rf Stabilization of Ballooning Modes in Tokamaks

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The use of radio frequency waves in the ion cyclotron range of frequencies is shown to stabilize ballooning modes in tokamaks when the frequency and spatial wave structure are appropriately chosen. The physics of rf stabilization may provide access to high-β tokamak operation in the second stability regime and may also be useful in the suppression of edge-localized modes in present experiments. Estimates of the required power in the ion cyclotron range of frequencies are given for two examples of rf stabilization.

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Dynamic stabilization is a well established idea in plasma physics,[1] but only recently has a practical means of implementing it been proposed. Recent mirror experiments[2] have demonstrated that ion cyclotron range of frequencies (ICRF) waves can stabilize ballooning-interchange modes at relatively modest power levels. Subsequent theoretical work[3-5] showed that the stabilization is due to the ponderomotive force of the applied waves, modified by sidebands[6] at the sum and difference of the ICRF and interchange-mode frequencies.

There are several reasons for examining the physics of rf stabilization in tokamak geometry. First, there are a number of planned tokamak experiments with available ICRF power (for heating) in the megawatt range. Second, the stability and energy confinement of the tokamak are sensitive to the edge physics which can be strongly affected by ICRF antennas. In particular, ballooning modes driven unstable by the toroidal curvature tend to localize on the outside of the torus where ICRF antennas can create strongly evanescent waves (with large ponderomotive forces). Finally, the existence of a second region of ballooning stability[7] at high β suggests the possibility of applying the rf power as a means of accessing the high-β regime. It is important to emphasize that in the limit of no resonant particles the ponderomotive effect is essentially reactive, i.e., no energy transfer from wave to plasma is required to obtain stabilization of the plasma. In this case, the rf power required to obtain stability is set by other sources of dissipation (in the antenna, walls, etc.).

Our starting point is the modified energy principle,[4] $\delta W=\delta W_{\rm MHD}+\delta W_{\rm rf}$, where

$$\delta W_{\rm rf}=\tfrac{1}{2}\int dV\sum_\mu[-\delta\epsilon_\mu\boldsymbol{\xi}^*\cdot\nabla(|E_\mu|^2/16\pi)+(\delta|E_\mu|^2/16\pi)\boldsymbol{\xi}^*\cdot\nabla\epsilon_\mu]. \quad (1)$$

Here, $\delta W_{\rm MHD}$ is the usual energy principle[8] for a plasma with isotropic pressure p and displacement $\boldsymbol{\xi}$. The term $\delta W_{\rm rf}$ describes the change in potential energy of the plasma due to work done against the (perturbed) ponderomotive force of the applied rf wave, $\mathbf{E}_{\rm rf}\equiv\mathbf{E}_\omega=\sum_\mu E_\mu\hat{\mathbf{e}}_\mu$ where the sum is over polarizations (left, right, and parallel). The first term in $\delta W_{\rm rf}$ is the "direct ponderomotive term" with $\delta\epsilon_\mu=-\boldsymbol{\xi}\cdot\nabla\epsilon_\mu$, where ϵ_μ are the elements of the (diagonal) Stix dielectric tensor.[9] The second ("sideband force") term in Eq. (1) is due to the distortion of the applied rf wave structure,

$$\delta|E_\mu|^2=E^*_{\omega,\mu}E_{\omega+\omega_s,\mu}+E_{\omega,\mu}E^*_{\omega-\omega_s,\mu},$$

by the plasma motion at the magnetohydrodynamic (MHD) frequency $\omega_s\ll\omega$, resulting in the generation of sidebands at $\omega\pm\omega_s$. The sidebands satisfy a wave equation driven by the beat current of the applied wave with the MHD mode, i.e.,

$$(c^2/\omega^2)\nabla\times\nabla\times\mathbf{E}_{\omega+\omega_s}-\boldsymbol{\epsilon}\cdot\mathbf{E}_{\omega+\omega_s}=-\boldsymbol{\xi}\cdot\nabla\boldsymbol{\epsilon}\cdot\mathbf{E}_\omega. \quad (2)$$

The form of $\delta W_{\rm rf}$ given in Eq. (1) assumes no wave-particle resonances, small inverse aspect ratio $\varepsilon=a/R$ with $\epsilon_\mu|E_\mu|^2/16\pi\simeq\varepsilon p$, and $\nabla|E_\mu|^2=\nabla\psi\partial|E_\mu|^2/\partial\psi$ where $2\pi\psi$ is the poloidal flux. In writing Eq. (2) we have assumed no perturbation of the antenna current $\mathbf{J}$ by the MHD mode, i.e., $\mathbf{J}_{\omega+\omega_s}=0$.

