Magnetic Form Factors of ^{205}Tl and ^{207}Pb: Establishing the Limits of Mean-Field Theory

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The ground-state magnetization densities of ^{207}Pb and ^{205}Tl have been studied by 180° elastic electron scattering. The magnetic form factors were found to be quenched substantially relative to the predictions of the independent-particle model. Microscopic calculations indicate that core polarization can account for part of the observed suppression but it is not sufficient to reproduce the data. The quenching observed at high momentum transfers is comparable to that observed in related electron-scattering studies in the lead region, and can be explained in terms of partial occupancy of shell orbits in ^{208}Pb.

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The ground-state magnetic properties of nuclei neighboring doubly closed shells provide sensitive tests of nuclear models. The magnetic moments of these nuclei offer a well understood and easily accessible meeting ground between theory and experiment, but they offer little guidance on the physics that must be added to remedy the observed shortcomings of theoretical calculations. The comparison of theoretical and experimental results for the distributions of magnetization that give rise to these moments (or, equivalently, for their form factors) has proven to be much more informative.[1] We report here the first measurement of the elastic magnetic form factors of ^{207}Pb and ^{205}Tl. The comparison of these data with available calculations provides new quantitative tests of our understanding of nuclear structure in the lead region. Specifically, we will assess through such comparisons the adequacy of the mean-field description and the degree to which the introduction of partially occupied shell orbits in ^{208}Pb can reconcile theory and experiment.

The elastic magnetic form factors of ^{207}Pb and ^{205}Tl arise from unpaired nucleons in the $3p_{1/2}$ neutron and the $3s_{1/2}$ proton valence orbitals of the ^{208}Pb core. In the case of ^{205}Tl the absence of an additional pair of neutrons from the $2f_{5/2}$ orbit of the ^{208}Pb core introduces additional degrees of freedom that are important but well understood; their influence can be estimated through configuration-mixing calculations. (^{207}Tl would be preferable to ^{205}Tl, but it is unstable and therefore not accessible by electron-scattering experiments.) The magnetic form factors of these nuclei are unique in two important ways. First, both nuclei have spin-$\frac{1}{2}$ ground states. As a consequence, only a single multipole, $M1$, can contribute to the transverse elastic electron scattering, making the connection between the measured form factor and the nuclear current density unambiguous. Second, both the $3p_{1/2}$ and $3s_{1/2}$ orbitals are characterized by a high principal quantum number, $N=3$, which implies rapid oscillations and a concentration of strength in the nuclear interior. As a result their magnetic form factors have distinctive structure with substantial components at high momentum transfer, permitting easy detection and identification.

The experiment was performed with use of the 180° scattering facility[2] at the Bates Linear Accelerator Center. Cross sections were measured for effective momentum transfers $q_{\rm eff}$ ranging from 1.3 to 2.6 fm^{-1}. Sets of enriched metallic foils 10, 52, and 78 mg/cm^2 thick were used as targets for each isotope. Beam currents as high as 40 μA were used to measure cross sections as low as 10^{-36} cm^2/sr. It was not possible to isolate the transverse cross section at momentum transfers below 1.3 fm^{-1} since, even at 180°, the Coulomb form factor greatly exceeds the transverse form factor because of the finite solid angle of the spectrometer and multiple scattering in the target. At intermediate momentum transfers (1.3 fm$^{-1} < q_{\rm eff} <$ 1.9 fm^{-1}) we accounted for the Coulomb contribution by measuring the cross section of ^{208}Pb at 180° under identical conditions and correcting for the precisely known ratios[3] of ^{207}Pb/^{208}Pb and ^{205}Tl/^{208}Pb charge scattering. The deduced elastic magnetic form factors are shown in Figs. 1 and 2.

The predictions of the independent-particle model (IPM) are also shown in Figs. 1 and 2. The calculations were performed in distorted-wave Born approximation with Hartree-Fock wave functions.[7] An alternative choice of the Hartree-Fock effective interaction[8] yields almost indistinguishable results, while other reasonable choices for the wave functions yield cross sections that exhibit almost identical momentum-transfer dependence with variations in amplitude that do not exceed 20%.

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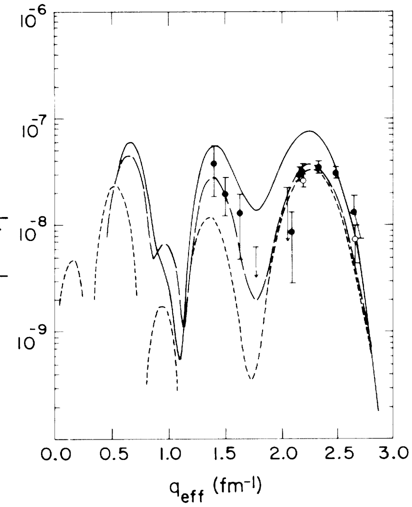

FIG. 1. The ^{207}Pb elastic-magnetic data compared to the independent-particle model (solid line), the core-polarization calculation of Ref. 4 (long dashes), and the core-polarization (Ref. 5) and meson-exchange calculation (Ref. 6) of Suzuki and co-workers (short dashes).

