\Delta V(L_1 + L_2) \leq \Delta V(L_1) + \Delta V(L_2), \quad (1)$$

$$\Delta V(L_1) \leq \Delta V(L_1 + L_2) + \Delta V(L_2). \quad (2)$$

In analogy with the classical addition of fluctuations discussed above we assume that the voltage fluctuations have a power-law dependence on L , $\Delta V(L) \propto L^\alpha$. Considering $L_1 = L_2$ in Eq. (1), we obtain $\alpha \leq 1$. Taking the limit as $L_1 \rightarrow 0$ with L_2 kept constant in Eq. (2) we find $\alpha \geq 0$. Thus, the length dependence of the total voltage fluctuations must be bounded by a constant value and a linear variation.

Experiment¹⁴ and theory¹⁶⁻¹⁹ have shown that in any four-wire measurement upon reversal of the magnetic field, the measured voltage (or conductance) fluctuations

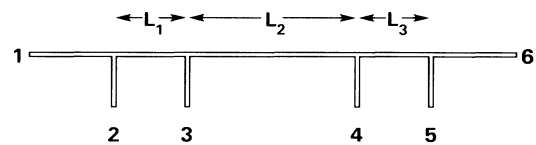


FIG. 1. Typical six-lead sample configuration.

contain a symmetric part ΔV_S and an antisymmetric part ΔV_A . This is also true in classical electrodynamics,^{14,18} where in general for an inhomogeneous conductor the elements of the resistivity tensor contain both symmetric and antisymmetric components. The transport properties that we measure are extremely sensitive to the microscopic details of the impurity configuration, and on the length scale of L_ϕ our samples cannot be considered as homogeneous. One important property of the fluctuations is that the antisymmetric part changes sign when

$$V_{16,34}(H) - V_{16,34}(-H) = V_{16,23}(-H) + V_{16,45}(-H) + V_{25,31}(H) + V_{25,64}(H) + V_{34,12}(-H) + V_{34,56}(-H). \quad (3)$$

The left-hand side of this equation is by definition twice the antisymmetric part of the voltage measured over a distance L_2 . By taking the rms value of the fluctuations for the antisymmetric part of Eq. (3) we obtain $2\Delta V_A(L_2) \leq 6\Delta V_A(L_i)$ with $L_i \leq \max(L_1, L_3)$.

We assume that both $\Delta V_A(L)$ and $\Delta V_S(L)$ have a power-law dependence on L but allow for each to have a different exponent. In order to be able to use the same function $\Delta V_A(L)$ to describe the fluctuations after reversal of the leads, all the leads must be of the same width w and thickness t . The above inequality is valid for all pairs L_2, L_i with $L_2 \geq L_i \geq w$; therefore the antisymmetric part of the voltage fluctuation cannot be a power-law function of the length L with a positive exponent. Since α cannot be less than 0, we arrive at our primary result that the antisymmetric part of the voltage fluctuation must be constant for both cases $w < L < L_\phi$ and $L > L_\phi$. For perfect point voltage probes with the separation $L < w$ we cannot make any prediction except that there must be some correlation length $\xi \leq w$ such that when $L < \xi$, the fluctuations ΔV_A start to decrease. Since experimentally¹⁴ $\Delta V_A \approx \Delta V_\phi$ for $L \approx L_\phi$, we have $\Delta V_A = c\Delta V_\phi$ and $\Delta G_A = c(e^2/h)(L/L_\phi)^{-2}$, where c is a constant of order unity. The total rms fluctuation can never be smaller than the antisymmetric contribution; therefore a dependence $\Delta V \propto L^\alpha$ with $\alpha > 0$ is not allowed for $L < L_\phi$. The final result is that the total measured voltage fluctuation must be constant for $w < L < L_\phi$,²⁰ and all our results are summarized in Table I.

We have measured the voltage fluctuations in long

TABLE I. Summary of the results for the total and antisymmetric contributions to the voltage and conductance fluctuations in a line as a function of the ratio L/L_ϕ . The rms voltage fluctuations are normalized to ΔV_ϕ and the conductance fluctuations are in units of e^2/h . a , b , and c are constants of order unity.