The system of Eqs. (1) and (2) (plus the analogous wave equation for $\mathbf{E}_{\omega-\omega_s}$) describes the nonlinear coupling of the applied rf wave to any MHD instability. For high-mode-number interchange-ballooning modes we let $\boldsymbol{\xi}=X(\mathbf{k}_\perp\times\hat{\mathbf{b}}/B)\exp(iS)$, where $\mathbf{k}_\perp=\nabla_\perp S$ and the eikonal function satisfies $\hat{\mathbf{b}}\cdot\nabla S=0$. The energy δW is minimized in the limit $S\to\infty$ following the standard analysis[10] and making use of an eikonal solution of the sideband wave equations (2). The result is the ballooning equation at marginal stability,

$$\hat{\mathbf{b}}\cdot\nabla(k_\perp^2/B)\hat{\mathbf{b}}\cdot\nabla X+(8\pi/B^3)(\hat{\mathbf{b}}\times\mathbf{k}_\perp\cdot\nabla p)(\hat{\mathbf{b}}\times\mathbf{k}_\perp\cdot\boldsymbol{\kappa})X-(1/4B^3)(\hat{\mathbf{b}}\times\mathbf{k}_\perp\cdot\nabla)(\mathbf{E}_\omega\mathbf{E}^*_\omega):(\hat{\mathbf{b}}\times\mathbf{k}_\perp\cdot\nabla)\boldsymbol{\epsilon}X$$
$$-(1/2D_\omega B^3)[|\mathbf{k}_\perp\cdot(\hat{\mathbf{b}}\times\mathbf{k}_\perp\cdot\nabla)\boldsymbol{\epsilon}\cdot\mathbf{E}_\omega|^2/k_\perp^2]X=0, \quad (3)$$

where $\boldsymbol{\kappa} = \hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}}$ is the curvature and

$$D_\omega \equiv \hat{\mathbf{b}} \times \mathbf{k}_\perp \cdot \boldsymbol{\epsilon} \cdot \hat{\mathbf{b}} \times \mathbf{k}_\perp / k_\perp^2 - K_\parallel^2 c^2/\omega^2$$

with $K_\parallel$ the parallel wave number of the rf wave. The terms in Eq. (3) represent the magnetic line bending, pressure-weighted curvature drive, direct ponderomotive force, and sideband coupling, respectively. The third term is written in a form valid for arbitrary rf-wave polarization, but for economy of presentation the last term was derived under the assumption that $E_\parallel = \hat{\mathbf{b}} \cdot \mathbf{E}_{\rm rf} = 0$. (The sideband term is negligible in the limit where $E_\parallel \gg E_\perp$.) For ICRF waves with $\omega > \Omega_i$, the fourth term is always destabilizing.

In order to illustrate the basic concepts, we adopt the "standard" model equilibrium[7,11] in which the inverse aspect ratio ε is small, the flux surfaces are shifted circles, and the plasma β ($\beta = 8\pi p/B^2$) is small but has a finite gradient localized radially in a thin layer. Equation (3) then takes the form

$$(d/d\theta) f dX/d\theta + gX = 0, \tag{4}$$

$$f = 1 + \Lambda^2,$$

$$g = \alpha(\Lambda \sin\theta + \cos\theta) - \alpha_{\rm rf},$$

$$\Lambda = s(\theta - \theta_0) - \alpha(\sin\theta - \sin\theta_0),$$

where $s = rq'/q$, $q = rB_t/RB_p$, $\alpha = -Rq^2\beta'$, B_t and B_p are the toroidal and poloidal components of the magnetic field, and primes denote d/dr. Here, θ is the extended poloidal coordinate[10] and θ_0 must be chosen to yield the most unstable mode. The effect of the rf terms in Eq. (4) is given by

