Amplification of Magnetic Modes in Laser-Created Plasmas

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(Received 2 February 1987)

The amplification of magnetic (Weibel) modes in laser-plasma interaction is investigated by use of unperturbed distribution functions given by Fokker-Planck simulations and a dispersion relation valid for all collisionality regimes. In the five cases studied, a strongly growing mode is found in the underdense plasma, where $\bar{v}_x^2 < \frac{1}{2}\bar{v}_\perp^2$, and the usual slowly growing one in the overdense plasma. The first mode grows convectively outwards by more than 10^4. The convection velocities are found to be very different from Nernst values.

PACS numbers: 52.50.Jm, 52.25.Fi, 52.35.Py

Generation of magnetic fields (in the megagauss range) in laser-produced plasmas is an important subject in relation to spherical pellet experiments; those fields lead to inhomogeneities of the compression which could prevent an isotropic implosion and a central ignition of the fuel. One possible mechanism for such a generation is the Weibel instability.

It has been shown by Weibel[1] that anisotropic electron-velocity distributions drive unstable magnetic modes. Let us assume that the angular distribution is symmetric about the x axis. If $\bar{v}_x^2 < \frac{1}{2}\bar{v}_\perp^2$, the most unstable mode has a wave vector **k** along x; if $\bar{v}_x^2 > \frac{1}{2}\bar{v}_\perp^2$, its wave vector is perpendicular to the x axis. We will therefore refer to these as the "k_x mode" and the "$k_\perp$ mode" respectively.

The first instability analysis for such magnetic modes in laser-created plasmas was performed by Ramani and Laval[2] in the collisionless limit ($k\lambda \gg 1$;

$$\lambda = 4\pi(\epsilon_0 k_B T_e)^2/(z+1)n_e e^4 \ln\Lambda$$

is the mean free path); they studied mainly the $k_\perp$ mode, driven by the heat flux in the overdense plasma, which here gives a positive second anisotropy. The same mode, but in a collisional limit ($k\lambda \ll 1$), was extensively studied by Epperlein and co-workers.[3,4] The latest results[4] indicate relatively weak instability (growth rate of order 10^9 sec^{-1}). Recently, the $k_\perp$ mode was shown to be unstable in the corona, in spherical situations.[5] But this analysis does not overlap with ours, as it concerns the very underdense plasma.

We present, in this Letter, the first investigation of the k_x mode in laser-produced plasmas, and show that it grows in the underdense plasma near critical density much more strongly than the $k_\perp$ mode in the overdense region. The transport physics which produces the required anisotropy is fairly straightforward: The heating of the plasma results in an electron heat flow from the underdense hot plasma into the cold dense plasma, supported by electrons with large $|v_x|$. Thus, we have a depletion of these electrons in the corona, which creates a strongly negative second anisotropy while the surplus in the cold dense plasma amounts to a positive but weaker second anisotropy, as the collisionality is higher. (The cold return current contributes only weakly to the second anisotropy).

In the present study, we have used electron-distribution functions computed by the Fokker-Planck International (FPI) kinetic code of Matte and co-workers,[6,7] and the semicollisional dispersion relation recently derived by Bendib and Luciani.[8] The FPI code performs a one-dimensional, time-dependent simulation of laser-plasma interaction, including the effects of transport, self-consistent electric field ensuring quasineutrality, electron-ion and electron-electron collisions, inverse-bremsstrahlung absorption of laser light in normal incidence, and cold-ion hydrodynamic motion. The electron-distribution function is expanded in Legendre polynomials:

$$f(x,v_x,v_\perp,t) = \sum_{l=0}^{N} f_l(x,v,t)P_l(\mu), \qquad (1)$$

where $v = (v_x^2 + v_\perp^2)^{1/2}$, $\mu = v_x/v$, and $P_l(1) = 1$. We have used $N=3$ and $N=5$.

