Stochastic Electron Acceleration in Obliquely Propagating Electromagnetic Waves

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Stochastic electron acceleration in intense electromagnetic waves, propagating obliquely to an ambient magnetic field, is considered. It is shown that when the waves' parallel phase velocities are supraluminous, the Hamiltonian surfaces are topologically open and, as a consequence, electrons can gain large energies. The results indicate that state-of-the-art, ground-based rf transmitters can accelerate ionospheric electrons to multimegaelectronvolt energies.

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Particle acceleration by electromagnetic waves is of great importance to laboratory and astrophysical plasmas.[1,2] It has long been known that the coherent interaction of an electromagnetic wave with electrons can lead to rapid, unlimited acceleration under ideal conditions.[3,4] These are as follows: (1) The wave propagates parallel to the ambient magnetic field B_0; (2) the index of refraction $n \equiv kc/\omega = 1$; and (3) the electrons satisfy the resonance condition

$$\omega - \omega_c/\gamma - kv_z = 0, \tag{1}$$

where $\omega_c = eB_0/mc$ is the nonrelativistic gyrofrequency ($-e$ is the electron charge and m is the rest mass), v_z is the electron's velocity parallel to the magnetic field, and γ is the relativistic factor.

Coherent acceleration has been conjectured to play a role not just in laboratory experiments, but also in space, for instance in type-III radio bursts[5] and pulsar magnetospheres.[6] Unfortunately, even slight violation of any of the above conditions leads to substantial deterioration in the obtainable electron energies. Although such control can be achieved in the laboratory, it is not practically possible when one is doing remote electron acceleration in the ionosphere from ground-based rf transmitters. In particular, one has no control over the initial electron energies, and satisfying Eq. (1) even approximately appears difficult, if not impossible. In astrophysical contexts, the situation is even more extreme; one has no control over anything, and major violation of all three conditions is expected.

We are, thus, led to contemplate stochastic acceleration mechanisms. In this Letter we show that when electrons interact with intense electromagnetic waves whose parallel phase velocities are supraluminous, i.e., $k_\parallel c/\omega < 1$, the Hamiltonian surfaces are topologically open, and it is possible to accelerate them to large energies; this contrasts to the well-studied subliminous case[7,8] where the Hamiltonian surfaces are topologically closed and the maximum energy gain is severely limited. Relativistic effects play a crucial role in this change of topology. The mechanism proposed here is quite robust; electrons gain large energies over a wide range in plasma parameters, wave characteristics, and initial particle energies—including zero initial kinetic energy—in contrast to the coherent mechanism described earlier. On the other hand, the energy gain is diffusive and thus occurs over longer length and time scales. Nonetheless, our results indicate that it is possible to obtain substantial fluxes of multimegaelectronvolt ionospheric electrons with use of the state-of-the-art, ground-based transmitters, thus opening up a new avenue for magnetospheric probing. Other applications to astrophysical plasmas will be discussed elsewhere.

The relativistic electron Hamiltonian, under the assumption of a right circularly polarized wave, is

$$H = c[(\mathbf{p} + e\mathbf{A}/c)^2 + m^2c^2]^{1/2}, \tag{2}$$

where

$$\mathbf{A} = A\left\{\frac{k_\parallel}{k}\sin\psi\hat{\mathbf{e}}_x + \cos\psi\hat{\mathbf{e}}_y - \frac{k_\perp}{k}\sin\psi\hat{\mathbf{e}}_z\right\} + xB_0\hat{\mathbf{e}}_y, \tag{3a}$$

$$\psi = k_\perp x + k_\parallel z - \omega t, \tag{3b}$$

B_0 is chosen in the z direction, and $\mathbf{p}$ is the *canonical* momentum. Any feedback of the particles on the wave is ignored. We note that $k_\perp = k\sin\alpha$ and $k_\parallel = k\cos\alpha$, where α is the angle of propagation. We shall further assume that ω and k are related through the dispersion relation

$$kc/\omega = [1 - \omega_p^2/\omega(\omega - \omega_c)]^{1/2}, \tag{4}$$

where ω_p is the plasma frequency. This approximation is adequate for the cases considered here.

To analyze Eqs. (2) and (3) further, we make a canonical transformation eliminating the explicit time dependence. The new variables are

$$z'=z-(\omega/k_{\parallel})t, \quad p_z'=p_z', \qquad H'=H-(\omega/k_{\parallel})p_z, \tag{5}$$

or, dropping the primes,

$$H=mc^2\gamma-(\omega/k_{\parallel})p_z, \tag{6}$$

where

$$\gamma=[1+(\mathbf{p}/mc+e\mathbf{A}/mc^2)^2]^{1/2} \tag{7}$$

is the relativistic factor, and $\psi=k_{\parallel}z+k_{\perp}x$ in Eq. (3a). Physically, this transformation is equivalent to a Lorentz transformation into the frame where the z dependence is eliminated.