$$\alpha_{\rm rf} = \frac{R^2 q^2}{4B^2} \sum_\mu \left\{ \epsilon_\mu \frac{d\ln N}{dr} \frac{d|E_\mu|^2}{dr} + 2\left(\frac{d\ln N}{dr}\right)^2 T_s \right\}, \tag{5}$$

where we let $d\epsilon_\mu/dr = \epsilon_\mu d\ln N/dr$, and T_s represents a term of the order $(\epsilon_\mu |E_\mu|)^2$ whose exact form depends on the choice of rf wave. Although $\alpha_{\rm rf}$ is not necessarily positive since $\epsilon_\mu(\omega, \mathbf{K}) d|E_\mu|^2/dr$ carries a sign, we will show that in two different applications the rf parameters $(\omega, \mathbf{K})$, and hence the radial wave structure, can be chosen to obtain stability.

One can recast Eq. (4) into the canonical form $d^2\hat{X}/d\theta^2 = V(\theta)\hat{X}$, where $\hat{X} = \sqrt{f}X$ and

$$V(\theta) = [(s - \alpha\cos\theta)^2 + f(\alpha_{\rm rf} - \alpha\cos\theta)]/f^2.$$

The condition $V > 0$ for all θ yields the *sufficient* stability condition $\alpha_{\rm rf} \gtrsim \alpha$. This gives the threshold estimate

$$E_{\rm rf} = C a^{1/2} R^{-1} N^{-3/4} B^2, \tag{6}$$

where we assumed that $\beta \simeq a/R$ and $L_{\rm rf} \equiv (d\ln|E_{\rm rf}|^2/dr)^{-1}$ scales like the appropriate skin depth, c/ω_{pi} for $E_\perp$ and c/ω_{pe} for $E_\parallel$. Here the constant is $C = 2.1 \times 10^{13}$ V cm$^{7/4}$ kG^{-2} for $E_\perp$ and 7.4×10^{10} V cm$^{7/4}$ kG^{-2} for $E_\parallel$, respectively.

In Fig. 1, we plot the marginal-stability boundaries corresponding to the numerical solution of Eq. (4) with $\alpha_{\rm rf}$ as a parameter. This procedure is strictly valid when $L_{\rm rf} \ll L_N \equiv (d\ln N/dr)^{-1}$ so that the sideband term is negligible. We observe that the effect of increasing the ponderomotive force ($\alpha_{\rm rf}$) is to increase both the first and second stability regions. Also note that for a fixed q profile (so that the maximum value of s is fixed), there is a critical value of $\alpha_{\rm rf}$ which provides stability for all α. Within the context of the s-α model, it is possible to provide access from the first to the second stability regime by means of rf stabilization, after which the rf could be turned off. The scaling of Eq. (6) suggests that the entrance to second stability should be made at low B.

We now turn to two specific examples of rf stabilization. First, motivated by the rf stabilization results in mirrors,[2] we consider the case of fast-wave eigenmodes. We solve the wave equation for the applied wave $E_\omega(r) \propto \exp[-i(m\theta + K_\parallel z)]$ in a cylindrical plasma model neglecting $E_\parallel$, and compute the direct ponderomotive and sideband contributions to $\alpha_{\rm rf}(r,\theta)$. The result of a number of computer runs varying the rf parameters and the density profile is that $\alpha_{\rm rf}$ is stabilizing for $\omega > \Omega_i$ when the density gradient is localized to the plasma edge. For a given rf eigenmode, Eq. (4) is solved on each field line to determine the global stability properties of the specified equilibrium.

The resulting stability diagram is shown in Fig. 2 for the profiles $q(r) = 1 + (q_a - 1)(r/a)^2$, $N(r)/N_0 = \beta(r)/\beta_0 = 1 - (r/a)^8$ (where a is the plasma radius), and the parameters $m = 1$, $\omega/\Omega_i = 1.3$, $N_0 = 1 \times 10^{13}$ cm^{-3}, $B = 5$ kG, and several values of $E_{\rm rms} \equiv (\int dr\, r|\mathbf{E}|^2/\int dr\, r)^{1/2}$.