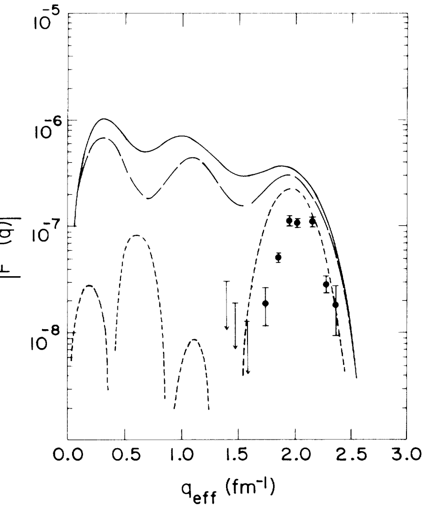

FIG. 2. The ^{205}Tl elastic-magnetic data together with the predictions of the single-particle model and the calculations of Ref. 4 and Suzuki and co-workers (Refs. 5 and 6). The conventions are the same as in Fig. 1.

The momentum-transfer dependence predicted from the radial shape of the $3s_{1/2}$ and $3p_{1/2}$ IPM Hartree-Fock wave functions[7,8] is in good agreement with our measured form factors. This indicates that the relative magnitudes of the intermediate- and high-q components of these wave functions are essentially correct as concluded in many other previous investigations.[1,8] However, the magnitude of the observed form factors is considerably smaller than the IPM predictions.

The ^{207}Pb magnetic cross section is quenched relative to the IPM prediction by a factor of $Q^2=0.5\pm0.1$; to the accuracy of the experiment no momentum-transfer dependence was observed. (Here Q is defined to be the apparent quenching of the IPM amplitude.) In the case of ^{205}Tl a strong momentum-transfer dependence is observed. The quenching in the region of the last observed maximum is $Q^2=0.3\pm0.1$. At intermediate momentum transfers the reduction observed is at least a factor of 10. The uncertainties quoted above do not include systematic error, which is estimated to be substantially smaller than the statistical uncertainty.

Deviations of elastic magnetic scattering form factors from the IPM predictions have traditionally been attributed[1] to core polarization and meson-exchange currents. Hamamoto, Lichtenstadt, and Bertsch[4] explain the quenching observed in the high-spin states of ^{208}Pb and in the elastic magnetic form factor of ^{207}Pb by first-order core polarization calculated with use of a zero-range, purely central, effective interaction. The neutron-proton interaction was neglected in their calculation and no attempt was made to estimate the meson-exchange contributions. They obtain good agreement with the form factors measured for the high-spin states and for the ^{207}Pb magnetic form factor (see Fig. 1), but their calculation does not provide a satisfactory description of the q dependence of the ^{205}Tl magnetic form factor. The dramatic quenching observed in ^{205}Tl at intermediate momentum transfers (see Fig. 2) is not reproduced. This calculation has been criticized[9] as relying on an artificially strong interaction to produce the required quenching.

The most complete calculation to date has been performed by Suzuki and co-workers.[5,6] These authors used the M3Y interaction,[10] which is of finite range and includes tensor components, to calculate the effects of core polarization and meson-exchange corrections. In the momentum-transfer region explored by our experiment, core polarization was found to be the dominant process, resulting in a net reduction from the IPM value (see Figs. 1 and 2). Good agreement is obtained with the ^{207}Pb data at high momentum transfers ($q_{\rm eff}\sim2.0$ fm^{-1}), but for lower q values the data exceed the calculation by a factor of 2. For ^{205}Tl this calculation provides a considerable improvement over the predictions of Hamamoto, Lichtenstadt, and Bertsch; still the agreement with the data is not satisfactory. The quenching at high momentum transfers ($q_{\rm eff}\simeq2$ fm^{-1}) is underestimated. However, the cancellation between single-particle and core-polarization amplitudes at intermediate momentum transfer is correctly predicted; it is attributed[1,11] partly to the tensor part of the interaction and partly to the $\nu(3p_{3/2}^{-1},3p_{1/2})_{M1}$ excitation.

