Healing Length of Superfluid ^{3}He

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Using fourth-sound techniques, we have measured the depression of the superfluid fraction of ^{3}He confined in small pores of packed powder over wide temperature $(0.2 < T/T_c < 1)$ and pressure $(0.5 < p < 20$ bars) ranges. From the analysis of the data we determine the magnitude and the pressure dependence of the healing length. The healing length decreases from 520 Å at 0.5 bar to 200 Å at 20 bars. The values are in fair agreement with those expected from the BCS expression.

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When a superfluid is confined in a sufficiently restricted geometry, the macroscopic order parameter describing the superfluid state becomes distorted by the presence of the boundary wall.[1-3] The nature of distortion will be determined by the particular boundary conditions that the wall imposes on the order-parameter function. Theoretical and experimental studies aimed at probing the boundary conditions for superfluid ^{3}He are currently under active research.[4] Understanding that the boundary conditions will be important in the design of an appropriate weak link for observing the long-sought analog of the Josephson effect in superfluid ^{3}He.[5,6] Interpretation of experiments on superfluid-^{3}He films will be related to the boundary conditions. An important parameter in these phenomena is the healing length which determines the characteristic distance over which the order parameter decreases from its bulk value to zero near a wall. In this paper we describe a systematic determination of the healing length over a wide pressure range from the analysis of the measured superfluid fraction of ^{3}He confined in the pores of packed powder.

Ambegaokar, de Gennes, and Rainer[1] showed that the nature of scattering of quasiparticles at a solid wall plays a crucial role in the determination of the boundary conditions. If the scattering is completely diffusive, they showed that the perpendicular (to the wall surface) component of the order parameter vanishes at the wall and that the parallel component is depressed from the bulk value by a factor of $\sim\xi_0/\xi(T)$. Here ξ_0 is the zero-temperature coherence length and $\xi(T)$ is the temperature-dependent healing length. Recently, Buchholtz[7] showed that if the surface is bumpy at random on the scale of inverse Fermi wave number (~ 5 Å) the parallel component also vanishes at the surface. It seems reasonable to assume that the powder used in our experiment presents diffuse scattering surfaces and that all components of the order parameter vanish at the surface. The Ginsburg-Landau (GL) equations, including the gradient terms for superfluid ^{3}He, have been derived by Ambegaokar, de Gennes, and Rainer.[1] The GL equations define a natural length scale given by the healing length for the spatial variation of the order parameter. Minimizing the GL free-energy functional by variational calculation, Ebisawa and Arai[8] have shown that if a cylindrical pore of radius R is filled with superfluid ^{3}He-B whose order parameter obeys the above boundary conditions, the average superfluid density in the pore, ρ_{sp}, can be written as

$$\rho_{sp}/\rho_{sb} = 1 - k\xi(T)R, \tag{1}$$

where ρ_{sb} is the bulk superfluid density. The numerical coefficient k is found to depend slightly on the ratio $\xi(T)/R$ and to vary between 2.2 and 2.7. The same problem has been treated for superfluid ^{4}He confined in cylindrical tube by Bot, Schubert, and Zimmermann,[9] who showed that the same depression of superfluid density is expected as given by Eq. (1) with $k=2.4$. If the confining geometry is in the shape of rectangular slabs, the depression of superfluid density is still given by Eq. (1) to a good approximation.[8] In our analysis we took k as a constant equal to 2.4.

If the quasiparticle scattering at the wall is specular reflection, then only the perpendicular (but not the parallel) component of the order parameter is expected to vanish and the transition temperature would be same as that for bulk.[1] It would appear difficult to prepare a surface which is flat to the scale of a few angstroms.

