Comparison of the Microscopic Potential with the Optical Potential in the $\alpha + {}^{16}O$ System

T. Wada
Research Institute for Fundamental Physics, Kyoto University, Kyoto 606, Japan

and

H. Horiuchi
Department of Physics, Kyoto University, Kyoto 606, Japan
(Received 2 February 1987; revised manuscript received 15 April 1987)

The $\alpha + {}^{16}O$ resonating-group–method calculation which fits well the spectroscopic data of the ${}^{20}Ne$ levels is shown to fit well also the scattering cross section up to high energy. The equivalent local potential derived from the resonating-group method is found to be very similar to the optical potential of Michel *et al.* Thus it is concluded that a significant part of the energy dependence of the real part of the $\alpha + {}^{16}O$ optical potential is due to the internucleus antisymmetrization.

PACS numbers: 25.55.Ci, 21.60.Gx, 24.10.Cn, 24.10.Ht

It is now believed that the discrete ambiguity of the optical potentials for light-ion projectiles such as ${}^{3}He$ and α can be removed by use of the high-energy scattering data exhibiting the nuclear rainbow effect.[1] Therefore any microscopic theory of the internucleus potential is now required to reproduce at least the essential features of these light-ion optical potentials.

The present authors and their collaborators have been studying the internucleus potentials microscopically by deriving them by the use of the resonating-group method (RGM).[2] We have shown that the basic properties of the derived potentials are in good agreement semiquantitatively with those of the real parts of the light-ion optical potentials.

In order to advance further the microscopic study of the internucleus potential, we make in this paper a detailed quantitative comparison in the $\alpha + {}^{16}O$ system between the microscopic potential derived from the RGM and the real part of the optical potential of Michel *et al.*[3] which fits the scattering cross sections very well for a wide energy range. Our aims are to see to what extent the real part of the optical potential can be reproduced by RGM and particularly to what extent the energy dependence of the real part of the optical potential comes from the effect of the internucleus antisymmetrization.

The internal wave functions of α and ${}^{16}O$ are both described by closed-shell configurations of the harmonic-oscillator shell model with the common oscillator parameter $\nu = m\omega/2\hbar = 0.16\ \mathrm{fm}^{-2}$. As the effective two-nucleon force we adopt the Hasegawa-Nagata-Yamamoto force[4] with the ${}^{3}E$ strength parameter of the medium-range part, $V_m({}^{3}E) = -546 + \Delta$ MeV, being given by $\Delta = 21.3$ MeV. In Matsuse, Kamimura, and Fukushima[5] it was already shown that the $\alpha + {}^{16}O$ RGM with these parameters reproduces very well the spectroscopic data of the low-lying states of ${}^{20}Ne$. Figure 1 shows the good correspondence of the rotational energy levels between theory and experiments where the 0^+ level is the ground state.

In order to calculate the scattering cross section, we introduce a phenomenological imaginary potential into the RGM equation of motion as follows:

$$[H + \sqrt{N}\, iW(r) \sqrt{N} - E_r N]\chi = 0, \tag{1}$$

where H and N are the Hamiltonian and norm integral kernels, respectively, E_r the scattering energy, and $W(r)$ is of the squared Woods-Saxon form

$$W(r) = W_0/\{1 + \exp[(r - R_I)/2a_I]\}^2. \tag{2}$$

Following Ref. 3 we fix the values of W_0 and a_I to be -25 MeV and 0.65 fm, respectively, while the value of

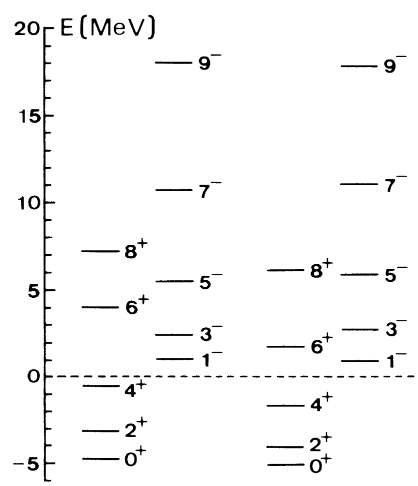

FIG. 1. Comparison of the observed spectra (obs.) (Ajzenberg-Selove, Ref. 6) of the ground ($K^\pi = 0^+$) and excited ($K^\pi = 0^-$) rotational bands in ${}^{20}Ne$ with those calculated (cal.) by $\alpha + {}^{16}O$ RGM.