The dispersion relation derived in Ref. 8 gives the growth rate $\gamma(k)$ and phase velocity $v_\phi(k)$ in the fluid frame of reference and is valid for all values of $k\lambda$:

$$v_\phi(k) = \int_0^\infty dv\, f_1(v) v^6 F/D, \qquad (2)$$

$$\gamma(k) = \left[-\frac{3}{8\pi}\left(\frac{kc}{\omega p}\right)^2 n_e v_t^4 + \frac{6}{5}\frac{\lambda^2}{v_t^4}(\epsilon k_\perp^2 - k_x^2)\int_0^\infty dv\, f_2(v) v^{10} G \right](\lambda D)^{-1}, \qquad (3)$$

where

$$D=-\int_0^\infty dv\, \partial_v f_0 v^6 F, \quad (4)$$

where $\epsilon=1$ for the $k_\perp$ mode [B perpendicular to the (k,x) plane], and $\epsilon=0$ for the k_x mode [B in the (k,x) plane]. F and G involve continuous fractions, but the following fits, accurate to better than 5% over the whole range,[8] have been used:

$$F=F(k,x,v)=\tfrac{1}{2}[1+(a/\delta)^2]^{-1/2},$$

$$G=G(k,x,v)=\tfrac{2}{3}(1+\alpha a^2)F^2/(1+\beta a^2)(1+2F),$$

$$a=2k\lambda(v/v_t)^4, \quad v_t=(k_B T_e/m_e)^{1/2}, \quad (5)$$

$$\delta=\tfrac{3}{2}\pi, \quad \alpha=30\beta/\delta^2,$$

and

$$\beta=(\tfrac{5}{252}\delta^2-\tfrac{3}{4})/(\delta^2-30).$$

In formula (3), smaller terms in f_1^2 have been dropped. Two points are readily seen on inspection of the dispersion relations. As k varies from the collisionless to the collisional limit, the phase velocity changes not only in magnitude but also in sign.[8] Second, the driving term in Eq. (3) is a moment of f_2. This is the "second anisotropy" referred to above.

We have performed five different simulations with the FPI code, with different laser wavelengths (λ_L), intensities (I), pulse lengths (t_L), and initial density-profile scale lengths (L_{n0}). The first case is the simulation described in Ref. 7. Slight differences are due both to improved numerics, and to the use of $N=5$. ($N=3$ was used for the $\lambda_L=0.353$ μm cases.) In all cases, initially, the ions were at rest, the electrons were Maxwellian at 100 eV, and the density profile consisted of two plateaus (5×10^{19} and 2×10^{22} cm^{-3} for $\lambda_L=1.06$ μm, 2.5×10^{20} and 10^{23} cm^{-3} for $\lambda_L=0.353$ μm), joined by an exponential ramp of scale length L_{n0}. The laser intensity was constant in cases 1 and 3 of Table I, but gradually increased in cases 2, 4, and 5.

The results of our studies are summarized in Table I where we give, for the five cases simulated, simulation parameters, relevant numbers for the final plasma state, and brief instability analyses for the k_x mode and the $k_\perp$ mode. The table gives the following data on the final plasma state: maximum temperature and temperature at the critical surface ($T_{\max}$ and T_{nc}), maximum ratio of heat flux to the free-streaming value (q/q_{fs}), density scale length at the critical surface (L_n), distance (in the underdense plasma) over which the heat flux falls to $\frac{1}{2}$ of its maximum value ($L_{q/2}$), and velocity of the critical surface (v_{nc}).

For the instability analysis, we have computed the growth rate γ and the phase and group velocities v_ϕ and v_g for several k values and for both modes as functions of space. For both modes, the maximum value of γ ($\gamma_{\max}$), and the corresponding wave vector (k_{opt}) and phase and group velocities ($v''_{\phi\text{opt}}$) and $v''_{g\text{opt}}$ are given. We give also the extent of the growth region (x_2-x_1), and the corresponding maximum and minimum densities (n_1, n_2).

For the k_x mode, the group velocity is always directed towards the underdense plasma; therefore, the relevant quantity is E, the number of convective e-foldings, which

TABLE I. (A) Simulation parameters and final plasma state, (B) k_x-mode analyses, (C) $k_\perp$-mode analysis.

