Measurement of Charge-Density Differences in the Interior of Pb Isotopes

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The ground-state charge densities of $^{204,206-208}$Pb have been accurately determined by elastic-electron scattering. Mean-field calculations fail to describe the individual charge densities in the central region of the nucleus. However, they properly account for polarization mechanisms in the nuclear interior, as they accurately describe the corresponding charge-density differences. We show that these data are sensitive to the value of the nuclear compressibility κ and that they are compatible with the value $\kappa=230$ MeV.

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The ability of mean-field theory to describe the shape of single-particle wave functions has been recently investigated in detail by electron scattering.[1,2] The comparison of observed and calculated fluctuations of the charge and transition charge densities in the nuclear interior has provided insights into the limitations of present theoretical descriptions. The concept of single-particle orbit in a mean field generated by all of the nucleons was found to be valid in the nuclear interior.[1] The observed transition strengths and occupation numbers are reduced considerably relative to single-particle values, presumably through ground-state correlations.[3]

In this paper we investigate a complementary aspect of mean-field theory: the response of the mean field to the successive removal of neutrons. As pointed out recently by Co' and Speth,[4] this polarization of the density distribution can be used to obtain a sensitive measure of the nuclear compressibility. We have studied this collective readjustment of the nucleus by measuring the isotopic charge-density differences of $^{204,206-208}$Pb by high-momentum-transfer elastic electron scattering. This experiment extends the maximum momentum transfer of available data[5] from $q_{max}=2.7$ to 3.5 fm^{-1}; it also increases significantly the accuracy of data available at lower q. A careful analysis of electron-scattering and muonic-x-ray data now determines the charge densities of all stable Pb isotopes with an accuracy of $\pm 1\%$ in the nuclear interior.

Elastic-electron-scattering cross sections were measured at an incident energy of 502 MeV by use of the standard experimental setup of the Saclay HE1 end station.[6] Isotopically enriched targets ($>95\%$) of 100-mg/cm^2 thickness were employed. Their average thicknesses were determined by measurement of their weights and areas, while their density profiles were determined by γ-ray absorption. The central target thickness was known to better than 0.5%. The targets were mounted on a rotating wheel which allowed us to perform measurements on different isotopes in rapid succession. Systematic errors largely cancel with this procedure, and yield only a small contribution to the overall uncertainty in the ratios of cross sections. To avoid thermal deformation, the targets were wobbled and cooled by a jet of hydrogen gas. Scattered electrons were analyzed with the Saclay 900-MeV/c high-resolution magnetic spectrometer in the energy-loss mode. An overall energy resolution of 100 keV was achieved; this was sufficient to separate elastic and inelastic scattering cleanly for all Pb isotopes. Background was absent. The absolute efficiency of the detection system was determined to be 0.96 ± 0.03 by our measuring well-known cross sections of ^{208}Pb.[7]

Angular distributions were measured for all isotopes for scattering between 25° and 65°. For larger angles, up to 81°, cross sections were determined only at those angles where we expected maximal isotopic differences

on the basis of the known behavior of the ^{208}Pb form factor.[7] At these high momentum transfers, the isotopic cross sections were found to be identical to the ^{208}Pb cross sections within the statistical accuracy ($\sim$30%).

The experimental cross sections have been extracted by use of a line-shape fitting technique with the standard expressions for radiative effects. The cross sections for ^{207}Pb have been corrected for small contributions from magnetic scattering.[8]

The data for ^{206}Pb, which are typical of the data obtained, are shown in Fig. 1 together with earlier data from Euteneuer, Friedrich, and Voegler.[5] The charge densities have been determined according to the model-independent analysis described in Sick.[10] Our cross sections were combined with five generalized moments derived from muonic-x-ray transition energies,[11] and the 119- and 199-MeV data of Ref. 5. We have omitted the 289-MeV data of Ref. 5 for all isotopes. We found these data to be inconsistent with our new measurements, as we previously observed for ^{208}Pb.[1,7] Systematic uncertainties ($\pm$3% for our data, $\pm$1% for those of Ref. 5) have also been accounted for in the analysis. The dominant uncertainty in the muonic-x-ray data, a 30% uncertainty in the nuclear polarization correction, has also been included. The charge density was expanded as a sum of Gaussians (SOG).[10] In this expansion the amplitude of each Gaussian is determined by a fit to the experimental data. The principal limitation in this "model-independent" analysis technique is due to the use of Gaussians of finite width. This restricts the amplitude of unmeasured high-frequency ($q > q_{max}$) Fourier components of the charge density. The value $\gamma = 1.388$ fm used for the width parameter allows us to reproduce theoretical charge densities in the Pb region to an accuracy of better than 0.1%. Therefore, this method provides enough flexibility to reproduce any fine structure in the charge density compatible with present theoretical concepts. The error band of the inferred densities includes a realistic estimate of the uncertainty associated with the lack of data beyond q_{max}. Because of the high value of $q_{max} = 3.5$ fm^{-1} in our experiment, the uncertainty associated with this truncation is less than 1%. An independent analysis of our previous data on ^{208}Pb with use of the inverse Fourier transform technique was found to yield results identical to those of the SOG technique for the charge density and its uncertainty in the nuclear interior. The results of our analysis for ^{206}Pb are shown in Fig. 1, together with mean-field theory predictions of Dechargé and Gogny. The experimental density is smaller in the center of the nucleus and has less structure than the theoretical prediction. A similar result has been found for all Pb isotopes.

