Bounds on the Electron Electric Dipole Moment in a Wide Class of Models

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It is shown that is a wide class of models that give an interestingly large electric dipole moment for the electron, d_e can be bounded from d_n or $B(\mu \to e\gamma)$, provided that certain assumptions are made about the pattern of flavor mixing. Types of models not satisfying these assumptions are also discussed.

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In spite of the vast literature on tests and models of *CP* nonconservation comparatively little theoretical attention has been given to the electric dipole moment (EDM) of the electron. In part this is because it is so small in the standard model, where the weak contributions arise only at three loops[1] and are proportional to neutrino masses. However, in recent years several authors[2–4] have pointed out that a substantial EDM for the electron (henceforth denoted d_e) is possible in models with "new physics." Moreover, on the experimental front, efforts are underway by Fortson[5] and his collaborators to improve the current bound[6] (10^{-24} $e\cdot$cm) by up to four orders of magnitude. These considerations have motivated the present work.

Our main result is that, while it is possible in a wide variety of models to obtain a measurable d_e, one can place certain fairly reliable upper bounds on it that are quite model independent. (As will be seen, however, these bounds depend for their validity on certain assumptions about the pattern of flavor mixing. They should be regarded, therefore, as expectations rather than rigorous bounds.) The essential point is that the EDM of any fermion not only violates *CP* invariance but, involving as well a chirality flip, is proportional to some fermion mass. Some contributions are proportional to the mass of the electron itself, but these usually are too small to be interesting experimentally. What is generally required for d_e to be substantial is a virtual fermion of large mass. Surveying the kinds of models that have occupied the attention of theorists in recent years, there are, in the main, three ways this can arise: flavor mixing, left-right models, and supersymmetry.

In flavor-mixing models the electron can be converted into a τ, b, t, or other heavy (possibly exotic) fermion by the emission of a scalar or vector boson. In left-right models the e_L can convert into a ν_L by emission of a W_L which turns into a W_R and is reabsorbed by the ν_R to give e_R. It is thus the Dirac mass of the neutrino that enters. (We could include left-right models under the heading of flavor mixing. We single them out since they contain naturally a mechanism, not involving mixing with heavy generations or exotic particles, that can give a large contribution to d_e. Of course, left-right models can also involve flavor changing in the above sense.) In supersymmetric models the electron can become a scalar electron by the emission of a virtual photino, which is probably heavy.

In the case of flavor mixing, usually whatever new physics gives the dominant contribution to d_e also leads in a similar way to $\mu \to e\gamma$, thus allowing us to bound d_e if we make reasonable assumptions about the flavor-mixing pattern. We find that

$$d_e/e \cong (4\times 10^{-21}\ \text{cm})[B(\mu \to e\gamma)]^{1/2}\delta, \tag{1}$$

where δ is a ratio of unknown couplings that one can plausibly argue is likely to be small compared with unity. In that case, given the present bound,[7] $B(\mu \to e\gamma) \lesssim 4.9\times 10^{-11}$,

$$d_e/e \lesssim 2.8\times 10^{-26}\ \text{cm}. \tag{2}$$

A discovery of $d_e \neq 0$ should strongly motivate a redoubled effort in the search for $\mu \to e\gamma$.

In left-right models[4] an interesting d_e can arise through ν_e, which does not involve generation changing, and is thus unrelated to $\mu \to e\gamma$. In Ref. 4 it is argued that d_e/e can be as large as the experimental bound if the right-handed neutrino mass is sufficiently large. We have nothing to add here to the analysis of those authors. We remark, however, that left-right models constitute the major exception to the bounds derived in this paper and given in Eq. (2) and Eq. (3). The reasons such a large d_e/e is possible in left-right models are, first, that a one-loop gauge contribution exists since the W_R couples to e_R, and, second, that no flavor changing is required, thus escaping the constraints from $\mu \to e\gamma$.

In the case where a substantial d_e arises through a large (and complex) photino mass the story is quite a bit different. Here no relation to $\mu \to e\gamma$ exists but the limit[8] on the neutron EDM proves to be a strong constraint. We find that at best [in the very artifical case that $\arg(Am_{\tilde{\gamma}}) \gg \arg(Am_{\tilde{g}})$] $d_e/e \lesssim 6\times10^{-25}$ cm; but much more probably [with $\arg(Am_{\tilde{\gamma}}) \sim \arg(Am_{\tilde{g}})$]