Both calculations[4,6] show better agreement with ^{207}Pb than with ^{205}Tl. This is due to their neglect of configuration mixing in the ground state of ^{205}Tl. Transfer reactions and shell-model calculations indicate that the ground state of ^{205}Tl is best described as

$$|\tfrac{1}{2}^+\rangle_{205} = \alpha\,|3s_{1/2}^{(-)}\rangle \times |0^+\rangle_{206} + \beta\,|2d_{5/2}^{(-)}\rangle \times |2^+\rangle_{206} + \gamma\,|2d_{3/2}^{(-)}\rangle \times |2^+\rangle_{206}.$$

Recent $(d,{}^3\mathrm{He})$ measurements[12] find $\alpha^2=0.89\pm0.08$ in reasonable agreement with a recent $^{206}\mathrm{Pb}(e,e'p)$ measurement,[13] which gives $\alpha^2=0.79\pm0.02$. The perturbation calculation of Zamick, Klemt, and Speth[14] yields $\alpha^2=0.74$. Such a description of the ^{205}Tl ground state will produce at high momentum transfers ($q_{\mathrm{eff}}\simeq 2.0$ fm^{-1}) a form factor nearly identical to the one labeled "single particle" but quenched by a factor α^2. The form factor for the $|3s_{1/2}^{(-)}\rangle \times |0^+\rangle_{206}$ component of the ground-state wave function dominates in the region of the last maximum ($q_{\mathrm{eff}}\simeq 2$ fm^{-1}). This is due to rapid falloff of the ^{206}Pb quadrupole form factor[15] and to the fact that the $|3s^{(-)}\rangle$ single particle form factor extends to substantially higher momentum transfers than do the $|2d^{(-)}\rangle$ single-particle form factors because of its more rapid oscillation in coordinate space. Detailed calculations show that the $(2d_{5/2}^{(-)},2d_{5/2})_{M1}$ and $(2d_{3/2}^{(-)},2d_{3/2})_{M1}$ contributions to the cross section are an order of magnitude smaller than the $(3s_{1/2}^{(-)},3s_{1/2})_{M1}$ contribution. Their influence is further diminished when multiplied by the corresponding small spectroscopic factors, β^2 and γ^2. Inclusion of this extra quenching will improve the agreement between mean-field calculations[4,6] and ^{205}Tl data in the region of last maximum.

The above comparisons show that the most complete microscopic mean-field calculations available are quite sensitive to model parameters and lead only to a qualitative description of the data. The situation is even less satisfactory when one requires a consistent description of all the available electron data in the lead region.[16] This difficulty has been attributed[16,17] to the basic assumption of mean-field calculations, that shell orbits are fully occupied in ^{208}Pb. Partial occupancy can explain the reduction of single-particle strength that is observed at high momentum transfer. For a number of transitions, electron scattering at high q is predominantly sensitive to the single-particle component of the wave function.[16] As in the case of the configuration-mixing terms discussed above, the multiparticle, multihole components, arising because of correlations, contribute weakly at high momentum transfers because of the rapid falloff characteristic of their collective nature. In the case of ^{207}Pb and ^{205}Tl, the high-momentum-transfer behavior of the $M1$ form factors arising from the rapid oscillations of the $3p_{1/2}$ and $3s_{1/2}$ wave functions makes the isolation of this single-particle component of the total wave function feasible. The high-q suppression of the form factors of the components of the ^{205}Tl and ^{207}Pb ground-state wave functions not associated with the $3p_{1/2}$ and $3s_{1/2}$ valence orbitals explains the fact that static integral properties like the magnetic moment of ^{207}Pb agree rather well with mean-field calculations while, at the same time, the $M1$ form factor is found to be quenched at the last maximum.

The form factors of ^{205}Tl and ^{207}Pb at the region of the last measured maximum thus allow us to test whether the observed reduction can be explained by partial occupancy of ^{208}Pb orbitals. Partial occupancy in ^{208}Pb implies[17] that the single-particle magnetic form factors of neighboring nuclei are quenched with respect to the IPM prediction by a factor $Z=n_- - n_+$, where n_+ and n_- denote the occupation probabilities of orbitals lying immediately above and below the Fermi energy. The quenching factor $Q=0.7\pm0.07$ obtained by comparing the IPM prediction to our measured ^{207}Pb form factor determines the Z factor of the neutron shell directly. The quenching factor $\tilde{Q}=0.55\pm0.07$ observed in ^{205}Tl is due to both partial occupancy and configuration mixing. For this nucleus we can write $\tilde{Q}=\alpha^2Q=\alpha^2Z$. Taking $\alpha^2=0.79\pm0.02$ from recent $(e,e'p)$ measurements,[13] we find $Z=0.7\pm0.07$ for the proton shell as well. Our value, $Z=0.7\pm0.07$, for the discontinuity of occupation numbers at the Fermi energy differs significantly from the value used in mean-field calculations ($Z=1.0$). Given this value of Z, one can expect mean-field theory to predict single-particle properties of nuclei in the lead region with an accuracy of order 70%.