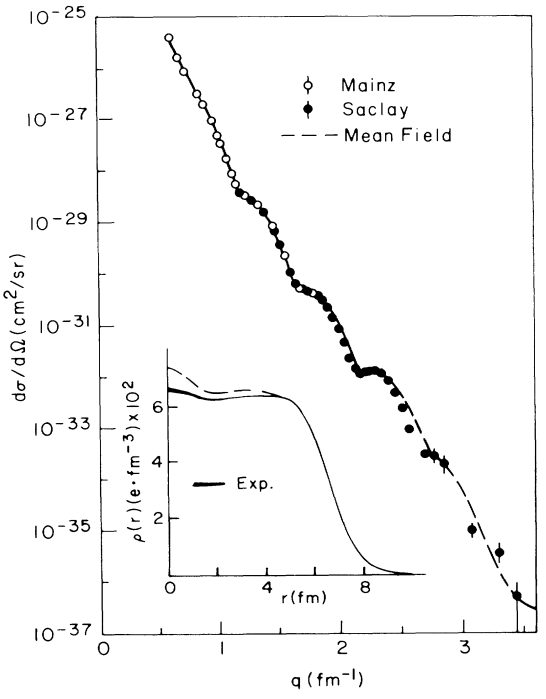

FIG. 1. Elastic cross section for ^{206}Pb at $E_e = 502$ MeV as a function of effective momentum transfer. The dashed curve is a HFB (Ref. 9) prediction. Inset: The ground-state charge density together with the HFB prediction (dashed curve).

A different procedure has been followed in extracting charge-density differences. The systematic uncertainties in the relative isotopic measurements are significantly smaller (typically < 1%) than those of the absolute cross sections. Only this uncertainty has been taken into account in the determination of charge-density differences. Furthermore, uncertainties due to the nuclear polarization correction are estimated to cancel in the comparison of neighboring nuclei. The charge-density differences resulting from this analysis are shown in Fig. 2. The error bands again take into account the contributions of statistical, systematic, and model uncertainties. This error band is compatible with the result obtained from the direct Fourier-transform integral as a function of the upper integration limit.[7]

The charge-density differences exhibit a number of features that can be understood qualitatively. Significant fluctuations are observed in the nuclear interior. The structure of these fluctuations is reminiscent of the characteristic shape of the $3s_{1/2}$ density; apparently the $3s_{1/2}$ proton orbit is most affected by the removal of $3p_{1/2}$ neutrons because of their large overlap in the nuclear interior. Martorell and Sprung[14] have shown that for ^{207}Pb the electromagnetic spin-orbit contribution[15] due to the $3p_{1/2}$ neutron also has an important role. The increase of the average charge density between 2 and 6 fm reflects the decrease in size. Charge conservation is satisfied by the negative lobe in $\Delta\rho(r)$ at large radii ($r > 6$ fm).

Also shown in Fig. 2 are the predictions of different mean-field calculations performed with effective interac-

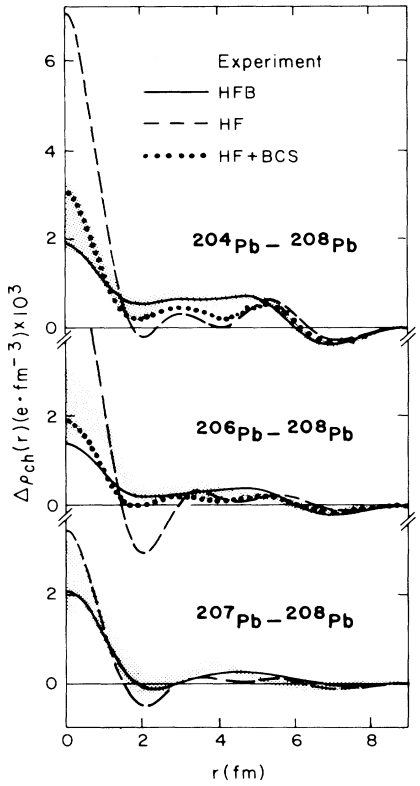

FIG. 2. Experimental charge-density differences, together with mean-field predictions HFB (Ref. 9) (solid lines), HF (Ref. 12) (dashed lines), and HF+BCS (Ref. 13) (dotted lines).

tions derived from the Brueckner G matrix. The effect of the electromagnetic spin-orbit interaction[13,14] has been taken into account in both predictions. The curve labeled HF is a Hartree-Fock calculation[12,13] with a finite-range interaction derived from the Reid soft-core potential. In this calculation the properties of finite nuclei are fitted by use of the local-density approximation, but no pairing correlations are taken into account. A related calculation (denoted by HF+BCS) includes the effect of pairing correlations in the framework of BCS theory.[14,15] A constant pairing force was used in the BCS formalism and pairing excitations were restricted to particle states close to the Fermi surface. Pairing correlations induce a substantial damping of the charge-density fluctuations and improve considerably the agreement with experiment; only small differences remain.