The quantities H and p_y are now constants of the motion. We may eliminate p_y from the Hamiltonian by making a canonical transformation which absorbs it into x, reducing the problem to two degrees of freedom. Next, we make the action-angle transformation $(p_x,x)\to(J,\theta)$, where $p_x=(2eB_0J/c)^{1/2}\cos\theta$ and $x=(2cJ/eB_0)^{1/2}\sin\theta$. It will be useful to define $p_{\perp}=(2eB_0J/c)^{1/2}$, $p_{\parallel}=p_z$, and $\rho=(2cJ/eB_0)^{1/2}$. Expanding the Hamiltonian in powers of $\epsilon=eA/mc^2$, we find through first order

$$H=H_0+\epsilon H_1, \tag{8}$$

where, defining

$$\gamma_0=[1+(p_{\perp}^2+p_{\parallel}^2)/m^2c^2]^{1/2}, \tag{9a}$$

$$a_l=\frac{mc^2}{\gamma_0}\left\{\frac{\omega_c\rho}{2c}[J_{l-1}(k_{\perp}\rho)(1+\cos\alpha)-J_{l+1}(k_{\perp}\rho)(1-\cos\alpha)]-\frac{p_{\parallel}}{mc}\sin\alpha J_l(k_{\perp}\rho)\right\}, \tag{9b}$$

we have

$$H_0=mc^2\gamma_0-(\omega/k_{\parallel})p_{\parallel}, \tag{10a}$$

$$H_1=\sum_{l=-\infty}^{\infty}a_l\sin(k_{\parallel}z+l\theta). \tag{10b}$$

From Eq. (10a), one finds that the zero-order Hamiltonian surfaces are hyperbolas in $p_{\parallel}$-$p_{\perp}$ space when $n=k_{\parallel}c/\omega<1$ and are ellipses when $n=k_{\parallel}c/\omega>1$.

We now consider the resonance surfaces. There will be resonances when

$$k_{\parallel}z+l\theta=0, \tag{11}$$

or, in other words, when

$$\omega\gamma_0/\omega_c-(k_{\parallel}/\omega_c)p_{\parallel}/m=l. \tag{12}$$

Comparing Eq. (10a) with Eq. (12), one readily verifies that the resonance surfaces are elliptic when the zero-order Hamiltonian surfaces are hyperbolic and vice versa. When both are parabolic, they lie on top of each other.

Finally, we determine the wave amplitude required for stochastic electron acceleration by computing the resonance widths and finding the conditions for overlap. This approach is known to provide good, albeit not exact, estimates for the onset of stochasticity.[9] We then find that the effective width of the lth resonance is given by

$$w=2\pi|\epsilon Ma_l|^{1/2}, \tag{13}$$

where

$$1/M=(1/m\gamma_0)(1-\omega^2/k_{\parallel}^2c^2) \tag{14}$$

is the inverse of the electron's effective mass. In cases where M becomes infinite, this approach breaks down with consequences that are presently being studied; however, for supraluminous waves, this approximation is quite good.

In order to examine the consequences of this theory, we plot what we call resonance diagrams. Typical examples are shown in Fig. 1 for $\omega/\omega_c=2.0$, $\omega_p/\omega_c=0.3$, and $\epsilon=0.03$. The value $\omega_p/\omega_c=0.3$ corresponds roughly to the night-time ionosphere at 130 km, while $\omega/\omega_c=2$ corresponds to a 2.6-Mhz wave. The value $\epsilon=0.03$ corresponds to a power flux of 500 mW/cm^2. The solid lines indicate the zero-order Hamiltonian surfaces, the small dots indicate the resonance surfaces, and the resonances widths are indicated by large dots on the Hamiltonian surfaces. Resonance overlap is visible as a continuous series of large dots stretching from resonance to resonance. Electrons can move freely along any portion of the Hamiltonian surfaces continuously connected by large dots. The numerical advantage of our using resonance diagrams is enormous. To determine the equivalent information from calculations of single-particle orbits would require several orders of magnitude more computer time.