Pandharipande, Papanicolas, and Wambach[17] have estimated occupation probabilities in ^{208}Pb by adding RPA corrections to nuclear-matter results; they estimate a value for $Z=0.6\pm0.1$. Partial occupancy of shell orbitals in ^{208}Pb can account for the systematic failures of mean-field theory to account for the lack of single-particle strength throughout the lead region[16,17] and for the central depression in the charge densities of the lead isotopes.[3,18] Similar conclusions have been reached by Mahaux and co-workers[19,20] who analyzed low-energy neutron-scattering data using dispersion relations; they estimate that Z is between 0.7 and 0.8. The good agreement between these theoretical estimates and the value of $Z=0.7\pm0.07$ inferred from this measurement can be taken as further support for the need to account for partial occupancy in future theoretical descriptions of nuclei.

To summarize, the measurement of the elastic magnetic form factors of ^{205}Tl and ^{207}Pb has revealed substantial deviations from the predictions of the independent-particle model. At high q, after we account for configuration mixing in ^{205}Tl, both the ^{207}Pb and the ^{205}Tl form factors exhibit a quenching of 0.7. Core polarization appears to be an important mechanism responsible for the observed quenching. At lower q, the unusually high suppression of the ^{205}Tl form factor relative to the IPM prediction and its attribution to the tensor component of the interaction needs further exploration. The partial occupancy of ^{208}Pb orbitals, expected on the basis of many-body correlations, provides a satisfactory explanation of the observed suppression of the magnetic form factors at high q is compatible with other electron-scattering data available in the lead region. On the basis of the measurements reported here, we deduce a Z factor (the discontinuity of occupation numbers at the Fermi energy) of 0.7 ± 0.07 for both the neutron and the proton shells in ^{208}Pb, in excellent agreement with recent theoretical estimates.[17,20]

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[1]T. W. Donnelly and I. Sick, Rev. Mod. Phys. **56**, 461 (1984).

[2]G. A. Peterson, J. B. Flanz, D. V. Webb, H. DeVries, and C. Williamson, Nucl. Instrum. Methods **160**, 375 (1979).

[3]J. M. Cavedon *et al.*, Phys. Rev. Lett. **49**, 978 (1982); also B. Frois *et al.*, Nucl. Phys. **A369**, 409c (1983); J. M. Cavedon *et al.*, Phys. Rev. Lett. **58**, 195 (1987).

[4]I. Hamamoto, J. Lichtenstadt, and G. Bertsch, Phys. Lett. **93B**, 213 (1980); also I. Hamamoto, private communication.

[5]T. Suzuki, M. Oka, H. Hyuga, and A. Arima, Phys. Rev. C **26**, 750 (1982).

[6]T. Suzuki and H. Hyuga, Nucl. Phys. **A402**, 491 (1983).

[7]J. W. Negele and D. Vautherin, Phys. Rev. C **5**, 1472 (1972).

[8]J. Decharge and D. Gogny, Phys. Rev. C **21**, 1568 (1980).

[9]S. Krewald and J. Speth, Phys. Rev. Lett. **45**, 417 (1980).

[10]G. Bertsch, J. Borysowicz, H. McManus, and W. G. Love, Nucl. Phys. **A284**, 399 (1977).

[11]T. Suzuki, private communication.

[12]P. Grabmayr *et al.*, Phys. Lett. **164B**, 15 (1985).

[13]E. N. M. Quint *et al.*, Phys. Rev. Lett. **57**, 186 (1986).

[14]L. Zamick, V. Klemt, and J. Speth, Nucl. Phys. **A245**, 365 (1975).

[15]C. N. Papanicolas *et al.*, Phys. Rev. Lett. **52**, 247 (1984).

[16]C. N. Papanicolas, in *Nuclear Structures at High Spin, Excitation, and Momentum—1985*, edited by H. Nann, AIP Conference Proceedings No. 142 (American Institute of Physics, New York, 1986), p. 110, and references therein.

[17]V. Pandharipande, C. N. Papanicolas, and J. Wambach, Phys. Rev. Lett. **53**, 1133 (1984).

[18]B. Frois *et al.*, Phys. Rev. Lett. **38**, 152 (1977).

[19]M. Jaminon, C. Mahaux, and H. Ngô, Phys. Lett. **158B**, 103 (1985).

[20]C. Mahaux and H. Ngô, Nucl. Phys. **A410**, 271 (1983).