Case	1	2	3	4	5
A. I (10^{14} W/cm^2); t_L (10^{-12} s)	3.0; 600	6.0; 600	4.5; 200	10.0; 250	10.0; 100
λ_L; L_{n0} (μm)	1.06; 46	1.06; 46	0.353; 23	0.353; 23	0.353; 9.2
$T_{\max}$; T_{nc} (keV)	2.4; 2.3	2.9; 2.8	1.9; 1.7	2.6; 2.4	1.9; 1.8
q/q_{FS}; v'_{nc} (10^7 cm/s)	0.15; 2.2	0.17; 2.3	0.073; 1.2	0.098; 2.1	0.115; 1.8
L_n; $L_{q/2}$ (μm)	103; 45	103; 46	39; 16	47; 16	16; 4.5
B. (1) Fixed k_x					
k_{opt} (10^3 cm^{-1}); $\gamma_{\max}$ (10^{11} s^{-1})	5.0; 2.0	5.0; 3.6	7.0; 0.54	10.0; 1.36	12.0; 3.2
n_{opt} (10^{21} cm^{-3}); T_{opt} (keV)	0.88; 2.4	0.80; 2.9	8.1; 1.8	8.7; 2.4	8.1; 1.9
$V''_{\phi\text{opt}}$; $V''_{g\text{opt}}$ (10^7 cm/s)	7.4; 8.0	9.2; 9.7	2.4; 3.2	5.4; 6.7	3.4; 5.4
x_2-x_1 (μm)	182	255	60	57	22
n_1; n_2 (10^{21} cm^{-3})	1.44; 0.36	1.61; 0.31	11.0; 2.9	10.7; 3.7	10.5; 3.2
(2) Space-varying k_x (WKB)					
k_{opt} (10^3 cm^{-1}); $E_{\max}$	3.0; 32	3.0; 53	7.0; 7.9	6.0; 11.5	8.5; 11.7
x_2-x_1 (μm)	173	279	115	124	45
n_1; n_2 (10^{21} cm^{-3})	1.52; 0.30	1.69; 0.26	10.7; 1.26	11.3; 1.52	10.6; 1.39
C. k_{opt} (10^3 cm^{-1}); $\gamma_{\max}$ (10^9 s^{-1})	1.0; 2.4	1.5; 3.4	2.5; 3.8	2.5; 5.8	3.0; 7.4
n_{opt} (10^{21} cm^{-3}); T_{opt} (keV)	5.0; 1.1	7.3; 1.1	21.8; 1.2	20.6; 1.6	21.4; 1.2
x_2-x_1 (μm)	180	178	40	56	21
n_1; n_2 (10^{21} cm^{-3})	22; 1.8	27; 2.4	60; 13.0	112; 14.0	73; 15.0
v'_{g1}; v'_{g2} (10^7 cm/s)	−1.8; 3.1	−2.2; 3.9	−1.8; 0.20	−2.1; 1.0	−2.4; −0.8

we have computed by a static WKB approach:

$$E(k_0) = \int_{x_1(k_0)}^{x_2(k_0)} dx\, \gamma(k(k_0,x),x) v_g''(k(k_0,x),x).$$

k_0 is the value of k_x at the critical surface, and $k(k_0,x)$ varies so as to keep kv_ϕ'' constant. This analysis was done in the frame where we expect the slowest temporal evolution, i.e., the critical-density frame. Thus $v_g'' = v_g + v_{\rm ion} - v_{nc} = v_g' - v_{nc}$, $v_\phi'' = v_\phi + v_{\rm ion} - v_{nc} = v_\phi' - v_{nc}$. In all cases, $k_{\rm opt}L_n > 13$, so that the WKB treatment should be accurate. Note that while the time over which substantial exponentiation takes place is only little less than the simulation time, we believe that our result is at least qualitatively correct, because the value of E hardly changes between 100 and 250 ps in cases 4 and 5.

We show in Table I the maximum number of e-foldings ($E_{\rm max}$), and the corresponding values of k ($k_{\rm opt}$), the size of the growth region ($x_2 - x_1$), and the densities at its edges (n_1, n_2).