$$d_e/e \lesssim 4\times10^{-27}\ \text{cm}. \tag{3}$$

We first discuss flavor-changing models. This includes models of many types: models with leptoquarks,[9] dileptons,[3] horizontal interactions,[2] mirror fermions,[10] and nonminimal Higgs bosons, all of which can give a large d_e through diagrams like Figs. 1(a) and 1(b). We will concentrate in detail on a particular leptoquark model for illustration, then generalize the discussion. The model we disquss has a scalar leptoquark (denoted ϕ) which is a $(3,2,\frac{7}{6})$ under $SU(3)_c\otimes SU(2)_L\otimes U(1)_Y$. The upper weak-isospin component has electric charge $+\frac{5}{3}$ and couples the charge -1 leptons to the charge $+\frac{2}{3}$ quarks. Let us denote its Yukawa coupling by $\phi\{\lambda_{32}(\bar{t}_R\mu_{\bar{L}})+\lambda'_{32}(\bar{t}_L\mu_{\bar{R}})+\lambda_{31}(\bar{t}_Re_{\bar{L}})+\lambda'_{31}(\bar{t}_Le_{\bar{R}})+\cdots\}$. Note that this leptoquark couples to both chiralities of lepton. For this model the virtual boson and fermion in Fig. 1 represent the leptoquark and heavy charge $\frac{2}{3}$ quarks, respectively. Evaluating these diagrams gives

$$d_e/e \cong \frac{1}{16\pi^2}\sum_k \mathcal{I}(\lambda'^*_{k1}\lambda_{k1})\frac{m_k}{M^2}\left\{Q\ln\frac{m_k^2}{M^2}+2Q+\tfrac{1}{2}\right\}, \tag{4}$$

where $Q=\frac{2}{3}$; $k=u$, c, or t; and M is the leptoquark mass. If we take $\lambda_{ki}=\lambda'_{ki}=(m_km_i)^{1/2}/(300\ \text{GeV})$, $M=300$ GeV, and $m_t=40$ GeV then the top quark greatly dominates and $d_e/e\cong10^{-26}$ cm.

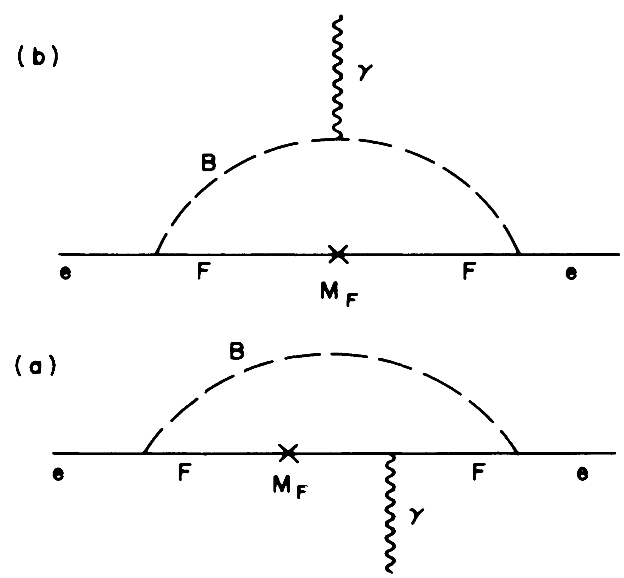

FIG. 1. Typical diagrams contribution to d_e. F is some heavy fermion, B a vector or scalar boson. $\{F;\ B\}$ could be $\{t;$ leptoquark$\}$; or $\{\tau;$ horizontal boson, or dilepton, or neutral nonminimal Higgs boson$\}$; or $\{\nu;\ W^{\pm}_{L,R}\}$; or $\{\tilde{\gamma};\ \tilde{e}\}$.

The critical observation is that similar diagrams lead to $\mu\to e\gamma$: One simply replaces one of the external electron lines in Fig. 1 by a muon line. The connection is even closer than suggested by the diagrams, since the decay $\mu\to e\gamma$ proceeds through the μ-e transition magnetic and electric dipole moments.[11] Defining the lepton, electromagnetic form factors in the conventional way (see Eq. 2.20 of Ref. 11 for instance) we have

$$|d_e/e| = (1/e)\,|(F_2^A)_{ee}/2m_e|, \tag{5}$$

and

$$B(\mu\to e\gamma) \cong \frac{24\pi^2}{G_F^2m_\mu^4}\left[\sum_{i=V,A}|(F_2^i)_{\mu e}/(m_\mu+m_e)|^2\right]. \tag{6}$$