	$\Delta V/\Delta V_\phi$	$\Delta G(e^2/h)^{-1}$
Total $L < L_\phi$	a	$a(L/L_\phi)^{-2}$
$L > L_\phi$	$b(L/L_\phi)^{1/2}$	$b(L/L_\phi)^{-3/2}$
Antisymmetric	c	$c(L/L_\phi)^{-2}$

either the magnetic field or the voltage leads are reversed as required by the Onsager relations. It has been shown that if both the current and voltage leads are reversed,^{14,18} $V_{ij,kl}(H) = V_{kl,ij}(-H)$. The first two subscripts designate current leads and the second two the voltage leads. Both ΔV_S and ΔV_A are of the same order of magnitude^{14,19} for a sample of length L comparable to L_ϕ . For the sample of Fig. 1, by using the Onsager symmetries and voltage additivity ($V_{ij,kl} = V_{ij,km} + V_{ij,ml}$), we obtain

lines made from antimony and gold, with typical widths and thicknesses ranging from 0.03 to 0.1 μm . Voltage leads of similar dimension and material were connected alternately to each side of a line at distances varying from 0.2 to 1.8 μm . One of our samples is shown in the inset to Fig. 2(a). By selecting different leads, we can measure the fluctuations for fifteen different lengths between 0.2 and 4 μm . The devices were lithographically

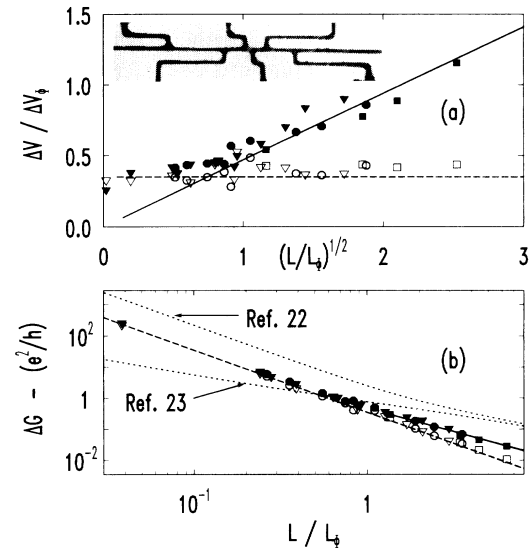


FIG. 2. (a) Measured rms voltage fluctuations normalized by ΔV_ϕ , as a function of $(L/L_\phi)^{1/2}$. The symmetric contributions are represented by solid symbols. The solid line represents the expected behavior for $L > L_\phi$. The antisymmetric part of the voltage fluctuations is represented by the open symbols and the dashed line is the predicted constant behavior. The symbols refer to different samples and temperatures: circles, Sb at $T=40$ mK and $L_\phi=1.05$ μm ; inverted triangles, Sb at $T=300$ mK and $L_\phi=0.60$ μm ; squares, Au at $T=40$ mK and $L_\phi=2.0$ μm . Inset: A photograph of the Sb sample. (b) Conductance fluctuations in units of e^2/h on a logarithmic scale for the data displayed in (a). Dotted lines are weak-localization predictions for two different boundary conditions.

patterned in a modified high-resolution scanning transmission electron microscope.²¹ The fluctuations are measured at low temperatures by sweeping of the magnetic field over a ± 3 T range. In order to compare different voltage fluctuations we must correct for any variations in linewidth. On the assumption that the line has a uniform thickness, the voltage fluctuations ΔV_ϕ can be written as $\Delta V_\phi = IR_\square^2 (e^2/h) L_\phi^2/w^2$, where R_\square is the resistance per square of the metallic film and w is the width of the line.