The technique of fourth sound was employed to measure the depression of superfluid density in the pores made by the interstices of packed powder. The fourth-sound resonance techniques used to measure the superfluid density have been described in detail elsewhere.[10] Three fourth-sound cylindrical resonator cavities (length = 14 mm and diameter = 8 mm), A, B, and

C, packed with alumina powders[11] of nominal grain sizes, 3, 1, and 0.3 μm, respectively, were all immersed in a chamber filled with liquid ^{3}He which was cooled by a cooper nuclear demagnetization apparatus. The temperature was measured from the magnetic susceptibility of a Lanthanum-doped cerium magnesium nitrate pill immersed also in the liquid with good thermal contact to the resonators. The thermometer was calibrated against the Helsinki phase diagram of superfluid ^{3}He.[12] The relevant parameters of the resonators are given in Table I. The index of refraction n of the packed powder was measured in a separate experiment with superfluid ^{3}He as the filling fluid. There was no depression of superfluid density of ^{4}He in these relatively large pores (the healing length in ^{4}He is ~ 1 Å). The speed of fourth sound, c_4^*, was measured by use of the resonant frequency of plane-wave resonances and was then converted to the average superfluid density by[10]

$$\langle \rho_s/\rho \rangle = (c_4^* n/c_1)^2, \tag{2}$$

where c_1 is the speed of first sound.[13] A quality factor in excess of 300 was observed at our lowest temperature and pressure. The relatively high Q at low temperature was unexpected from the rather small Q observed near the transition temperature by Chainer, Morii, and Kojima.[10] The Q measurements are relevant to the hydrodynamic boundary conditions on the normal fluid flow in restricted geometries, which is of current interest in its own right.[14] This will be discussed in a separate report.

TABLE I. Some relevant parameters of the fourth-sound resonators.

Resonator	Nominal grain size (μm)	Porosity (%)	n	R_{av} (μm)
A	3.0	68.7	1.17	0.42
B	1.0	75.1	1.09	0.30
C	0.3	79.8	1.11	0.11

The measurements were carried out at pressures of 0.5, 2.0, 5.0, 10.0, 15.0, and 20.0 bars and temperatures down to a reduced temperature, $t = 1 - T/T_c$, of 0.8. Here T_c is the bulk superfluid-^{3}He transition temperature. All regions covered in the experiment correspond to the B-phase liquid in the bulk-phase diagram.[13] The measured superfluid fraction is shown as a function of reduced temperature by open symbols in Fig. 1. In converting the fourth-sound velocity to superfluid density, the index of refraction was kept constant for a given resonator for all temperatures and pressures.[16] The dashed lines show the bulk superfluid density measured by Parpia *et al.*[15] All resonators show substantial depression in superfluid density from the bulk value down to the lowest temperatures reached. As expected, a smaller nominal powder size produced a larger depression of superfluid density. The magnitude of depression is greater at lower pressure for the given value of reduced temperature, showing that the healing length is greater at lower pressures. Long tails near the transition temperature are due to the larger pores present in the