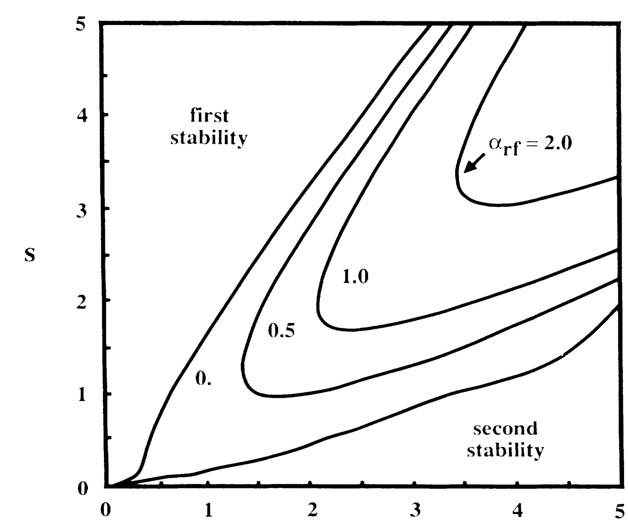

FIG. 1. Marginal stability curves in the plane of shear (s) and pressure gradient (α) for fixed values of the ponderomotive force term ($\alpha_{\rm rf}$). Curves shown are for the most unstable θ_0.

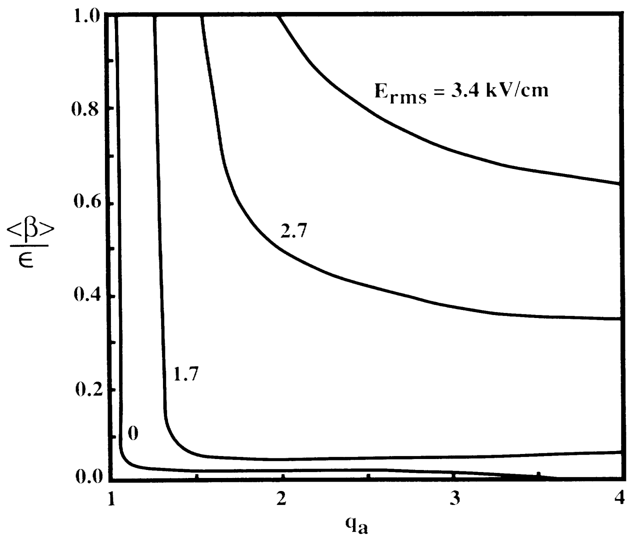

FIG. 2. Global marginal-stability curves in the plane of average toroidal β over inverse aspect ratio vs edge q for fixed amplitudes of the fast-wave eigenmode, $E_{\rm rms}$. Curves shown are for the most unstable θ_0.

We find that $E_{\rm rms}=1.7$ kV/cm is sufficient to increase the first stability region substantially while fields on the order of 3.4 kV/cm provide access to the high-β operating regime. Detailed results[12] show that the global stability boundaries are sensitive to profile shapes and that inclusion of $E_\parallel$ is required to stabilize the last flux surfaces $0.98<r/a<1.00$ where the $\mathbf{E}_\perp$ sidebands are large. The present analysis assumes that the fast-wave frequency can be chosen to avoid all plasma resonances and thereby attain large-amplitude toroidal eigenmodes. Thus, the spatial variation of B (and hence of the cyclotron resonances) from the inside to the outside of the torus restricts this particular application to small-ε devices.

The power requirement for fast-wave stabilization can be estimated from $P=\omega W/Q$, where W is the stored rf energy and Q is the rf quality factor. Estimating $\omega\approx\Omega_i$ and

$$W\approx(2\pi a)(2\pi R)L_{\rm rf}\epsilon_\perp|E_\perp|^2/16\pi$$

with $L_{\rm rf}\simeq c/\omega_{pi}$ and $\epsilon_\perp\simeq(\omega_{pi}/\Omega_i)^2$, and employing the sufficient condition $\alpha_{\rm rf}\simeq\alpha$ to estimate the required $E_\perp$, we obtain the scaling $PQ\simeq(64\ {\rm MW\ cm^{-1}\ kG^{-1}\ keV^{-1}})aBT$. For the parameters $a=40$ cm, $B=5$ kG, and $T=3$ keV (appropriate for a high-β experiment) this estimate gives $PQ\sim4\times10^4$ MW, compared with $PQ\sim10^2$–10^3 MW in present tokamaks.

