Spatial Dissipative Structures in Passive Optical Systems

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We consider a nonlinear, passive optical system contained in an appropriate cavity, and driven by a coherent, plane-wave, stationary beam. Under suitable conditions, diffraction gives rise to an instability which leads to the emergence of a stationary spatial dissipative structure in the transverse profile of the transmitted beam.

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A large variety of unstable phenomena have been reported in optics which lead to the appearance of organized behavior in time or both in time and in space. For example, it is well known that some optical systems, when subjected to stationary control parameters, may exhibit a pulsed, an oscillatory, or a chaotic output[1]; it has been found also in optical bistability that spatial patterns of transverse[2] and longitudinal[3] type may occur in the switching process from the lower to the upper branch of the hysteresis curve. To our knowledge, however, the possibility of soft-mode symmetry-breaking instabilities leading to the spontaneous formation of stationary spatial patterns (dissipative structures)[4] in an initially uniform system has never been pointed out in the field of optics. Such instabilities have drawn considerable interest in chemistry and in developmental biology[4,5] where they are commonly known as Turing instabilities. In these fields they arise generally from the coupling between nonlinear chemical reactions and diffusion. We show here on a simple optical model that analogous phenomena may arise from the coupling between light dispersion and diffraction in an appropriate optical cavity.

We call z the longitudinal coordinate and x,y the transverse coordinates. We consider a cavity formed by four mirrors, two orthogonal to the axis z with a distance L and transmission coefficient $T \ll 1$, and two orthogonal to the axis x with a distance b and 100% reflectivity. The cavity is filled with a medium with a nonlinear refractive index. A coherent, stationary, plane-wave field E_I is longitudinally injected into the cavity. We assume that both the input and the internal cavity field are linearly polarized in the y direction; hence, because of the transversality condition, the internal field is independent of y. We assume that it has the structure $E(x)\cos(K_z z)\exp(-i\omega_0 t)+\text{c.c.}$, where ω_0 is the frequency of the input field E_I and $K_z=\pi n_z/L$, with n_z being a positive integer. The field transmitted by the system is proportional to the normalized envelope function $E(x)$, which obeys the evolution equation

$$\frac{\partial E}{\partial \bar{t}} = -E + E_I + i\eta E(|E|^2 - \theta) + ia\frac{\partial^2 E}{\partial \bar{x}^2}. \qquad (1)$$

The variable E^* obeys the complex-conjugate equation. E_I is taken real and positive for definiteness. The independent variables are $\bar{x}=x/b$, $\bar{t}=kt$, where $k=cT/2L$ is the cavity linewidth. The parameter a is defined as $a=1/2\pi T\mathcal{F}$, where $\mathcal{F}=b^2/\lambda L$ is the Fresnel number and λ is the wavelength. The quantity η is defined as $+1$ or -1 in the case of self-focusing or self-defocusing nonlinearity, respectively, and $\eta\theta$ is the detuning parameter. This model can be derived from the Maxwell-Bloch equations for a two-level system by introduction of the mean-field limit $T \ll 1$, which reduces the dynamics to the single longitudinal mode n_z, the purely dispersive limit, and the adiabatic elimination of the atomic variables. Because $T \ll 1$ and we want a to be of order unity, we assume that $\mathcal{F} \gg 1$ (generalized mean-field limit). The model (1) holds also for a Kerr medium.

The cavity supports the transverse modes $\cos(\pi n\bar{x})$, with $n=0,1,\ldots,$ which corresponds to the reflecting boundary conditions $\partial E/\partial\bar{x}=0$ on the mirrors orthogonal to the axis x.

Equation (1) admits transversally homogeneous stationary solutions, governed by the well-known cubic steady-state equation[6]

$$E_I^2 = |E_s|^2\{1+(|E_s|^2-\theta)^2\}. \qquad (2)$$

For $\theta \le \sqrt{3}=\theta_c$ the steady-state curve $|E_s|^2(E_I^2)$ is single valued (Fig. 1), whereas for $\theta > \sqrt{3}$ it is S shaped and leads to a hysteresis cycle. In order to analyze the stability of these solutions, we introduce in Eq. (1) the decomposition $E=E_s+\delta E$ and neglect all terms nonlinear in the perturbation δE, δE^*. The *Ansatz*

$$\begin{pmatrix} \delta E(\bar{x},\bar{t}) \\ \delta E^*(\bar{x},\bar{t}) \end{pmatrix} = \exp(\lambda\bar{t})\cos(\pi n\bar{x}) \begin{pmatrix} \delta E_n \\ \delta E_n^* \end{pmatrix} \qquad (3)$$

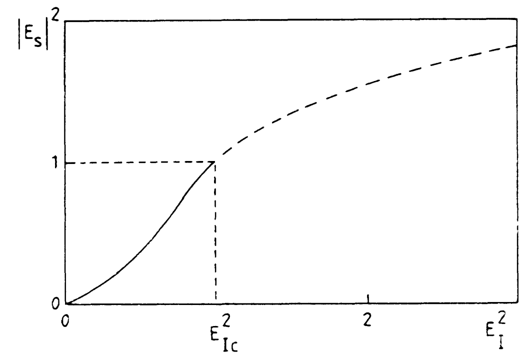

FIG. 1. Steady-state curve of transmitted vs incident intensity for $\theta=1$. The broken portion is unstable in the self-focusing case, when condition (6) is satisfied (see Fig. 2).