Figure 2(a) displays the symmetric and antisymmetric parts of the normalized voltage fluctuations $\Delta V/\Delta V_\phi$ as functions of $(L/L_\phi)^{1/2}$ for all our samples. The values of L_ϕ are estimated by a fit to the weak-localization behavior for the higher-temperature data on the Sb line. For the other temperatures and samples, L_ϕ is adjusted to obtain the same fluctuations at $L=L_\phi$. All measurements at different lengths on the same sample at constant temperature are represented by the same symbol. The data clearly show that for all values of the sample length L the antisymmetric part of the voltage fluctuations is constant but the symmetric part of the voltage fluctuations is independent of length only for $L < L_\phi$. Both these observations are consistent with our predictions. For $L > L_\phi$, the symmetric part of the voltage fluctuations increases and approaches the previously determined result¹² for the total fluctuation $\Delta V_S \approx \Delta V \propto (L/L_\phi)^{1/2}$. Each sample exhibits the same qualitative effect, independent of temperature and exact choice of L_ϕ . As $L \rightarrow 0$ the symmetric part of the fluctuations is constant and is the same order of magnitude as the antisymmetric part. Figure 2(b) displays the data of Fig. 2(a) as conductance fluctuations. The measured conductance fluctuations in a four-wire measurement can be much larger than e^2/h . The difference, however, between $(L_\phi/L)^{3/2}$ (solid line) and $(L_\phi/L)^2$ (dashed line) dependence is difficult to distinguish in this plot and was one of the problems in earlier work.¹²

The point corresponding to $L \sim 0$ in Fig. 2(a) was measured by our injecting current into leads 3 and 6 (see Fig. 1) and measuring the voltage between leads 1 and 2. The distance L_1 was $0.2 \mu\text{m}$ and, as expected classically, the average resistance was approximately zero. However, the measured rms voltage fluctuation $\Delta V_{36,12}$ was the same value as $\Delta V_{16,23}$, resulting in a " ΔG " = $2 \times 10^6 e^2/h$. The nonlocal nature of this voltage measurement is additional proof that the fluctuations in voltage are constant, independent of the position of the voltage probes for $L \ll L_\phi$. It also demonstrates the quantum nature of the transport we are measuring. Because of the wavelike properties of the electrons, the effective extent of sample is defined not by L but rather by L_ϕ . A similar result has been predicted for the case of weak localization in wires.²² It has been demonstrated²²⁻²⁴ that when L_ϕ becomes comparable to or greater than the distance between contact points along the sample the quantum

corrections to the measured conductance become larger. However, the exact functional form of the corrections depends crucially upon the exact choice of the boundary conditions on the diffusion propagator. For the case of open boundary conditions at the ends of the wire²² (appropriate for wire arrays), the corrections to the conductivity have the limiting form $\Delta G \propto (L_\phi/L)^2$ (ΔV independent of L). Recently, Doucot and Rammal's theory has been modified to include the boundary conditions appropriate for a four-terminal geometry.²³ Both corrections are shown in Fig. 2(b) as dotted lines. The magnitudes of these weak localization corrections differ by more than an order of magnitude in places from the data. This weak localization approach is not expected to describe the physics of our samples because in our system there is no self-averaging.²² However, a new approach has recently been used that includes both diagonal and off-diagonal contributions from the diffusion propagator and should be appropriate to the conductance fluctuations we measure.²⁵ In all cases it is clear that when $L_\phi > L$, L_ϕ sets the relevant length scale. The propagation of the electrons into the leads will result in additional contributions to the measured voltage fluctuations.

An interesting consequence of the power-law dependence of the rms voltage fluctuations, $\Delta V(L) \propto L^\alpha$, is that the correlation function of voltages measured between different sections of the wire depends on α . Referring to Fig. 1, we will consider the rms value of the fluctuating part of the voltage difference measured between two probes $V_{ij} = V_j - V_i$. From only the additivity of the voltages along the wire together with the definition of the rms value ($\langle V_{ij}^2 \rangle = \langle V_j^2 \rangle + \langle V_i^2 \rangle - 2\langle V_i V_j \rangle$), the correlation C between measured voltages V_{23} and V_{34} is

$$C = \frac{\langle V_{23} V_{34} \rangle}{(\langle V_{23}^2 \rangle \langle V_{34}^2 \rangle)^{1/2}} = \frac{(L_1 + L_2)^{2\alpha} - L_1^{2\alpha} - L_2^{2\alpha}}{2L_1^\alpha L_2^\alpha}. \quad (4)$$