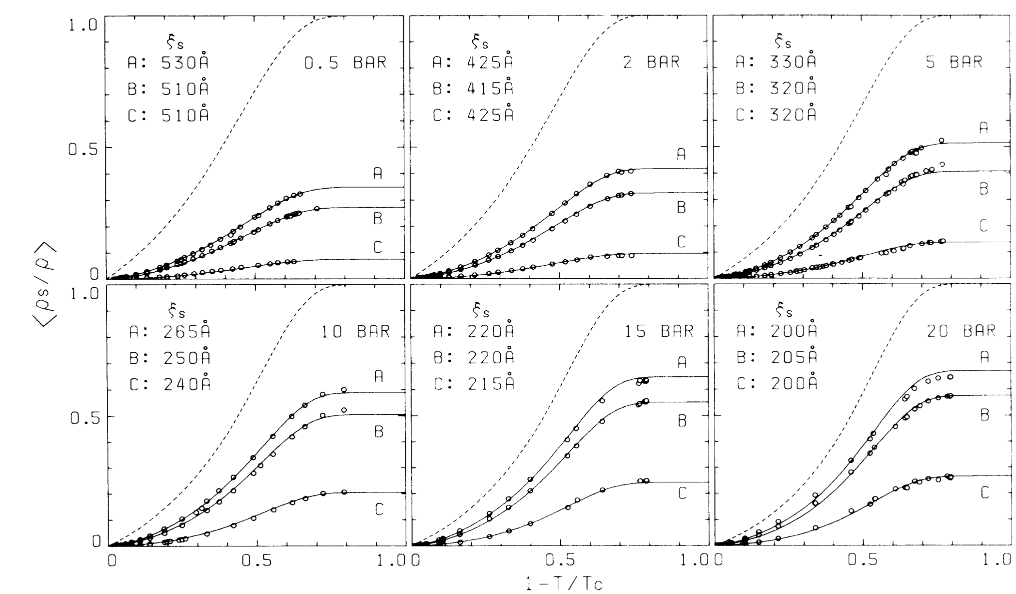

FIG. 1. The superfluid fraction as a function of pressure and reduced temperature. The open circles are the superfluid fractions determined from the measured speed of fourth sound in three resonators. The dashed lines are the bulk superfluid fractions obtained by Parpia *et al.* (Ref. 15). The solid lines are fits to the data made by a fitting of the healing length to the values indicated for each resonator.

packed powder. The presence of the tails makes it difficult to determine the depression in transition temperatures.

Scanning-electron-miscroscopy pictures of the alumina powders used to make the small pores indicate a complicated surface structure, as might be expected. In order to obtain a measure of distribution of pore sizes present in the packed powder, pore-size analyses were carried out for each of the packings by mercury intrusion. The results show a rather wide distribution of pores, probably caused by the relatively large pores created by agglomeration of smaller powder grains. The volume-averaged pore radius R_{av} is given in Table I. For the purpose of analysis we assume that the pores of packed powder present a homogeneous collection of interconnected, but more or less independent, cylinders with radii determined by the pore-size analysis. Each cylindrical pore contributes an amount given by Eq. (1) to the total superfluid density. The effective superfluid density is the sum of contributions from all pores. We rewrite the superfluid density in the cylinder of radius R as $\rho_s(R)/\rho=[\rho_s(R)/\rho_{sb}](\rho_{sb}/\rho)$. Summing over the pore-size distribution, the effective superfluid density is given by

$$\langle\rho_s/\rho\rangle=\sum[1-k\xi(T)/R](\rho_{sb}/\rho)[\Delta P(R)/\Delta R]\Delta R, \tag{3}$$

where $P(R)$ is the cumulative fraction of the pores having radius less than R.[17] In order to fit over a wide temperature range, we assumed for simplicity that the temperature dependence of $\xi(T)$ is inversely proportional to the BCS gap parameter, $\Delta(T)$. We follow Einzel[18] in interpolating the temperature dependence of the energy gap and write

$$\xi(T)=(\tfrac{3}{5})^{1/2}\frac{\alpha\xi_s}{\tanh(\gamma\{[8/7\zeta(3)](T_c-T)/T\}^{1/2})}, \tag{4}$$

where $\gamma=(\pi k_B T_c)/\Delta(0)$ was taken as the BCS weak-coupling value equal to 1.78. The constant $\alpha=\gamma[8/7\xi(3)]^{1/2}$ is taken such that the $\xi(T)$ coincides near T_c with the GL expression which is given by $\xi(T)=\xi_s/[\frac{5}{3}(T_c-T)/T_c]^{1/2}$. The BCS expression for ξ_s is given by

$$\xi_s=[7\xi(3)/48]^{1/2}\hbar v_F/\pi k_B T_c, \tag{5}$$

where $\xi(3)=1.202$, $\hbar$ is Planck's constant divided by 2π, v_F is Fermi velocity,[13] and k_B is Boltzmann constant. Note that $\xi(0)$ approaches the usual expression, $\hbar v_F/\pi\Delta(0)$. The magnitude of ξ_s is taken as an adjustable parameter in the fitting procedure.