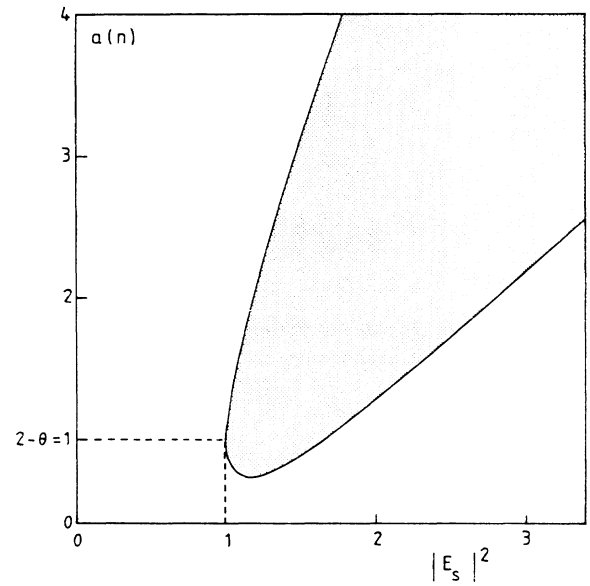

FIG. 2. Self-focusing case, $\theta=1$. The shaded region corresponds to the unstable domain in the plane of the variables $a(n)=a\pi^2n^2$ and $|E_s|^2$.

leads then to a homogeneous set of algebraic equations for δE_n and δE_n^*, and to a quadratic eigenvalue equation of the form $\lambda^2+2\lambda+\zeta(|E_s|^2,a(n))=0$, with $a(n)=a\pi^2n^2$. The stationary solution is unstable when one of the roots of this equation has a positive real part for at least one choice of n. This amounts to the condition that the constant term ζ be negative. Explicitly this condition reads

$$1+(|E_s|^2-\theta)(3|E_s|^2-\theta)+a(n)[a(n)-2\eta(2|E_s|^2-\theta)]<0. \quad (4)$$

One notes the following: (i) The first two terms in Eq. (4) are equal to the slope $dE_I^2/d|E_s|^2$ of the homogeneous steady-state curves as given by Eq. (2). Accordingly, instability is possible with respect to homogeneous perturbations, i.e., corresponding to $n=0$, only if the steady-state curves present a negative-slope portion, i.e., if $\theta>\theta_c=\sqrt{3}$. (ii) In the positive-slope portions of the steady-state curve, the system is unstable with respect to inhomogeneous perturbation modes lying in the interval

$$a^{(-)}(|E_s|^2)<a(n)<a^{(+)}(|E_s|^2), \quad (5)$$

where

$$a^{(\pm)}(|E_s|^2)=\eta(2|E_s|^2-\theta)\pm(|E_s|^4-1)^{1/2},$$

when

$$|E_s|^2\geq 1 \text{ and } \eta(2|E_s|^2-\theta)>0. \quad (6)$$

The unstable domain is shown in Fig. 2 for $\eta=1$, $\theta=1$ (see also Fig. 1). This robust instability arises for the following reasons. If we consider only the longitudinal modes of the cavity, for $T\ll 1$ the modes are well separated and therefore the input field selects the nearest mode (resonant mode). On the other hand, when the parameter a is of order unity there are transverse modes whose frequency distance from the resonant homogeneous mode is on the order of the modal width k. Therefore, these modes compete with the resonant mode, and via the instability give rise to a spatial coexistence of modes, quite different from the temporal coexistence that arises in several oscillatory behaviors.[1]