As a second example, and one that is more relevant to present experiments, we consider edge plasma stabilization using the evanescent near field of an ICRF antenna. For this purpose, it is preferable to use an rf wave with a large $E_\parallel$ component, such as that produced by ion Bernstein-wave couplers.[13] The ponderomotive force in this case is enhanced because of both the large parallel conductivity $[|\epsilon_\parallel|=(\omega_{pe}/\omega)^2\gg|\epsilon_\perp|]$ and the shorter radial scale length $(L_{\rm rf}\ll c/\omega_{pi})$. Furthermore, because $|L_N|\gg|L_{\rm rf}|$ the destabilizing sidebands are negligible. By Eq. (5), the direct ponderomotive term is stabilizing when $|E_\parallel|^2$ decays into the plasma. Although the stabilizing effect is confined to a narrow layer of thickness $L_{\rm rf}$, it is possible that this is sufficient to improve particle and energy confinement by stabilizing edge-localized modes, particularly in H-mode operation of divertor tokamaks.[14]

From Fig. 1 and Eq. (6) we find that "high-n" ballooning stability is easily obtained for reasonable fields $E_\parallel$, where n is the toroidal MHD mode number. For the more interesting intermediate-n modes such that $1\ll\sqrt{n}\ll a/L_{\rm rf}$, the ballooning eigenfunction has the usual $a/\sqrt{n}$ radial scale length[10] in the region $r<r_c$, where r_c denotes the location of the rf-stabilized layer, i.e., $\alpha_{\rm rf}(r_c)=1$. In the rf layer $r>r_c$, the dominant terms in δW are the ponderomotive force and the radial line bending term. The approximate intermediate-n ballooning equation

$$\hat{\mathbf{b}}\cdot\nabla(\hat{\mathbf{e}}_\psi\cdot\nabla)^2\hat{\mathbf{b}}\cdot\nabla X+(1/4B^2)(\hat{\mathbf{b}}\times\mathbf{k}_\perp\cdot\nabla\epsilon_\parallel)(\hat{\mathbf{b}}\times\mathbf{k}_\perp\cdot\nabla|E_\parallel|^2)X=0 \quad (7)$$

can be solved by separation of variables on the assumption of an exponential decay of $E_\parallel$ in the plasma. The solution[12] shows that X decays rapidly for $r>r_c$ to avoid the stabilizing ponderomotive contribution if $\alpha_{\rm rf}\sim(a/L_{\rm rf}nq_a)^2$, giving a quiescent layer at the plasma surface. Using this estimate and the definition of $\alpha_{\rm rf}$, one obtains the following scaling for the required $E_\parallel$:

$$E_\parallel=[(1.4\times10^5\ {\rm V\ cm^{1/4}\ kG^{-2}})/nq_a^2](2\pi R/L_A)^{1/2}(a/R)L_N^{1/2}N_e^{-1/4}B^2, \quad (8)$$

where N_e is the edge density and we assumed $L_{\rm rf}\simeq c/\omega_{pe}$ and a single antenna of length L_A. [In Eq. (8), the factor $(2\pi R/L_A)^{1/2}$ due to toroidal averaging is valid when the MHD parallel wavelength is much larger than $2\pi R$, i.e., when $q_a\gg1$.] For the case $a=40$ cm, $R=140$ cm, $B=10$ kG, $N_e=2\times10^{12}$ cm^{-3} (edge), $L_N=10$ cm, $q_a=5$, and $L_A=60$ cm, we find that $E_\parallel\approx200$ V/cm would be sufficient to exclude $n>8$ modes from the edge plasma.