Examination of many such resonance diagrams indicates that it becomes easier to accelerate electrons to high energies as the propagation angle increases. For the parameters of Fig. 1, electrons can be accelerated from zero kinetic energy to 5 MeV when $\alpha>60°$. In addition to requiring $\omega>\omega_p$ so that electron waves can propagate, we require $\omega>\omega_c$ so that resonances can easily overlap. As ω/ω_c increases above 2, the diagrams show a slow falloff of the maximum energy attainable at a given value of ϵ, which implies that the power flux needed to achieve a given maximum energy increases slightly faster than ω^2. Hence, the most favorable frequency is

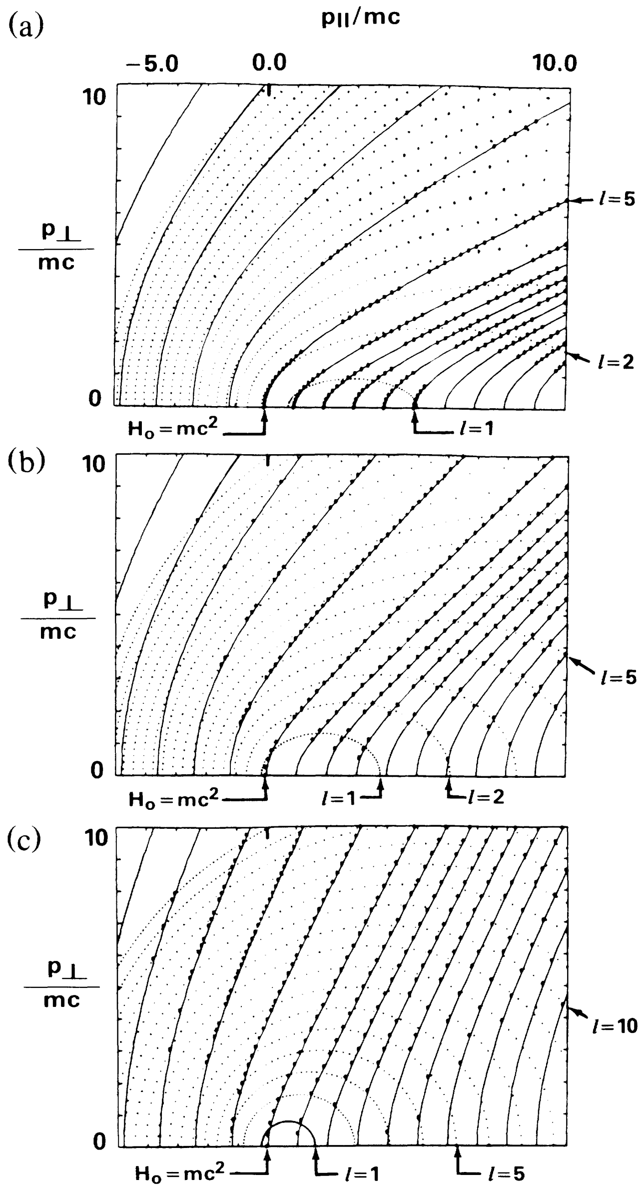

FIG. 1. Resonance diagrams at increasing angles ($\omega_p/\omega_c=0.3$, $\omega/\omega_c=2.0$, $\epsilon=0.03$). (a) $\alpha=20°$; the resonances on the $H_0=mc^2$ surface do not overlap. (b) $\alpha=40°$; the $l=1$ resonance does not overlap with the other resonances. (c) $\alpha=60°$; the resonances overlap and will carry electrons to high energy.