© 1987 The American Physical Society

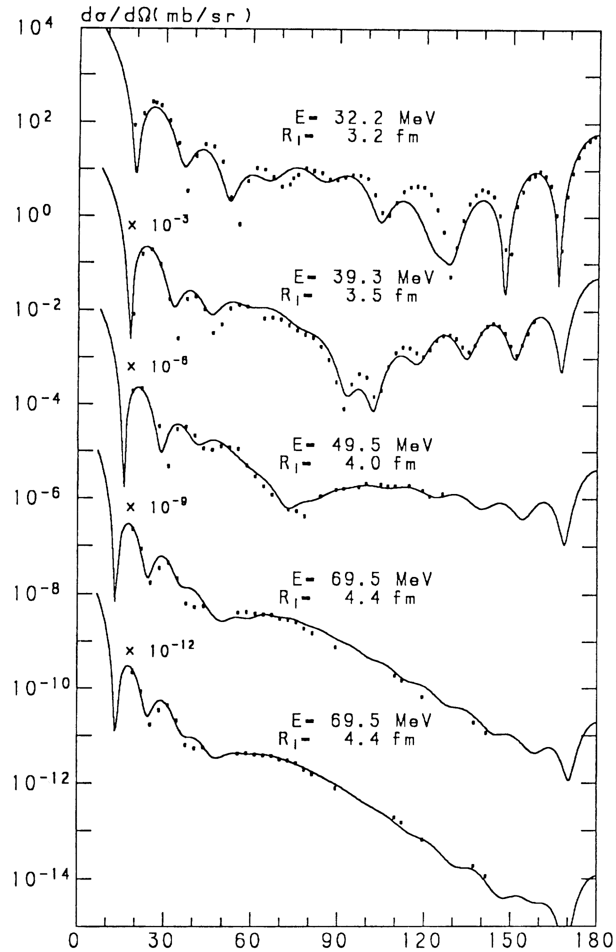

FIG. 2. Comparison of the experimental data (dots, Ref. 3 and Cowley and Heymann, Ref. 7) for $\alpha+{}^{16}O$ elastic scattering with $\alpha+{}^{16}O$ RGM calculations (solid lines) at four laboratory energies E of the α particle. The upper four solid curves are calculated with $\Delta=21.3$ MeV, while the last one for $E=69.5$ MeV is with $\Delta=35$ MeV.

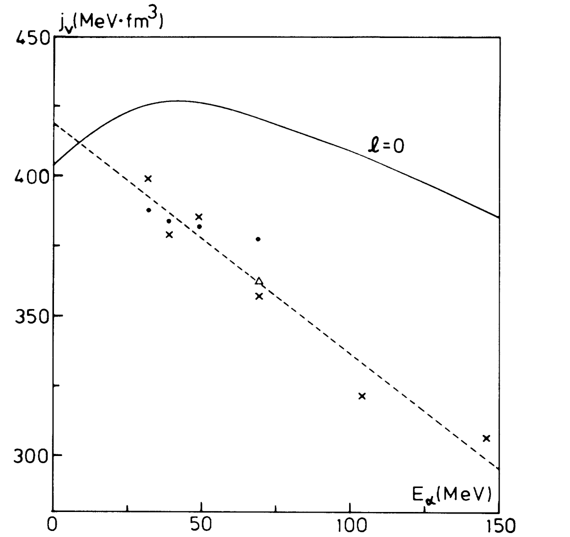

FIG. 3. Comparison of the volume integral per nucleon pair, j_v, of the real part of the $\alpha+{}^{16}O$ optical potential $V^{\rm opt}(r)$ (crosses, Ref. 3) with that of the l-averaged equivalent local potential $V^{\rm ELP}(r)$ of Eq. (1) (dots). The solid curve shows j_v of $\mathrm{Re}[V_l^{\rm ELP}(r)]$ with $l=0$. The dots and solid curve are calculated with $\Delta=21.3$ MeV. The triangle at $E=69.5$ MeV shows j_v of $\mathrm{Re}[V^{\rm ELP}(r)]$ with $\Delta=35$ MeV. The dashed straight line is the linear approximation of the energy dependence of the crosses (Ref. 3).

R_l is changed at each energy E_r so as to get a better fit to the data.

The comparison of the angular distributions between the RGM calculation and the data is shown in Fig. 2. Note that E means the α-particle energy in the laboratory frame while E_r is that in the center-of-mass frame. We can say that the data fitting by the theory is fairly good. (At $E=32.2$ MeV the data fitting around 60° and 120° is not so good, but this is also the case for the optical-potential fitting of Ref. 3, though the latter is of course better than the former.) Hence in the $\alpha+{}^{16}O$ system, the RGM with fixed effective two-nucleon force has proved to give a good fit to the data from bound states up to high-energy scattering states with $E\approx 70$ MeV. To the knowledge of the present authors this is the first case, except for very light systems like $\alpha+\alpha$, that the RGM has proved to give a good fit to the data in such a wide energy range as the present case. A possible way to get an even better fit to the data in the present RGM framework would be to change the value of the parameter Δ in $V_m({}^3E)$ for different energies E.