Dechargé and Gogny[9] followed a different approach using the D1 finite-range force, which has been optimized for a consistent treatment of pairing correlations. In their calculation (shown by the curve labeled HFB in Fig. 2) the nonlocality of the mean field and the pairing correlations are treated fully self-consistently in the Hartree-Fock-Bogoliubov (HFB) framework. In this procedure there is no restriction on the Fourier components of the particle-particle matrix elements. The oscillatory behavior of the HFB predictions closely follows the experimental results. The ^{204}Pb$-^{208}$Pb difference where pairing effects are largest is particularly well predicted. The ability of the HFB calculation to describe in detail the fluctuations of charge densities induced by pairing correlations demonstrates the advantage of the use of this formalism.

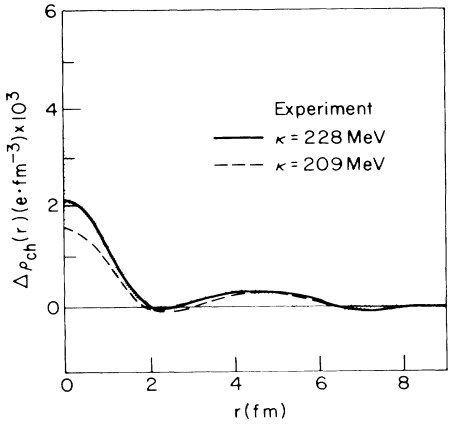

FIG. 3. Experimental ^{207}Pb$-^{208}$Pb charge-density difference together with HFB predictions (Ref. 9) for two values of the nuclear compressibility.

The isotopic density differences in the Pb region are mostly sensitive to three effects: the nuclear compressibility, neutron pairing, and the electromagnetic spin-orbit contribution. For the $^{204\text{-}208}$Pb and $^{206\text{-}208}$Pb differences, pairing has a large effect. For the particular case of the $^{207\text{-}208}$Pb difference, there is no pairing; ^{207}Pb is described by a $3p_{1/2}$ neutron hole in ^{208}Pb. Since the electromagnetic spin-orbit effect can be calculated reliably, quantitative information on the nuclear compressibility can be deduced from the $^{207\text{-}208}$Pb difference. Figure 3 shows a comparison between the experimentally determined density difference and the results of two HFB calculations performed with two versions of Gogny's D1 force which differ in their values of compressibility (κ=228 MeV, κ=209 MeV), but otherwise provide equally good fits to the set of observables[16] used to adjust the force parameters. The $\Delta\rho(r)$ in the nuclear interior shows that our results are compatible with κ=230 MeV. A change of κ by 30 MeV would lead to a serious disagreement with the measured isotopic difference. This finite-range force not only accurately reproduces the surface and the interior shape of the isotopic charge-density differences, but also accounts reasonably for a variety of bulk and structure properties of nuclei. One might reach significantly different conclusions by using unrealistic zero-range forces.[16] This value of κ is in agreement with the values of 210 ± 30 MeV and $\sim$260 MeV obtained by Blaizot, Gogny, and Grammaticos[16] and by Pandharipande and Wiringa,[17]

respectively, from the analysis of breathing-mode energies. It has recently been shown[18] that calculations that use realistic two- and three-nucleon interactions obtained by fitting both the binding energies of $A=3$ and 4 nuclei and the saturation density of nuclear matter also give $\kappa \sim 215$ MeV.

In summary, we have determined model-independent charge densities of $^{204,206-208}$Pb with a high precision by a combined analysis of muonic-x-ray and high-momentum-transfer electron scattering data. All have a central charge density which is significantly smaller than that of mean-field calculations. The same calculations do, however, satisfactorily described the density differences in the nuclear interior. This suggests that mean-field theory is able to account in detail for the small readjustments which occur in the central region of heavy nuclei, even when failing to give the complete description of their ground state. The large effects of pairing correlations predicted by Hartree-Fock-Bogoliubov calculations in ^{204}Pb and ^{206}Pb are observed in the experimental density differences.

The shape of the density difference $^{207}\mathrm{Pb} - {}^{208}\mathrm{Pb}$ is sensitive to nuclear compressibility at zero temperature. We find that the observed fluctuations are reproduced best by a density-dependent force having a compressibility of 228 MeV. This result agrees with the known properties of light nuclei and nuclear matter, and with the value derived from the energy of the breathing mode.

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