In the neighborhood of the bifurcation point at which the first unstable mode n_c appears, i.e., for $|E_s|^2=1$, $E_{Ic}=[1+(1-\theta)^2]^{1/2}$, $a(n_c)=\eta(2-\theta)$, the small-amplitude inhomogeneous stationary solutions of (1) can be calculated analytically by the methods of bifurcation theory. More precisely, by defining $E_1=\mathrm{Re}E$, $E_2=\mathrm{Im}E$, and setting $E_1=E_{1,s}+u(\bar{x})$, $E_2=E_{2,s}+v(\bar{x})$, where $E_{1,s}$, $E_{2,s}$ refer to the homogeneous stationary solution and u,v represent the inhomogeneous part of the new stationary solutions, one finds

$$u(\bar{x})=\pm\{(E_I-E_{Ic})/\phi(\theta)\}^{1/2}\cos(n_c\pi\bar{x})+\{(E_I-E_{Ic})/\phi(\theta)\}[f_1(\theta)+f_2(\theta)\cos^2(n_c\pi\bar{x})], \quad (7a)$$

$$v(\bar{x})=\pm\eta(2-\theta)\theta^{-1}\{(E_I-E_{Ic})/\phi(\theta)\}^{1/2}\cos(n_c\pi\bar{x})+\{(E_I-E_{Ic})/\Phi(\theta)\}[f_3(\theta)+f_4(\theta)\cos^2(n_c\pi\bar{x})], \quad (7b)$$

$$\Phi(\theta)=(41-30\theta)[1+(1-\theta)^2]^{1/2}/18\theta^2, \quad (8)$$

and $f_i(\theta)$, $i=1,2,3,4$, are rational functions whose explicit expressions will be given elsewhere.[7] The $\pm$ sign in front of the main term in Eqs. (7) describes the bifurcation of two nonhomogeneous stationary solutions at the critical point. For the values of θ such that $\Phi(\theta)>0$ [$\Phi(\theta)<0$] the two solutions exist for $E_I>E_{Ic}$ ($E_I<E_{Ic}$) and therefore are

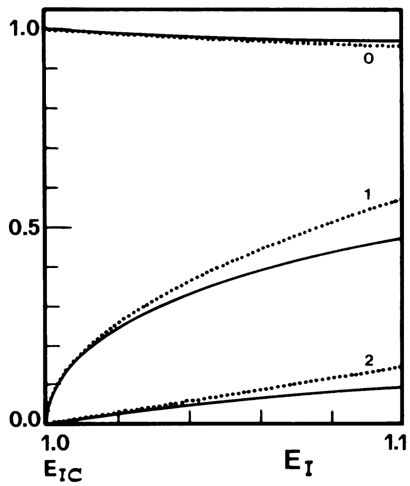

FIG. 3. The absolute values of the amplitudes of the modes $n=0,1,2$ are graphed as functions of the input field (solid line, numerical; dotted line, analytical).

stable (unstable) because the bifurcation is supercritical (subcritical). Note that the function $\Phi(\theta)$ is independent of η. In the self-focusing case $\eta=+1$ the bifurcation is supercritical for $\theta<\frac{41}{30}\simeq 1.4$; in the self-defocusing case $\eta=-1$ the instability arises for $\theta>2$ and therefore is always subcritical. Figure 3 compares, for $\theta=1$, $\eta=1$, $n_c=1$, the modal amplitudes obtained from Eqs. (7) and (8) (dotted line) with those obtained by solving Eq. (1) numerically (full line).

Clearly, several investigations are in order, e.g., a numerical analysis to calculate the large-amplitude stationary states that evolve far from the critical point, in the search for higher-order bifurcations. Our analysis is certainly related to the known results on self-focusing and filamentation of light beams in nonlinear media. It is especially related to the diffractive instabilities in passive optical devices described by McLaughlin, Moloney, and Newell.[8] Our model, however, is drastically different because it assumes the mean-field limit. Furthermore, the instability reported here does not require a bistable steady-state curve and leads to stationary instead of dynamical structures.

Our results predict that a plane-wave input field is spontaneously converted into a stationary beam which presents a transverse stripe structure. An experimental observation of this phenomenon would be of extraordinary interest. The reflecting boundary conditions in x can be realized, at least approximately, by coating of the sides of the sample, orthogonal to the axis x, by dielectric layers with a refractive index larger than that of the sample itself. We note furthermore that the polarization that we considered was selected because it allows for an exact treatment of the problem. On the other hand, the choice of an electromagnetic field linearly polarized with the *magnetic* field oriented in the y direction allows for a simple realization by means of a cavity with conducting walls. In this case, again Maxwell's equations imply that $\mathbf{E}$ does not depend on y. The mode configuration for the component E_x is[9] $\cos(\pi n x/b)\sin(\pi n_z z/L)$. For $n\neq 0$ there is also a component $E_z \propto \sin(\pi n x/b)\cos(\pi n_z z/L)$, but this is extremely small because $E_z/E_x \propto n/n_z \ll 1$, and the condition $n/n_z \ll 1$ is already assumed in the derivation of Eq. (1). With neglect of E_z, our previous treatment remains completely unchanged.

In order to obtain a plane-wave configuration for the input beam, it is necessary to magnify it by lenses and use only its central part. This means a loss of intensity. However, the power requirements for the observation of this instability are not severe because the instability threshold is lower than the bistability threshold.

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