Now we construct the equivalent local potential (ELP) from this RGM nonlocal interaction by the method of Ref. 2 and Horiuchi[8] and compare it with the optical potential $V^{\rm opt}(r)$ of Ref. 3. Since the ELP, $V_l^{\rm ELP}(r)$, depends slightly on the angular momentum l of the relative motion, in order to compare the ELP with the angular-momentum-independent optical potential, we use the l-averaged ELP, $V^{\rm ELP}(r)$, defined as follows:

$$V^{\rm ELP}(r)=\textstyle\sum_l I_l V_l^{\rm ELP}(r)/\sum_l I_l, \qquad (3)$$

$$I_l=(2l+1)\,|1-\exp(2i\delta_l)|^2,$$

where δ_l denotes the nuclear phase shift. Figure 3 displays the comparison of the volume integral per nucleon pair, j_v, between $\mathrm{Re}[V^{\rm ELP}(r)]$ and $\mathrm{Re}[V^{\rm opt}(r)]$, where j_v of $\mathrm{Re}[V_l^{\rm ELP}(r)]$ for $l=0$ is also shown to see the l dependence of $V_l^{\rm ELP}(r)$. Here $\mathrm{Re}[V(r)]$ means the real part of $V(r)$. We see in this figure that j_v for $\mathrm{Re}[V^{\rm ELP}(r)]$ and $\mathrm{Re}[V^{\rm opt}(r)]$ are close to each other, although at $E=69.5$ MeV the difference between the two is somewhat large. The comparison of the shape of $V^{\rm ELP}(r)$ with that of $V^{\rm opt}(r)$ is given in Fig. 4 at $E=32.2$ and 69.5 MeV. We see here also that $V^{\rm ELP}(r)$

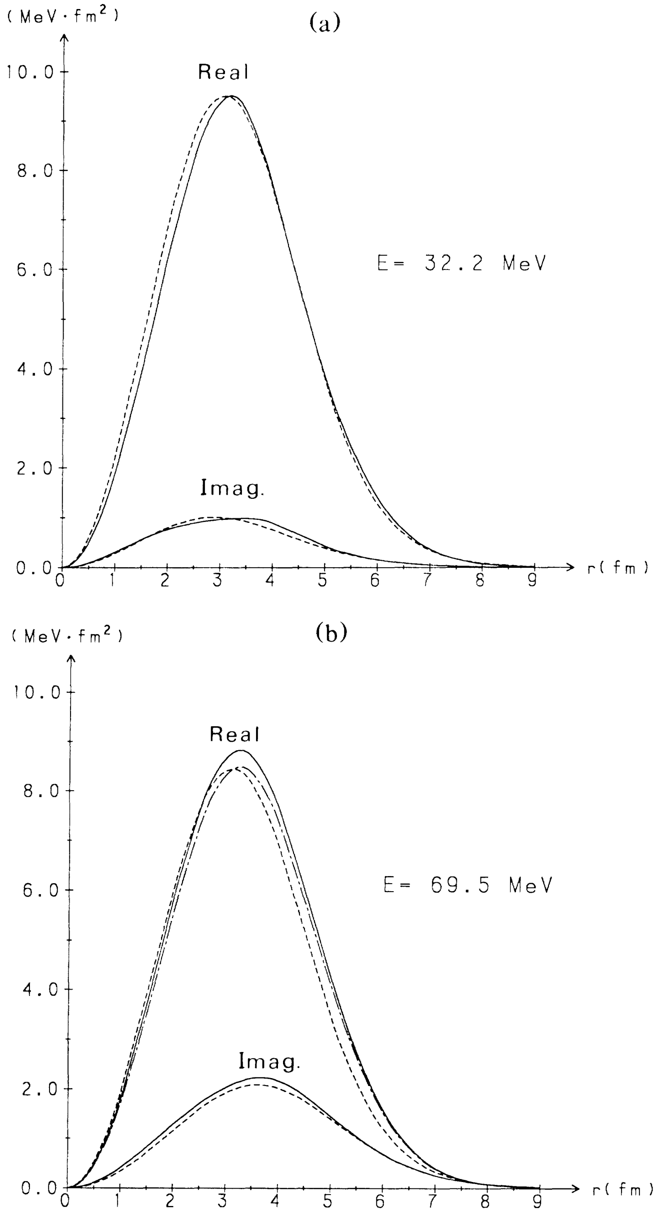

FIG. 4. Comparison of the shape of $r^2V^{\rm opt}(r)/64$ (dashed curve) with that of $r^2V^{\rm ELP}(r)/64$ (solid curve) calculated with $\Delta=21.3$ MeV. The dash-dotted curve at $E=69.5$ MeV shows the theoretical values obtained with $\Delta=35$ MeV.

is close to $V^{\rm opt}(r)$ although at $E=69.5$ MeV the difference between the two in the outer region is somewhat large.