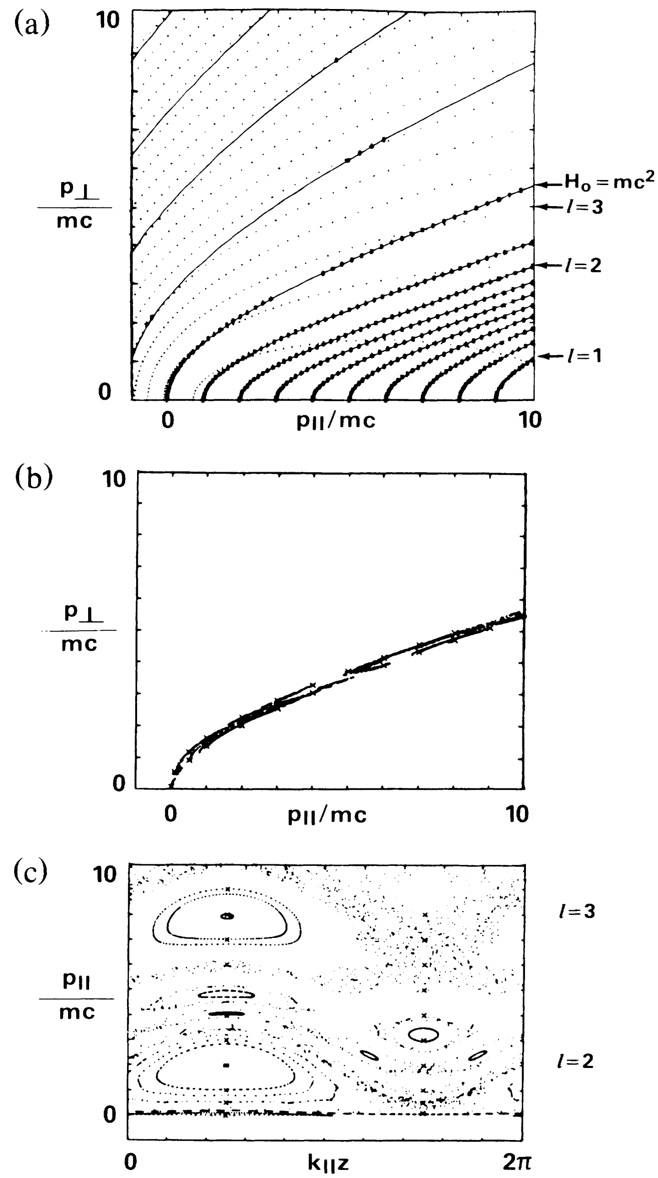

FIG. 2. Comparison of a resonance diagram and a surface-of-section plot ($\omega_p/\omega_c=0.3$, $\alpha=10°$, $\omega/\omega_c=1.8$, $\epsilon=0.1$). (a) Resonance diagram, (b) z-$p_\parallel$ surface of section, (c) $p_\parallel$-$p_\perp$ surface of section. A cross indicates the starting point of each trajectory. There are twenty trajectories.

near $\omega/\omega_c\simeq 2$.

We have verified the results from the resonance diagrams by solution of single-particle orbits in selected instances. Shown in Fig. 2 is a comparison between a resonance diagram and the corresponding surface-of-section plots[10] for $H=mc^2$. Excellent agreement between the $H_0=mc^2$ surface of the resonance diagram and the surface-of-section plots is clearly visible. The finite width of the $H=mc^2$ surface is also visible in Fig. 2(b).

In regions of phase space where resonances overlap, the electron energy increases diffusively. From simple dynamical arguments, it follows that $\Delta p\sim\epsilon^{1/2}mc$ during one diffusion time $\Delta t\sim 1/\omega\epsilon^{1/2}$, so that $D=(\Delta p)^2/\Delta t\sim m^2c^2\omega\epsilon^{3/2}$. Noting that $v_z=[(\gamma-1)n/\gamma]c\cos\alpha\simeq c\cos\alpha$, when $H=mc^2$ and $n\simeq 1$, we conclude that $D_z=(\Delta p)^2/\Delta z=mc\omega\epsilon^{3/2}/\cos\alpha$.[11] Therefore, the acceleration length s needed to achieve a given average particle energy scales as $(\cos\alpha)\epsilon^{-3/2}$. The exact coefficient has been determined by simulations.

As noted earlier, the robustness of this acceleration mechanism makes it ideal for settings outside the laboratory, where control of the ambient parameters is limited or altogether absent. In particular, our study indicates that it is possible to use state-of-the-art, ground-based transmitters for active ionospheric and magnetospheric probing. To date, such probing has been carried out with use of energetic electrons from rocket-borne accelerators, with all the drawbacks inherent in rocket-borne experiments.[12,13] Recalling that the parameters of Fig. 1 correspond to typical night-time ionospheric conditions at 130 km and a power flux of 500 mW/cm^2 at $f=2f_c\simeq 2$ MHz we find that electrons can be accelerated from zero kinetic energy to above 5 MeV. For injection angles $\alpha\simeq 80°$, simulations and the diffusion scaling we previously discussed indicate that about 1% of the electrons achieve this energy within a length of about 15 km. A ground-based transmitter, emitting 1-msec pulses at power levels of 1–10 GW would be sufficient to satisfy the power-flux requirement in an area of 10^9–10^{10} cm^2.

In this Letter, we have studied electron acceleration in intense, obliquely propagating electromagnetic waves using resonance diagrams, supported by studies of single-particle orbits. We have found that the zero-order Hamiltonian surfaces are hyperbolic when the waves' parallel phase velocities are supraluminous, so that electrons can be accelerated to substantial energies. The mechanism we have considered here is capable of accelerating electrons from zero initial energy and is quite robust, being insensitive to changes in the wave parameters. With use of state-of-the-art transmitters, it is possible to accelerate ionospheric electrons to 5 MeV and more.

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