The near-field value of $E_\parallel$ on the plasma surface can be related to the current I_A of a model field-aligned antenna located at $r=r_A$, and the power dissipation computed from $P=R_LI_A^2$, where R_L is the antenna loading resistance. Taking $\omega\approx\Omega_i$ we obtain the scaling

$$P=(2.8\times10^{-4}\ {\rm MW\ cm^2\ V^{-1}\ \Omega^{-1}\ kG^2})g^2R_LE_\parallel^2B^{-2}, \quad (9)$$

where the dimensionless factor g is determined by the antenna-plasma coupling.[15] Using the illustrative values $L_A = 60$ cm, $R_L = 2$ Ω, and $g = 3$, Eqs. (8) and (9) give the estimate $P \approx 134$ MW/n^2 so that $n > 8$ modes could be suppressed with about 2 MW power. A self-consistent numerical solution of the full wave equation for $\mathbf{E}_\omega$ to be reported elsewhere[12] indicates that a quiescent layer of about a centimeter in thickness is achievable for the parameters used here when ω lies above the Alfvén frequency at the plasma edge.

Two examples of ICRF ponderomotive effects in tokamaks have been presented. Other applications, e.g., control of kink modes and equilibrium modifications, are under study. Independent investigations[16] of rf stabilization in tokamaks have also recently been drawn to our attention.

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[1]See, for example, the reviews of early work in S. M. Osovets, Plasma Phys. **6**, 421 (1964); G. Berge, Nucl. Fusion **12**, 99 (1972).

[2]J. R. Ferron, N. Hershkowitz, R. A. Breun, S. N. Golovato, and R. Goulding, Phys. Rev. Lett. **51**, 1955 (1983); Y. Yasaka and R. Itatani, Phys. Rev. Lett. **56**, 2811 (1986), and references therein.

[3]P. L. Similon, A. N. Kaufman, and D. D. Holm, Phys. Fluids **29**, 1908 (1986).

[4]D. A. D'Ippolito and J. R. Myra, Phys. Fluids **29**, 2594 (1986), and references therein.

[5]J. R. Myra, D. A. D'Ippolito, and G. L. Francis, Phys. Fluids **30**, 148 (1987).

[6]J. B. McBride, V. Stefan, and N. A. Krall, Phys. Rev. Lett. **54**, 42 (1985).

[7]B. Coppi, A. Ferreira, J. W.-K. Mark, and J. J. Ramos, Nucl. Fusion **19**, 715 (1979); J. M. Greene and M. S. Chance, Nucl. Fusion **21**, 453 (1981).

[8]I. B. Bernstein, E. A. Frieman, M. D. Kruskal, R. M. Kulsrud, Proc. Roy. Soc. London, Ser. A **244**, 17 (1958).

[9]T. H. Stix, *The Theory of Plasma Waves* (McGraw-Hill, New York, 1962).

[10]J. W. Connor, R. J. Hastie, and J. B. Taylor, Proc. Roy. Soc. London, Ser. A **365**, 1 (1979); Y. C. Lee and J. W. Van Dam, in *Proceedings of the Finite Beta Theory Workshop, Varenna, 1977,* edited by B. Coppi and W. L. Sadowski (U.S. Department of Energy, Washington, DC, 1977), p. 93; D. Dobrott, D. B. Nelson, J. M. Greene, A. H. Glasser, M. S. Chance, and E. A. Frieman, Phys. Rev. Lett. **39**, 943 (1977).

[11]J. W. Connor, R. J. Hastie, and J. B. Taylor, Phys. Rev. Lett. **40**, 398 (1978).

[12]D. A. D'Ippolito, J. R. Myra, and G. L. Francis, to be published.

[13]M. Ono *et al.*, in Proceedings of the Eleventh International Conference on Plasma Physics and Controlled Nuclear Fusion Research, Kyoto, Japan, 1986, to be published, paper F-I-3; M. Porkolab *et al.*, *ibid.*, paper F-II-2.

[14]F. Wagner *et al.*, Phys. Rev. Lett. **49**, 1408 (1982).

[15]For present ion Bernstein heating experiments, we estimate $g \approx 3$. The possibility of reducing g for optimizing rf stabilization rather than heating merits further study.

[16]X-H. Yang, private communication; A. Sen, P. K. Kaw, and A. K. Sundaram, in Ref. 13, paper E-II-2-2.