If the two adjoining segments have equal lengths ($L_1 = L_2$), the correlation function takes a particularly simple form, $C = \frac{1}{2} \times 2^{2\alpha} - 1$. For the case $\alpha = 1$, we find a complete correlation between measured voltages, $C = 1$. For $\alpha = \frac{1}{2}$, there is no correlation between voltages, $C = 0$. For $\alpha = 0$, the correlation is $C = -\frac{1}{2}$. A similar computation can be performed for the case of separate segments (L_1 and L_3) with the same results for the correlation in the cases $\alpha = 1$ and $\alpha = \frac{1}{2}$. For the case of $\alpha = 0$ with separate segments, there is no correlation between measured voltages ($C = 0$). Figure 3 displays three voltage correlation functions, for the case of adjacent segments measured over the magnetic field range of ± 3 T. This figure shows the transition from a region without correlation ($L > L_\phi$, $\alpha = \frac{1}{2}$) to a region with negative correlation ($L < L_\phi$, $\alpha = 0$). The correlation

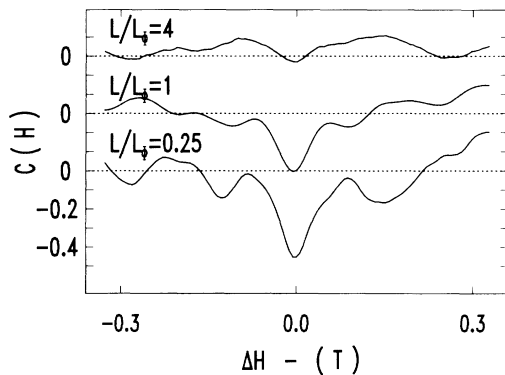


FIG. 3. Three magnetic field correlation functions between voltages measured along adjacent segments for three different ratios of L/L_ϕ computed from

$$C(H) = \frac{\int V_1(x)V_2(x+H)dx}{[\int V_1^2(x')dx' \int V_2^2(x)dx]^{1/2}}.$$

measured for the smallest ratio, $L/L_\phi=0.25$, is $C = -0.48 \pm 0.07$ ($c = -\frac{1}{2}$ theoretically for $\alpha=0$). All correlations measured for separate probes are zero within experimental error. This gives us an independent confirmation that $\alpha=0$ when $L < L_\phi$. The correlations are measured without any assumption for L_ϕ and the geometrical dimensions. The $-\frac{1}{2}$ value for the voltage correlation function arises only in the case $\alpha=0$ when the two voltage measurements share a common probe, and does not imply that the potentials are correlated. The potential fluctuations are completely uncorrelated at distances larger than some characteristic length ξ . From our comparison between segments in series with no common voltage leads, $\xi < 0.2 \mu\text{m}$.

In summary, using only the property of voltage additivity and the Onsager relations, we have shown that the total voltage fluctuations in small systems ($L < L_\phi$) are independent of the position of the voltage probes. This leads to a $(L/L_\phi)^{-2}$ divergence of the measured conductance fluctuations. The antisymmetric component of the fluctuations has the same behavior for all L_ϕ . Both results are confirmed by the experiments presented here. In addition, the observation of both nonlocal voltage fluctuations and a L/L_ϕ dependence of the correlation function provide independent confirmation of the length independent nature of voltage fluctuations when $L_\phi > l > \xi$.

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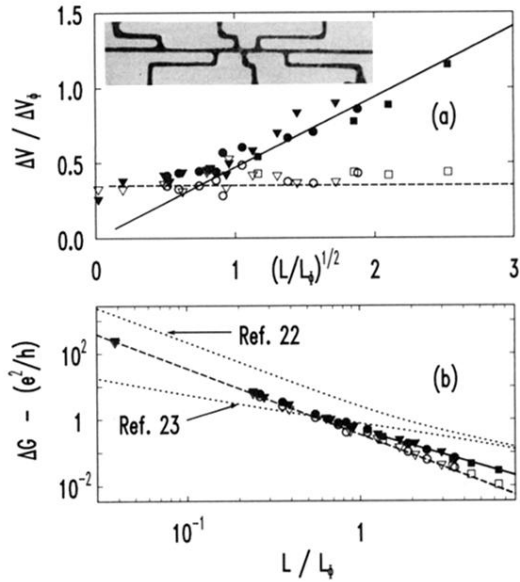


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