Exact Decimation-Type Functional Renormalization Group for Critical Wetting in 1+1 Dimensions

Lipowsky and Fisher[1] have recently studied the critical wetting problem using an approximate functional renormalization group (RG). They found that *both* the critical point and the unbound or delocalized phase are governed by nontrivial fixed points. This leads to a novel bifurcation structure at the upper critical dimension. Here I present an exact decimation-type[2] RG for the problem in one interface and 1+1=2 bulk dimensions, in which these two nontrivial fixed points can be found explicitly.

Consider the reduced interfacial Hamiltonian

$$\beta H=\sum_i[\tfrac{1}{2}(l_i-l_{i+1})^2+V(l_i,l_{i+1})], \tag{1}$$

where the $\{l_i\}$ are the transverse positions of the interface at equally spaced points $\{i\}$ and $V(l,l')$ is the interface potential. This Hamiltonian may be viewed as the "bare" Hamiltonian of a lattice problem or as an effective Hamiltonian obtained for a continuum system or a system with a much finer lattice by integration over all positions of the interface between the points $\{i\}$. The transfer operator for this system is simply

$$T(l,l')=\exp\{-[\tfrac{1}{2}(l-l')^2+V(l,l')]\}. \tag{2}$$

An exact decimation-type RG[2] with rescaling factor $b=2$ is simply written in terms of the square of the transfer operator as

$$T'(l,l'')=\int dl'\,T(\sqrt{2}l,l')T(l',\sqrt{2}l''), \tag{3}$$

where the rescaling factor of $b^{1/2}=\sqrt{2}$ is exact for wetting in 1+1 dimensions.[1] The fixed points satisfy $T'=cT$, where c is a constant.

For the critical wetting problem (strong fluctuation regime[1]) we consider potentials $V(l,l')$ that vanish sufficiently rapidly for $l,l'\to\infty$ and have a hard wall at $l=0$, so that $V(l,l')=\infty$ for $l<0$ or $l'<0$. The nontrivial fixed point for the unbound phase is (for $l>0$ and $l'>0$)

$$T(l,l')=e^{-(l-l')^2/2}-e^{-(l+l')^2/2}, \tag{4}$$

or

$$V(l,l')=-\ln[1-\exp(-ll')]. \tag{5}$$

This potential is, as expected,[1] purely repulsive, vanishing as $\exp(-l^2)$ for l large. For l or $l'\to 0$ it diverges as $-\log(ll')$; Lipowsky and Fisher also found a logarithmic divergence for $l\to 0$.[3]

The critical fixed point is then simply

$$T(l,l')=e^{-(l-l')^2/2}+e^{-(l+l')^2/2}, \tag{6}$$

or

$$V(l,l')=-\ln[1+\exp(-ll')], \tag{7}$$

which, perhaps surprisingly, is purely attractive with a hard wall at $ll'=0$. This differs from the critical fixed-point potential found by Lipowsky and Fisher,[1] which has a repulsive part that diverges smoothly for $l,l'\to 0$. This difference is not of concern, though, since the fixed-point Hamiltonian is not generally expected to be universal.

The flows of this exact RG are not very simply represented in terms of the transfer operator itself, but are quite simple in terms of its spectrum and eigenfunctions since the RG just squares the transfer operator and rescales space. Therefore the eigenvalues of the transfer operator are simply squared, while the eigenfunctions are rescaled by $\sqrt{2}$ under each iteration of the RG. However, if you must perform the RG this way then it is probably only of pedagogical rather than practical interest, because if you can diagonalize the transfer operator then you already know everything the RG can tell you. Hopefully, this type of RG can be made useful in a situation where the spectrum and eigenfunctions of the transfer matrix are not known.

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