If the raw mercury-intrusion pore-size distribution is used, qualitative features of the data can be reproduced by adjusting the value of ξ_s. In order to improve the fit the pore-size distribution was modified. The modification of the pore distribution was done at the highest pressure (i.e., 20 bars) data in which the depression of superfluid density is smallest and therefore the fourth sound probes the widest range of pores possible. Once determined, the pore distribution was kept fixed for the fitting analysis of all the other pressure data. The solid lines show the results of fitting with the best values of ξ_s, as indicated in the figure, for each resonator. The fit is excellent at all pressures and temperatures.

The values of ξ_s determined in each resonator by the above fitting procedure are shown as a function of pressure in Fig. 2. The uncertainties in the best-fit values of ξ_s are estimated to be ± 20 Å. Within the uncertainty the values of ξ_s from each resonator agree with each other. The solid line in Fig. 2 is the BCS expression for ξ_s and is in reasonable agreement with the experiment. If the recent measurement of T_c by Greywall[19] is used, the BCS value in Fig. 2 increases by 11% at all pressures. The present results are consistent with the healing length calculated from the measured depression of transition temperature from the bulk value in an array of small pores in a Nuclepore filter,[20] packed powders,[10] and a single submicron channel.[21]

Although Eq. (1) was derived based on the GL equations which are valid only near T_c, the fit obtained is good at temperatures much lower than T_c. There has been no calculation of the depression of superfluid density in small pores by solving the full nonlocal gap equation. However, Kjaldman, Kurkijarvi, and Rainer[3] calculated the depression of transition temperature by solving both GL equations and the full nonlocal gap equations as a function of the pore radius. They found that the two methods give the depression of transition temperatures within about 5% of each other for the values of pore radius to healing-length ratio greater than 2. Thus it is plausible that the GL equations give adequate description of the average superfluid density in the pores of the resonators in which pore radius is greater than about 1000 Å.

Recently Johnson, Koplik, and Schwartz[22] introduced a new length parameter Λ which alone can characterize

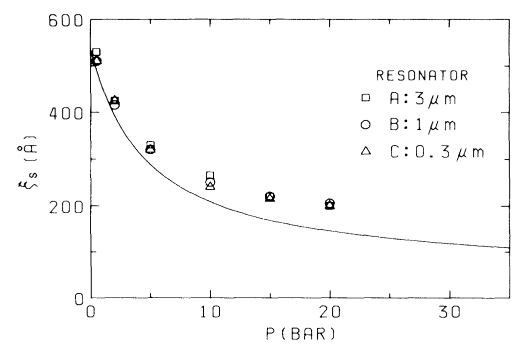

FIG. 2. The pressure dependence of the healing length determined by fitting the measured superfluid fraction (see text). The solid line is the healing length given by Eq. (5).

a variety of transport phenomena in porous media. They showed that the ratio of the average superfluid density in a given porous medius to that in the bulk is given by a formula very similar to Eq. (1), with the radius R replaced by the Λ parameter [see Eq. (6) of Ref. 22]. Since the healing length of ^{3}He turns out to be very large, especially at low pressures, the present type of experiment should provide a rather sensitive tool for measurement of the Λ parameter of porous media.

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[16]Since the healing length is much larger in ^{3}He, it may be argued that the fluid in pores smaller than a certain size which depends on the magnitude of ξ_s remains normal down to $T=0$ and that those pores do not participate in fourth-sound propagation. In this case, the index of refraction may be different in ^{3}He from that in ^{4}He. We take the point of view that the pore distribution is sufficiently homogeneous throughout the resonator and the "plugging" of some pores does not change n significantly. This view appears to be justified by the excellent fit that can be obtained for fixed n as well as the pore-size distribution over the range of pressure for which the healing length changes by a factor of 2.5.

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