[The factors $(m_\mu+m_e)^{-1}$ and $(2m_e)^{-1}$ are merely conventional in the definition of the form factors.] Then

$$|d/e| = [B(\mu\to e\gamma)]^{1/2}(G_Fm_\mu/2\sqrt{6}\pi e)\,\delta, \tag{7}$$

where e is the unit of charge and

$$\delta \equiv \frac{|(F_2^A)_{ee}/(2m_e)|}{[\sum_{i=V,A}|(F_2^i)_{\mu e}/(m_\mu+m_e)|^2]^{1/2}}. \tag{8}$$

In our leptoquark model, with the t quark dominating,

$$\delta \cong \sqrt{2}\,\mathcal{I}(\lambda'^*_{31}\lambda_{31})/(|\lambda'^*_{31}\lambda_{32}|^2+|\lambda'^*_{32}\lambda_{31}|^2)^{1/2}.$$

There are two reasons why we might expect δ to be smaller than unity. First, the numerator involves a *CP*-nonconserving phase which could be small. Second, our understanding of the Kobayashi-Maskawa matrix and quark and lepton masses suggests that flavor changing between the third and second generations is probably larger than between the third and the first. If, indeed, we take $\delta<1$ we get the result stated in Eqs. (1) and (2).

Before generalizing this result let us dwell on some features of this model. First, μ-e conversion in nuclei can happen at tree level.[9,12] Nevertheless, one probably gets a stronger bound on leptonic flavor changing (and hence on d_e/e) from $B(\mu\to e\gamma)$, rather than from $R_{eN}\equiv\Gamma(\mu N\to eN)/\Gamma(\mu N\to\nu N)$. We find[13] that

$$R_{eN}\cong 3\,|(Z+A)/Z|^2(1+3g_A^2)^{-1}G_F^{-2}M^{-4} \times\{|\lambda_{12}\lambda'^*_{11}|^2+|\lambda'_{12}\lambda^*_{11}+\tilde{\lambda}_{12}\tilde{\lambda}^*_{11}|^2\}.$$

$\tilde{\lambda}_{ij}$ are the couplings of the charge $\frac{2}{3}$ partner of $Q^{+5/3}$. Taking from experiment[14] $R_{eN}<4.5\times10^{-12}$ we find $\{|\lambda_{12}\lambda'^*_{11}|^2+\lambda'_{12}\lambda^*_{11}+\tilde{\lambda}_{12}\tilde{\lambda}^*_{11}|^2/M^4<8.7\times10^{-23}\ \text{GeV}^{-4}$. In comparison, from $B(\mu\to e\gamma)<4.9\times10^{-11}$ we find [see Eqs. (7) and (5), (6), (9), and (10)] that

$$\{|\lambda'^*_{31}\lambda_{32}|^2+|\lambda'_{32}\lambda^*_{31}|^2\}/M^4\lesssim 1.2\times10^{-23}\ \text{GeV}^{-4} \times(40\ \text{GeV}/m_t)^2.$$

$B(\mu \to e\gamma)$ is more sensitive than R_{eN} even if all the λ_{ij} are of the same order. A second point of interest is that d_e which is proportional to m_t may be much larger than d_n^{weak} which is proportional to m_τ. Finally, the *CP*-nonconserving phase may appear in the leptoquark mass matrix rather than couplings.

There were two assumptions that went into Eqs. (1) and (2): that d_e arose principally from flavor mixing with a heavy generation and that generation mixes more with the second generation than with the first. These assumptions are likely to be true if the heavy fermion is the τ, t, or a fourth-generation quark or lepton. There are a number of key exceptions to Eqs. (1) and (2) which we emphasize here. First, there may be a discrete symmetry allowing, say, e-τ but forbidding e-μ transitions. Second, and more likely, intergeneration mixing among leptons may be suppressed and d_e may arise through a heavy fermion which belongs to the electron family. One example is the ν_e in left-right models already noted.[4] A second example is in E_6 models which can have heavy leptons, $E^\pm$, mixing with $e^\pm$ through off-diagonal couplings of the Z or Z'.[15] These contributions to d_e will not exist (at one loop) unless a scalar with the gauge quantum numbers of a ν_L acquires a vacuum expectation value, and even then will not be larger than the bound of Eq. (2).