Since the fit to the data at $E=69.5$ MeV by $V^{\rm opt}(r)$ is better than that by the RGM with $\Delta=21.3$ MeV, we change the value of Δ at this E such that j_V of $\mathrm{Re}[V^{\rm ELP}(r)]$ becomes much closer to j_V of $\mathrm{Re}[V^{\rm opt}(r)]$. This aim is satisfied by the choice of $\Delta=35$ MeV as seen in Fig. 3. Furthermore, at $E=69.5$ MeV, for this new value of Δ, the shape of $V^{\rm ELP}(r)$ becomes much closer to that of $V^{\rm opt}(r)$, as shown in Fig. 4, and the RGM fitting to the cross section improves also, as shown in Fig. 2.

From the above investigations of the ELP, we can derive the following two conclusions in the system of $\alpha+{}^{16}\mathrm{O}$. First, the ELP constructed from the RGM is very close, in a wide energy range, to the optical potential which fits the data very well. Second, in order to get an even better fit to the data, which is achieved by our making $V^{\rm ELP}(r)$ much closer to $V^{\rm opt}(r)$, we need to introduce some slight energy-dependent modification into the RGM treatment such as a slight energy-dependent change of the parameters of the effective two-nucleon force.

When one makes a straight-line fit of the four dot points in Fig. 3, the magnitude of the slope of this line will be about 30% of that of the dashed line. According to our preliminary RGM fit of the data at $E=104$ and 146 MeV, the values of j_V of $\mathrm{Re}(V^{\rm ELP})$ with $\Delta=21.3$ MeV at these energies are about 370 and 360 MeV·fm^3, respectively. The inclusion of these preliminary values for a straight-line fit yields almost the same answer for the slope of the line. Hence we can say that in the $\alpha+{}^{16}\mathrm{O}$ system the following view is correct: A significant part of the energy dependence of the real part of the optical potential comes from the nonlocality of the internucleus interaction originating from the internucleus antisymmetrization. A similar study of the $\alpha+{}^{40}\mathrm{Ca}$ system, which is being pursued by the present authors, supports this view and so it is highly probable, as has been strongly suggested in Ref. 2, that this view holds true at least for weakly absorbed light-ion projectiles. For the full understanding of the energy dependence of the real part of the optical potential, we need to study other origins quantitatively, including the renormalization effects on the elastic channel of the nonelastic processes which have been extensively studied recently.[9] The net effect coming from the other origins is just the difference $D(r)=V^{\rm opt}(r)-V^{\rm ELP}(r)$ and therefore if a theory does not take into account the internucleus antisymmetrization the energy dependence to be reproduced by the theory is not that of $V^{\rm opt}(r)$ but that of $D(r)$. In the present $\alpha+{}^{16}\mathrm{O}$ system we have seen that $D(r)$ can be reproduced well by the energy-dependent change of the effective two-nucleon force in the RGM framework.

[1]D. A. Goldberg and S. M. Smith, Phys. Rev. Lett. **29**, 500 (1972); D. A.Goldberg, S. M. Smith, and G. F. Burdzik, Phys. Rev. C **10**, 1362 (1974).

[2]H. Horiuchi, in *Proceedings of the International Conference on Clustering Aspects of Nuclear Structure and Nuclear Reactions*, edited by J. S. Lilley and M. A. Nagarajan (Reidel, Dordrecht, 1985), p. 35; T. Wada and H. Horiuchi, J. Phys. Soc. Jpn. **54** Suppl. II, 53 (1985).

[3]F. Michel, J. Albinski, P. Belery, Th. Delbar, Gh. Gregoire, B. Tasiaux, and G. Reidemeister, Phys. Rev. C **28**, 1904 (1983).

[4]Y. Yamamoto, Prog. Theor. Phys. **52**, 471 (1974); A. Hasegawa and S. Nagata, Prog. Theor. Phys. **45**, 1786 (1971).

[5]T. Matsuse, M. Kamimura, and Y. Fukushima, Prog. Theor. Phys. **53**, 706 (1975).

[6]F. Ajzenberg-Selove, Nucl. Phys. **A392**, 1 (1983).

[7]A. A. Cowley and G. Heymann, Nucl. Phys. **A146**, 465 (1970).

[8]H. Horiuchi, Prog. Theor. Phys. **64**, 184 (1980).

[9]M. A. Nagarajan, C. Mahaux, and G. R. Satchler, Phys. Rev. Lett. **54**, 1136 (1985); C. Mahaux, H. Ngo, and G. R. Satchler, Nucl. Phys. **A449**, 354 (1986), and **A456**, 135 (1986).