For the $k_\perp$ mode, such a calculation of e-folding would be meaningless: The growth rate is too low, and the group velocity changes sign within the growth region (group velocities in the laboratory frame v_{g1} and v_{g2}, at the edges of the growth region, are given in Table I). Note that this does not imply absolute growth, because the velocities diverge from the neutral point.

Before we discuss the results further, we will note a few important points. One is that the k_x mode is nearly collisionless (at worst the error for the growth rate would be 30%). Another question is the role of ablation in the generation of anisotropies: The plasma expansion "cools" the electrons along the x axis, and then gives a negative contribution to the second anisotropy, but we have checked that this is small (about 10% in case 1) by continuing the simulation during 50 ps with immobile ions. We conclude that the main effect is that of the transport. Finally, the effect of a fast-electron source was found to be small, unless its share of energy absorption is comparable to that of inverse bremsstrahlung.

In view of these considerations, we are in a better position to understand the instability and its scaling. Taking the collisionless limits of Eqs. (2) and (3), optimizing k_x for maximum γ, noting that the growth region scales with $L_{q/2}$ (see Table I), and computing the second anisotropy from the heat flux via the Fokker-Planck equation, we obtain the following scaling parameter for the total e-folding:

$$\eta = (L_{q/2}/\lambda_L)(q/q_{fs})^{1/2}(\lambda/L_{q/2})^{3/2}.$$

Roughly $E \sim 60\eta$ for blue light, but $E \sim 30\eta$ for red light because WKB corrections are more important in the red than in the blue cases and also because collisional effects enhance E in the blue case. Note that, as the gradients are quite steep in these simulations, q and $L_{q/2}$ will not be correctly given by the Spitzer-Härm theory[9] but depend on the density scale length, delocalization effects,[10,11] and absorption.

Since fluctuation levels in laser-produced plasmas are far above thermal, amplification by a factor e^{10} should produce strong oscillating magnetic fields. The fields might not be coherent, because the k spectrum of the instability is quite wide. For example, in case 4 we found $E = 6.8$, 11.5, and 6.5 for $k_0 = 3000$, 6000, and 10000 cm^{-1}, respectively. A seed of a few tens of gauss at the critical surface could be amplified to approximately 1 MG, thus inhibiting transport in the corona by either quasilinear diffusion[2] or trapping, depending on the strength of the field and the width of the spectrum. The criteria for significant effect, which are roughly $r_{\rm Larmor} < \lambda$ and $kr_{\rm Larmor} < 1$, both imply a field of order 1 MG. The resulting shortening of the effective mean free path λ would also tend to saturate the instability by reducing η.

For the $k_\perp$ mode, we find low growth rates, in agreement with Epperlein, Kho, and Haines.[4] However, the convective properties turn out to be quite different because of the semicollisional dispersion relation [Eq. (2)] so that the magnetic fields can grow weakly and stagnate instead of being convected away at the Nernst velocity. We emphasize this convective aspect. We have found that, for example, fields generated near the critical surface by any magnetic field source will be deformed by dispersion: Fourier components with k below and above 500 cm^{-1} are convected inwards and outwards respectively.

Preliminary estimates for the Weibel instability in long-pulse-length experiments indicate that both modes ($k_\perp$ and k_x) could grow appreciably. By comparison with the present results (moderate pulse length), the lower growth rate could be more than compensated by the longer time combined with the stagnation effect discussed above. For the k_x mode, several competing effects need to be considered: lower growth rate due to lesser anisotropy, but lower convection velocity [v_g'' in Eq. (6), longer growth region, and higher temperatures. Quantitative studies will be reported in a future work.

The present work owes much to the help of Dr. J. Virmont. Useful discussions with him and Dr. P. Mora are gratefully acknowledged. The computations reported here were performed at both Institut National de la Recherche Scientifique—Energie and the Centre Inter-Régional de Calcul Electronique (Centre National de la Recherche Scientifique Computing Center). One of the authors (J.P.M.) thanks the Centre de Physique Théorique for its hospitality and the Centre Européen de Calcul Atomique et Moléculaire Workshop (Orsay, September, 1986) organized by Dr. M. G. Haines. This research was partially supported by the Ministère de l'Education du Québec and the National Science and Engineering Council of Canada.

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