We turn now to inherently supersymmetric contributions to d_e, which can be large. These are unrelated to $\mu \to e\gamma$, but we can get interesting bounds from a consideration of d_n. There can be one-loop contributions to d_e from Fig. 1(b) where B is a scalar electron ($\tilde{e}$), and the F is a neutralino, i.e., a photino ($\tilde{\gamma}$), Z gaugino ($\tilde{Z}$), or neutral Higgs fermion. Corresponding to these latter possibilities there are several diagrams, of which we focus on one, the $\tilde{\gamma}$ diagram. (The case where a $\tilde{\gamma}$-Higgs-fermion mixing occurs is considered by del Aguila *et al.*[16] They obtain a d_e of the same order as our result here.) In the photino diagram since there is a helicity flip (giving a factor $m_{\tilde{\gamma}}$) there must also be a mixing $m^2_{\tilde{e}_L\tilde{e}_R}$. Both $m_{\tilde{\gamma}}$ and $m^2_{\tilde{e}_L\tilde{e}_R}$ are, in general, complex parameters which appear in the effective soft-supersymmetry-breaking terms in the low-energy limit of $N=1$ supergravity models. The diagram in Fig. 1(b) yields

$$\frac{d_e}{e} = \frac{\alpha}{\pi} \frac{m_e \mathcal{J}(Am_{\tilde{\gamma}})}{m^3} f(x), \quad (9)$$

where m is the scale of low-energy supersymmetry breaking; $x \equiv m_{\tilde{\gamma}}/m$; A, which is complex and of order unity, enters the relation $m^2_{\tilde{e}_L\tilde{e}_R} = Am_e m$; and $f(x)$ can be gathered from Polchinski and Wise.[17] Taking $m_{\tilde{e}} \sim m_{\tilde{\gamma}} \sim m \sim 100$ GeV we find $d_e/e = (2\times 10^{-24}$ cm) times the *CP*-nonconserving phases which can be bounded from d_n. There are two cases. First, and most likely, $\arg(Am_{\tilde{\gamma}})$ is comparable to $\arg(Am_{\tilde{g}})$ where $\tilde{g}$ is the gluino. An analogous calculation of d_d from a gluino loop[18] yields $d_d/e = \frac{4}{9}(\alpha_s/\pi)[m_d \mathcal{J}(Am_{\tilde{g}})/m^3 f(x')$, where $x' \equiv m_{\tilde{g}}/m$. With $m_{\tilde{g}} \sim m$, and with use of $d_n \cong \frac{4}{3} d_d$ we get that $\arg(Am_{\tilde{\gamma}}) \sim \arg(Am_{\tilde{g}}) \cong (d_n/e)/(10^{-22}\text{ cm}) < 2\times 10^{-3}$. Thus $d_e/e \lesssim 4\times 10^{-27}$ cm, the result stated in Eq. (3), and perhaps reachable by projected experiments.[5] The bound may even be stronger since the $\tilde{\gamma}$ is probably lighter than the $\tilde{g}$. The second (and very artificial) case is that $\arg(Am_{\tilde{\gamma}}) \gg \arg(Am_{\tilde{g}})$ for some unknown reason. If that is so d_d/e like d_e/e may arise principally through the $\tilde{\gamma}$, and the upper bound on d_e/e is very much loosened to 6×10^{-25} cm.

If $Am_{\tilde{g}}$ is real then the one-loop supersymmetric contributions to d_e vanish, and in the supersymmetric extension of the standard model (SSM) (with R parity), d_e/e turns out to be terribly small[19] ($<10^{-32}$ cm). However, in SSM's with massive neutrinos d_e can be enhanced. In particular in such models with neutrino masses arising from the "see-saw mechanism" it has been shown that large lepton-number nonconservation is possible.[12] Then d_e/e can arise at two loops. (The analogous result was shown for the quark sector by Duncan.[20]) The resulting value of d_e depends on unknown parameters in the scalar-lepton mass matrix, but, at any rate, could conceivably be as high as 10^{-28} cm.

A final comment is in order on the possibilities for d_e/e in "superstring-inspired" low-energy models. We have already noted the possibility of effects from E_6 heavy leptons arising at one loop in a subset of such models. Another possibility is that the scalar partners of the new charge $-\frac{1}{3}$, isosinglet quarks that exist in E_6 ($\tilde{D},\tilde{D}^c$), if they have weak-scale mass, could give one-loop contributions. Too-rapid proton decay can be forbidden by a discrete symmetry[21] which has the effect of making the $\tilde{D},\tilde{D}^c$ couple as leptoquarks (but not as diquarks). The situation is similar to the leptoquark model discussed above, except that now there exists a supersymmetric partner of Fig. 1. The discussion leading to the results in Eqs. (1) and (2) applies here as well. With $m_{\tilde{D}} \sim 200$ GeV,[22] leptoquark couplings $\sim 10^{-3}$, the present experimental bound on $B(\mu \to e\gamma)$ saturated, and maximal *CP* nonconservation one could get a result as large as $d_e/e \sim 10^{-26}$ cm.

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